
Supplementary information

An entangling gate for dual-rail erasure qubits

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Supplementary Information for An entangling gate for dual-rail erasure qubits

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(Dated: June 5, 2026)

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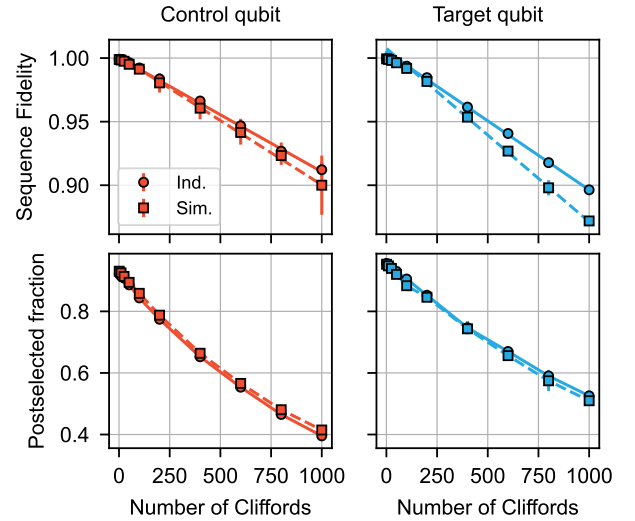
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I. SINGLE-QUBIT CHARACTERIZATION

A. 1Q gate characterization

Single-qubit gates are realized by four wave mixing parametric beamsplitter-type interactions implemented by a SQUID-transmon coupler [1]. We apply a flux drive to the SQUID coupler at half the frequency detuning between the two cavities (a and b) of a dual-rail

qubit to enable a parametric interaction via the Hamiltonian $\mathcal{H}/\hbar = \frac{g_{bs}}{2} (\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger)$. To characterize single-qubit gates, we perform both individual and simultaneous RB [2]. For these measurements we use a gateset consisting of $\{X_{\pm\frac{\pi}{2}}, Z, Z_{\pm\frac{\pi}{2}}\}$ from which we generate the 24 single-qubit Cliffords. For each Clifford sequence length we average over 20 different randomized sequences. Performing RB individually on control and target qubit, we extract a gate fidelity in excess of $> 99.98\%$ with a small decrease in postselected fraction of $< 0.1\%$ per Clifford detected by our end-of-the-line measurement (see Fig. S1). When operated simultaneously, the control and target qubit fidelities are observed to modestly decrease by 0.001% and 0.002% , respectively for the control and target qubits. We attribute this effect to a small measurable cross-Kerr between two dual-rail qubits (a_2 and b_1 cavities) of 6.64 kHz. Compared to our single-qubit beamsplitter rates of 1.78 MHz (2.55 MHz) for the control (target) qubit we estimate simultaneous operation infidelities to be limited by $(\chi_{ab}/g_{aa(bb)})^2 \sim 1.3 \times 10^{-5}$ per gate.



Supplementary Figure S1. **Single-qubit gate benchmarking** Individual (circles) and simultaneous (square) RB on the control qubit (left) and target qubit (right). Control qubit individual and simultaneous RB are measured to be $9.0(2) \times 10^{-5}$ and $1.01(1) \times 10^{-4}$, respectively. Target qubit individual and simultaneous RB are measured to be $1.08(1) \times 10^{-4}$ and $1.36(1) \times 10^{-4}$. From postselected fraction we quantify erasure rates of $0.0856(5)\%$ and $0.0592(6)\%$ for control and target qubit, respectively. Results show minimal change in gate fidelity when operating the two qubit simultaneously and that there are low cross-talk errors in this system.

	Description	Parameter	Units	Control qubit	Target qubit
System	Cavity b_1 -Coupler c cross-Kerr	$\chi_{bc}/2\pi$	MHz	-1.51	
	Cavity a_2 -Coupler c cross-Kerr	$\chi_{ac}/2\pi$	MHz	-1.26	
	Cavity a_2 -Cavity b_1 cross-Kerr	$\chi_{ab}/2\pi$	kHz	-6.64	
	Cavity a_2 -Coupler c photon exchange rate	$g_{ac}/2\pi$	MHz	4.23	
Cavity	Relaxation time	$T_1^{a(b)}$	μs	231, 411	652, 342
	Decoherence time, Ramsey	$T_{2R}^{a(b)}$	μs	433, 638	1001, 454
	Resonance frequency	$\omega_{a(b)}/2\pi$	GHz	7.03, 6.75	6.56, 7.21
Entangling coupler	Relaxation time	T_1^c	μs	70	
	Dephasing time, Ramsey	T_ϕ^c	μs	327	
	Dephasing time, Echo	T_ϕ^c	μs	1001	

TABLE I. Measured Hamiltonian and coherence properties.

Property	Parameter	Control qubit	Target qubit
SPAM [1 round]	Average misassignment	2×10^{-4}	2.3×10^{-4}
	Average leakage detection error	3.46×10^{-3}	3.90×10^{-3}
	Average erasure assignment	6.79×10^{-2}	6.43×10^{-2}
SPAM [2 rounds]	Average misassignment	7×10^{-6}	1.2×10^{-5}
	Average leakage detection error	1.5×10^{-4}	1.5×10^{-4}
	Average erasure assignment	1.81×10^{-1}	1.41×10^{-1}
Coherences	Ramsey time	3.1 ms	1.5 ms
	Echo time	4.0 ms	4.8 ms
	Bit-flip	520 ms	1100 ms
Single-qubit gates	Individual gate error [iRB]	9.0×10^{-5}	1.08×10^{-4}
	Simultaneous gate error [iRB]	1.01×10^{-4}	1.36×10^{-4}
	Erasure per $\sqrt{X}$ gate	8.56×10^{-4}	5.92×10^{-4}
	Total gate duration	208 ns	136 ns
	Beamsplitter rate	1.78 MHz	2.55 MHz

TABLE II. Measured dual-rail qubit properties.

B. State Preparation and Measurement (SPAM)

Dual-rail measurement are implemented by performing simultaneous single-shot cavity measurements that determine whether a photon is present in each cavity individually, extracting whether the system is in $|01\rangle$ or $|10\rangle$. This measurement is designed to also have an important property: it can detect for both cavity photon loss and also can flag additional hardware errors during the measurement protocol, such as decoherence or transmon readout misassignments, during the measurement itself. Detailed characterization of dual-rail SPAM are shown in Table II. In addition, this measurement can be repeated for multiple rounds and correlating the results can further suppress misassignment errors. This postselection capability comes at the cost of a first-order decrease in postselected fraction, however, we benefit from an exponential decrease in misassignments, which is highly beneficial for characterization of our CZ gate.

We benchmark performance for state preparation and measurement (SPAM) and extract logical misassignment errors of 0.023% and 0.020% for the target and control dual-rail qubit respectively. The overall SPAM erasure

errors, defined to be assigned erasure outcomes when attempting to prepare in either $|01\rangle$ or $|10\rangle$, are measured to be 6.79% and 6.43% for the target and control qubit. Finally we benchmark the leakage detection properties by initializing the dual-rail qubit in the $|00\rangle$ and we observe erroneous codespace assignments to be 0.39% and 0.34% for the target and control qubit. The system parameters used in this work are similar to what was used in [3], wherein a detailed error budget of the erasure, misassignment, and leakage detection errors can be found. The large erasure assignment rates are dominated by cavity photon loss and transmon decoherence during the long $4\mu s$ selective pulse used to map the cavity photon number to the measurement transmon. With relatively simple design modifications and using a square selective instead of a gaussian pulse, it is possible to shorten the photon number mapping pulse from $4\mu s$ to $1\mu s$. This, combined with reducing transmon readout times from $3\mu s$ to less than $1\mu s$, it is straight forward to achieve erasure assignment rates of $\lesssim 1\%$ in our system.

II. TWO-QUBIT GATE CALIBRATION

In this section, we discuss the calibration of the CZ gate in detail. The SWS gate can be calibrated via a straightforward procedure. In essence, it can be broken down into three pieces; i) swapping the photon from the control qubit cavity a_2 into the coupler c , ii) allowing the entangling phase from the dispersive interaction to accumulate over a specified delay time, and iii) swapping the photon back from the coupler into the control qubit cavity. In order to implement the CZ gate consisting of these pieces, we need to calibrate six parameters; the resonance frequency and duration of the parametric pump drive for cavity-coupler swap operation, the duration of the delay time needed to accumulate a π entangling phase, the rotation angle for the second cavity-coupler swap to restore the photon population into the two-qubit manifolds, and the deterministic Z -phase of each single qubit gained during the gate. It is important to note that all these parameters can be independently calibrated in a sequential procedure; the one exception is the delay time, which requires prior adjustment of the rotation angle for the second cavity-coupler swap.

The calibration procedure is illustrated in the flowchart in Fig. S2a, which starts with calibrating the photon swap operation between the control qubit cavity a_2 and the coupler c , referred to as the cavity-coupler swap. The underlying interaction for this cavity-coupler swap is identical to that used for the single-qubit gate. However, in this case, the parametric flux pump is applied at half the frequency difference of the Stark shifted coupler c and cavity a_2 . A coherent transfer of a single photon between the cavity and the coupler is shown in Fig. S2b where we also present the stroboscopic time evolution of the photon population in the cavity P_{cav} at the resonant pump frequency, which follows

$$P_{\text{cav}}(t) = \frac{1}{2}e^{-\kappa t} (1 + e^{-\kappa_\phi t} \cos(g_{ac}t)). \quad (1)$$

Here, g_{ac} represents the rate of the cavity-coupler swap, while κ and κ_ϕ denote the photon decay rate and the dephasing rate when the pump is turned on. As the first step in calibrating the cavity coupler swap operation, we find the amplitude A_p that maximizes the metric g_{ac}/κ_ϕ , which leads to the highest fidelity of the cavity-coupler swap operation when you postselect for photon loss. Following this, we calibrate the duration of the swap pulse t_{SWAP} by repeatedly applying the pulse after starting with a photon in cavity a_2 and minimizing the population in the cavity a_2 at the end of odd number of swap operations (see Fig. S2c). In this analysis, it is also helpful to introduce the notation $|a_1, a_2, c, b_1, b_2\rangle$, which describes the full state of the system in the number basis and includes both control and target dual-rail qubits and the coupler mode.

When the swap operation is used in the SWS gate sequence, an additional parameter needs to be specified which is the relative phase difference between the first

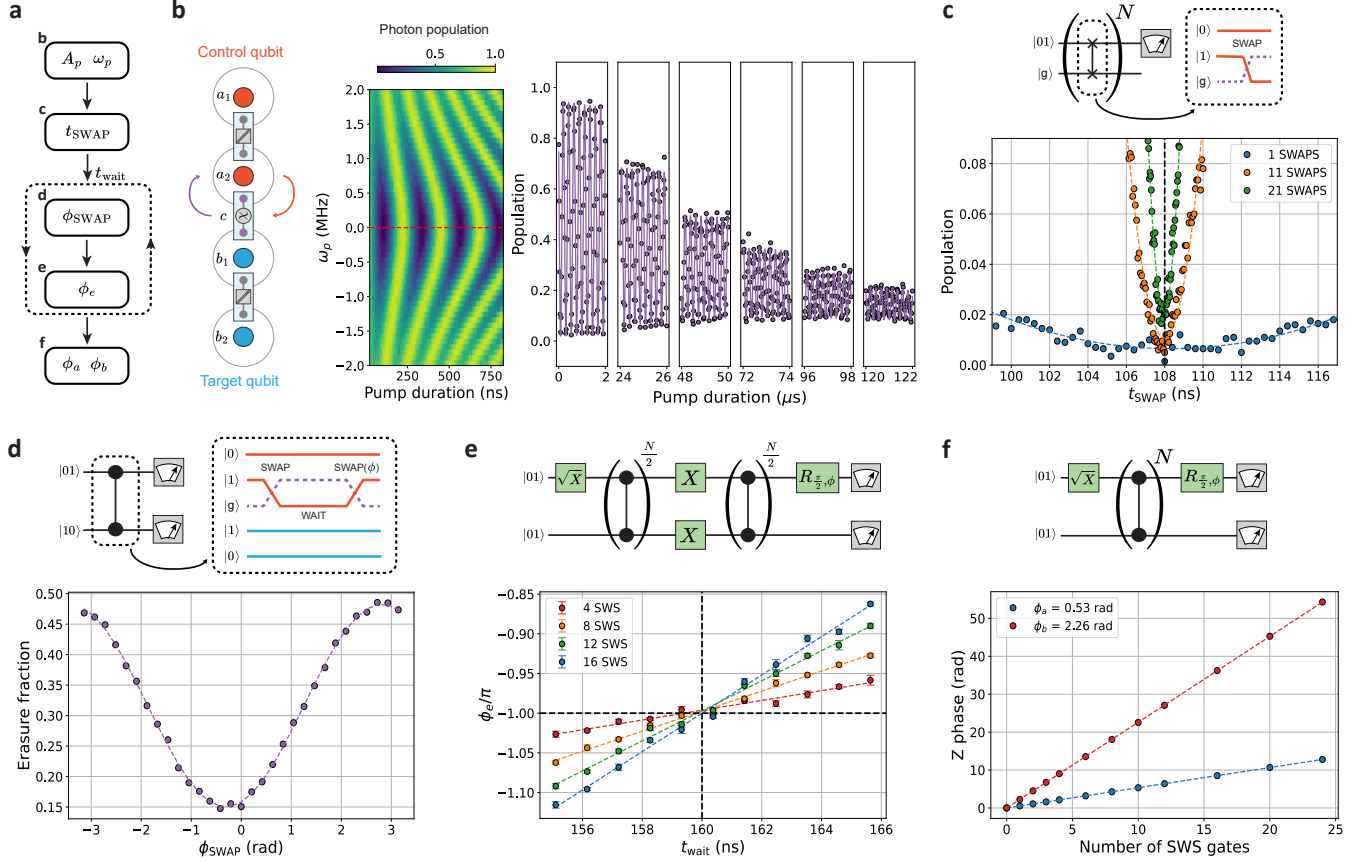
and the second swap operations. To understand why this is important, we first note that at the end of the CZ gate we want all the population in cavity a_2 to return to its original value irrespective of which two qubit state we start in. This condition is trivially satisfied for starting states $|10g01\rangle$, $|10g10\rangle$ and $|01g01\rangle$ since in these cases either no transfer of population takes place between cavity a_2 and coupler c or an on-resonant swap takes place, either way the phase of the second swap operation is irrelevant. The only non-trivial case occurs when we start in the state $|01g10\rangle$, in which case we perform a swap operation detuned by χ_{bc} (see Fig. S10). In this case we can close the Bloch sphere trajectory by choosing the correct phase of the second swap operation in the gate. To calibrate this parameter we initialize our system in the state $|01g10\rangle$ and apply the SWS gate sequence while sweeping the phase of the second swap pulse and measure the total erasure fraction of the circuit. We choose the swap back phase ϕ_{swap} that minimizes the erasure fraction (See Fig. S2d). Since our gate is excitation-preserving, every time we leave population in the coupler mode c we will find the control qubit in the vacuum state. Therefore, minimizing the erasure fraction is equivalent to maximizing the population in cavity a_2 at the end of the gate. It is important to note that as the delay time varies, the swap back phase should be calibrated and adjusted accordingly prior to measuring the corresponding entangling phase ϕ_e as illustrated in the flowchart, until the delay time is accurately calibrated. A detailed measurement of CZ gate induced coupler heating can be found in Appendix VIII.

With the cavity-coupler swap time and the swap back phase for a given delay time t_{wait} calibrated, the unitary of the SWS gate can be written as the following,

$$U_{\text{SWS}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_a} & 0 & 0 \\ 0 & 0 & e^{i\phi_b} & 0 \\ 0 & 0 & 0 & e^{i(\phi_a+\phi_b+\phi_e)} \end{pmatrix} \quad (2)$$

where ϕ_e is the entangling phase, ϕ_a and ϕ_b are respective Z -phase on each qubit accumulated during the gate. Next, to realize CZ unitary, we calibrate the wait time t_{wait} such that $\phi_e = \pi$. For cavity-coupler swaps with instantaneous pulse rise times and zero cavity-cavity cross-Kerr χ_{ab} between the cavities a_2 and b_1 , the wait time can be analytically calculated (see Eq. 62 in Appendix XIII). However, in the case of non-zero pulse rise time and χ_{ab} , we need to experimentally determine the wait time. To measure ϕ_e for a given wait time, we perform a Ramsey-like sequence [4] on either the control or the target qubit.

In this protocol, we start with both qubits in the state $|0_L\rangle = |01\rangle$ and apply a $\sqrt{X}$ gate to one of the qubits to prepare a superposition state $\frac{1}{\sqrt{2}}(|0_L0_L\rangle - i|1_L0_L\rangle)$, where $|1_L\rangle = |10\rangle$. Following that we repeatedly apply N CZ gates interrupted by X gates on both qubits in the middle. This protocol then creates the state $\frac{1}{\sqrt{2}}(e^{iN\phi_e/2}|1_L1_L\rangle - i|0_L1_L\rangle)$ where the phase $N\phi_e/2$ is



Supplementary Figure S2. **Calibration of CZ gate.** **a** Flowchart of the CZ gate calibration procedure. **b** Left: Schematic illustration of swap between the control qubit cavity a_2 (red) and coupler c (purple). Right: Photon population in the control qubit cavity as a function of parametric pump frequency ω_p and duration of the pump, and additionally the stroboscopic evolution at the resonant pump frequency marked by a red dashed line. **c-f**. Each Fig. shows the pulse sequence and the corresponding calibration result. **c**. Calibration of duration of cavity-coupler swap by measuring the population in the control qubit cavity after odd numbers of swap. **d**. Calibration of the phase of rotation for the second swap operation in the CZ gate by minimizing the erasure fraction. **e**. Calibration of the delay time of the CZ gate by measuring the entangling phase per CZ gate, which should corresponds to π for the calibrated delay time marked by dashed line. **f**. Calibration of single-qubit Z rotations per CZ gate. The Ramsey-like protocol should be performed on either the control or target qubit to measure Z rotation of each qubit.

accumulated as a relative phase shift between the states and the phases ϕ_a and ϕ_b are factored out as global phases. Note that we only accumulate entangling phase during one half of the sequence shown in Fig. S2e as the X gates in the middle turn off the entangling interaction. The entangling phase ϕ_e measured as a function of wait time t_{wait} and number of repeats is shown in Fig. S2e from which we find the optimal wait time that leads to $\phi_e = \pi$.

Once the delay time is precisely calibrated, the last two parameters to be calibrated are the Z-phase rotation for the control and target qubit ϕ_a and ϕ_b . These can also be measured via Ramsey-like protocol on the control and target qubit in similar manner as the entangling phase ϕ_e . The only difference is that we do not perform X gate in the middle of the sequence as the echo pulses will convert the phases ϕ_a and ϕ_b that we are interested in measuring

into global phases. Fig. S2f shows these local Z-phases as a function of number of repeated CZ gates for the wait time corresponding to entangling phase $\phi_e = \pi$ where the extracted slope of each trace corresponds to the phases ϕ_a and ϕ_b accumulated per gate.

III. CZ GATE ON-OFF RATIO

As described in the main text, the construction of our CZ gate is based on the dispersive interaction between the coupler and the target dual-rail, $\chi_{bc}\hat{b}_1^\dagger\hat{b}_1\hat{c}^\dagger\hat{c}$, where c and b_1 denote the coupler and target qubit cavity connected to the coupler, respectively. Therefore, once the interaction is activated, the entangling phase accumulates at a rate χ_{bc} , which can be readily measured by performing spectroscopy on the target qubit cavity b_1 .

Fig. S3a shows the results with the coupler prepared in either the ground or excited state with a choice of applying the cavity-coupler swap prior to spectroscopy. In the case where the coupler is in the excited state, the decay of the population in the coupler from the excited to the ground state leads to an additional dip at zero frequency detuning. By comparing the resonance frequency of the target qubit cavity b_1 depending on the state of the coupler, we can extract $\chi_{bc}/2\pi \sim 1.51$ MHz as marked in the figure. On the other hand, the dispersive interaction $\chi_{ab}\hat{a}_2^\dagger\hat{a}_2\hat{b}_1^\dagger\hat{b}_1$ between the control qubit cavity a_2 and target qubit cavity b_1 leads to *unwanted* entanglement between the two dual-rail qubits when they are idle. In order to accurately extract χ_{ab} , we apply the Ramsey sequence on the target qubit with a given delay time t_{wait} . We measure the probability of being in $|01\rangle$ after the sequence which oscillates as a function of the phase of the second $\pi/2$ pulse. Fig. S3b shows the associated phase shift in the oscillation for various delay times, with the control qubit cavity either empty or occupied with a photon during the Ramsey sequence. By taking the difference in the slopes of the linear fit to the respective results, we can extract $\chi_{ab}/2\pi \sim 6.64$ kHz, which leads to an on-off ratio of the CZ gate ~ 230 .

IV. INFERRING CZ GATE FIDELITIES AT SHORT GATE DEPTH

In this section we provide additional detail for how we estimate CZ gate fidelity in the short-depth limit.

Following the treatment in Ref. [5], consider the decay in RB survival probability as a function of the number of gates N , $F = Ap^N + B$, where p corresponds to the depolarizing parameter. We first substitute $p = 1 - x$, and expand to linear order in N

$$\begin{aligned} F &= A(1 - x)^N + B = A(1 - xN) + B \\ &= -Ax \cdot N + A + B \end{aligned}$$

We perform a linear fit to our data and extract a slope $m = -Ax$ and can see that the depolarizing parameter p can be computed as $p = 1 - |m|/A$. From this analysis, we see that the value of p depends on the amplitude and offset of the survival probability.

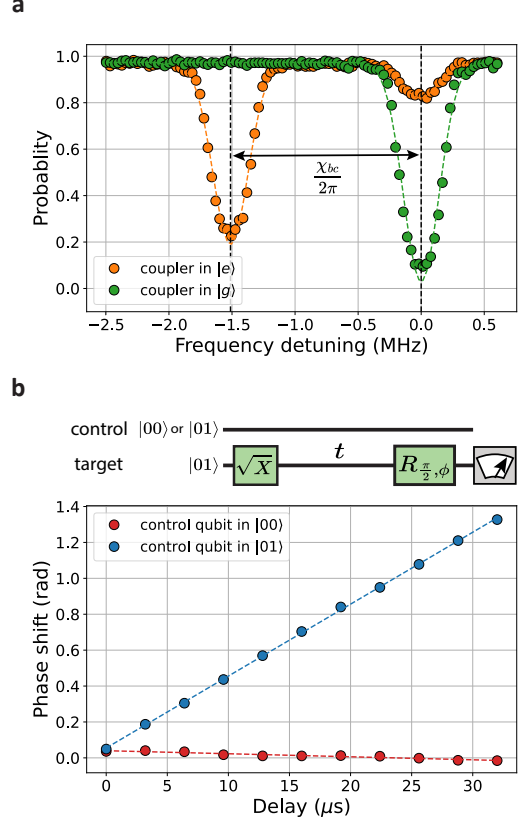
The average CZ gate error is computed from interleaved RB as:

$$r_{\text{CZ}} = \frac{(d-1)(1 - p_i/p_r)}{d} \quad (3)$$

with $d = 2^{N_q} = 4$, $p_{i,r}$ are the depolarizing parameter for interleaved and reference RB, respectively.

For two-qubit RB we expect $A = \frac{3}{4}$ and $B = \frac{1}{4}$, and thus

$$r_{\text{CZ}} = \frac{3}{4} \left(1 - \frac{1 - \frac{4}{3}|m_i|}{1 - \frac{4}{3}|m_r|} \right) = \frac{3}{4} \frac{|m_i| - |m_r|}{\frac{3}{4} - |m_r|}, \quad (4)$$



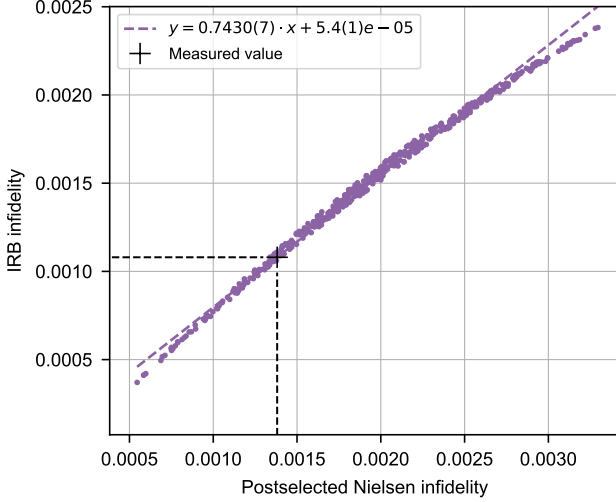
Supplementary Figure S3. **Measuring interaction strength for the CZ gate.** **a.** Spectroscopy on the target qubit cavity with the coupler either in the ground (green) or excited state (orange). **b.** Circuit diagram for the Ramsey sequence on the target dual-rail and the associated phase shift as a function of delay time with the control qubit cavity either empty (red) or occupied with a photon (blue).

where $m_{r,i}$ are the extracted slopes for the reference and interleaved RB data, respectively. In experiment, we extract $m_r = 2.79(1) \times 10^{-3}$ and $m_i = 3.86(4) \times 10^{-3}$, from which we extract the error per CZ gate of $r_{\text{CZ}} = 1.08(5) \times 10^{-3}$.

For one-qubit RB, $A = B = \frac{1}{2}$ we extract the average gate error as

$$r_{1q} = \frac{d-1}{d}(1-p) = \frac{1}{2} \frac{A}{|m|} = \frac{1}{4|m|} \quad (5)$$

We also consider extracting gate fidelity from decay of Bell state fidelity. For our CZ gate in the limit of large number of repeated gates N , the state fidelity will saturate to $B = 1/4$. Using $A = 3/4$, then we find the CZ gate fidelity $p = 1 - \frac{4|m|}{3}$.



Supplementary Figure S4. CZ gate infidelity inferred from simulated IRB experiments vs. the infidelity computed from Eq. 10. 500 random dephasing and leakage rates are chosen uniformly distributed with $p_Z^{\text{control}}, p_Z^{\text{target}} \in [1 \times 10^{-4}, 2 \times 10^{-3}]$, $p_{ZZ} \in [5 \times 10^{-5}, 2 \times 10^{-4}]$, and $p_{\text{leakage}}^{\text{control}}, p_{\text{leakage}}^{\text{target}} \in [1 \times 10^{-4}, 4 \times 10^{-3}]$. Each IRB fidelity simulation uses the exact same random gate sequence as the measurement in Fig. 2. The RB sequence fidelity is fit to the linearized exponential discussed in Appendix IV out to 50 Cliffords. The subtle curvature of the simulated points arises from the limited range of validity for the linearization: as error rates increase, second order terms become relevant. The dashed lines guide the eye to convert our measured CZ infidelity to the infidelity when error detection immediately follows the gate.

V. EVALUATING THE ACCURACY OF IRB WITH POSTSELECTION

Appendix IV utilizes IRB to estimate CZ gate fidelities at short depths with a linear approximation of the decay exponential. Here, we demonstrate that the experimental results are a reliable estimator for the true underlying gate errors. The addition of error detection to RB is novel and, to our knowledge, only utilized here and in neutral atom platforms [6]. These error detection measurements are high-fidelity [3], but leakage misassignment errors do occur at 10^{-4} rates. Additionally, the infidelity of our CZ gate exceeds those of single qubit rotations by more than one order of magnitude. This produces an asymmetry between some Cliffords and the interleaved CZ gate that can negatively impact the CZ fidelity estimate [7]. Between the addition of error detection, gate asymmetry, and limited circuit depth, it is valid to worry that IRB may not provide an accurate estimate of CZ gate fidelity. In this section, we define a parametrized channel for our CZ gate, add postselection to the standard definition of average gate fidelity [8], and simulate IRB under different CZ error rates to evaluate its estimates of the postselected gate fidelity.

First, we discuss a simplified leakage model for a DR qubit. For a single DR, we represent the logical and leakage states with qutrit states $|0\rangle = |0_L\rangle$, $|1\rangle = |1_L\rangle$, $|2\rangle = |*_L\rangle$. We also define unconditional reset of qubit x to the leakage state with the following channel:

$$\Lambda_{r,x}(\rho) = |2\rangle\langle 0|_x \rho |0\rangle\langle 2|_x + |2\rangle\langle 1|_x \rho |1\rangle\langle 2|_x + |2\rangle\langle 2|_x \rho |2\rangle\langle 2|_x \quad (6)$$

Thus, for each control or target qubit $x \in \{c, t\}$ in a two DR system, the leakage channel is:

$$\Lambda_{\text{leakage}}^x(\rho) = (1 - p_{\text{leakage}}^x)\rho + p_{\text{leakage}}^x \Lambda_{r,x}(\rho) \quad (7)$$

Detectable leakage errors form the dominant component of the CZ error channel, but residual dephasing arises from intrinsic sources and control qubit leakage during gates. To form the dephasing error channel, note that dephasing in the computational subspace $\{|0_L\rangle, |1_L\rangle\}$ is generated by the σ_z -like Gell-Mann matrix λ_3 [9] with non-zero diagonal $(1, -1, 0)$. The resulting dephasing Kraus operator on a single qutrit is $K_3 = \exp\{-i\pi\lambda_3/2\}$. This produces the two qutrit dephasing channel:

$$\begin{aligned} \Lambda_{\text{dephas.}}(\rho) = & (1 - p_Z^c - p_Z^t - p_{ZZ})\rho \\ & + p_Z^c K_3^c \rho (K_3^c)^\dagger + p_Z^t K_3^t \rho (K_3^t)^\dagger \\ & + p_{ZZ} (K_3^c \otimes K_3^t) \rho (K_3^c \otimes K_3^t)^\dagger. \end{aligned} \quad (8)$$

The Z subscripts serve as a reminder that dephasing occurs in the qubit subspace.

Finally, we can write a simple CZ channel $\mathcal{E}$ as the composition of leakage and dephasing channels

$$\mathcal{E}(\rho) = \Lambda_{\text{dephase}} \circ \Lambda_{\text{ideal}} \circ \Lambda_{\text{leakage}}^c \circ \Lambda_{\text{leakage}}^t(\rho) \quad (9)$$

where $\Lambda_{\text{ideal}}(\rho) = \text{CZ}\rho\text{CZ}^\dagger$. This channel captures target qubit dephasing resulting from control qubit leakage during the CZ gate, albeit more simply than the detailed picture presented in Appendix VI. The rates parametrizing each individual error channel are derived from the error-per-gate results in Fig. 3.

Our RB circuits only include leakage detection at the end of a circuit, unlike other approaches that interleave checks with batches of gates [6, 10]. Here we explain why this technique is valid for our system. DR qubit seepage via cavity photon heating occurs at a rate at least 1000 times slower than leakage [3], obviating the need for mid-circuit checks here or other more sophisticated techniques [11, 12]. This means that the fidelity of a standalone gate immediately followed by leakage detection should be nearly equivalent to the fidelity of a gate with delayed leakage detection. An RB experiment with postselection should then infer the fidelity of a postselected gate, up to misassignment errors in the leakage detection. This motivates a modification to the standard entanglement fidelity definition found in [8]:

$$F_e(\mathcal{E}, \text{CZ}) = \sum_{j,k} \frac{\alpha_{jk} \text{Tr}(\text{CZ} U_j \text{CZ} (M \mathcal{E}(\rho_k) M^\dagger))}{d^3 \text{Tr}(M \mathcal{E}(\rho_k) M^\dagger)}. \quad (10)$$

The $U_j = \sum_k \alpha_{jk} \rho_k$ form a unitary basis on the two qubit space [8, 13] with pure state representation ρ_k . Using a state basis here is essential for postselection after propagation through the CZ channel $\mathcal{E}$ as the U_j can be traceless. The postselection measurement operator M projects into the two-qubit codespace, with misassignments characterized in Table II that appropriately penalize the gate fidelity [3]. The denominator in Eq. 10 renormalizes the fidelity contribution of each ρ_k by its postselected fraction. This is similar to the approach taken in [14], where the entanglement fidelity is divided by the average postselected fraction with the intent to preserve channel linearity.

Now that we can compute the postselected fidelity of our CZ channel, we compare this to the fidelity inferred from IRB simulations across a range of CZ error rates. We follow a standard technique used in [5, 7, 8] using the superoperator formalism to propagate a density matrix through a random sequence of gates. There are two more gates required to generate the Clifford group: $X_{\frac{\pi}{2}}$ and virtual-Z. We compute the superoperator implementing both individual and simultaneous $X_{\frac{\pi}{2}}$ operations using the coherences and gate durations in Tables I and II. It is important to include the residual $a_1 - b_2$ cross-Kerr to capture unitary gate errors. Virtual-Z gates are perfect unitaries in the simulation.

Having described our model for all operations, we proceed to simulate IRB and extract two-qubit gate fidelities. All of our simulations implement the same random gate sequence used for the measurement in Fig. 2. Due to finite sampling of random sequences (20 at each depth), we expect noise in the inferred infidelity particular to each sequence. We fit the sequence fidelity over the first 50 Cliffords to the linearized exponential in Appendix IV to extract the RB fidelities, just as in Fig. 2. Using error rates centered around estimated values from Fig. 3, we find that IRB systematically underestimates infidelities across the range, including by about 22% at our measured value (marked on Fig. S4). In absolute terms, this only adjusts the infidelity from 0.108(5)% up to 0.138(7)%, confirming a high-fidelity gate. Other investigations have found comparable fractional errors at similarly high gate fidelities, albeit under maximally deleterious unitary errors [7]. The underestimate persists even when using perfect leakage detection measurements in the simulation, suggesting that the bias may be a product of this particular set of 20 random sequences. Simulations of other random sequences show variance in the extracted vs. actual infidelity, supporting this claim.

VI. LEAKAGE PROPAGATION

In this section we will provide more details supporting our investigation of the leakage propagation properties of our CZ gate. We consider leakage before and during the CZ gate and derive error models for both cases.

Control qubit action	Effect on target qubit
start in $ 10\rangle$	I
start in $ 01\rangle$	Z
start in leakage state $ 00\rangle$	I
start in $ 10\rangle$ and leaks during the gate	I
start in $ 01\rangle$ and leaks during the gate	$Z(\theta), \theta \in [0, \pi]$

TABLE III. Leakage propagation properties for CZ gate

A. Leakage before a CZ gate

We first consider the simpler case when leakage occurs before the CZ gate and either the control or target qubit starts in the ground state, $|00\rangle$. In this situation the dispersive interaction between coupler and target qubit is never switched on; this interaction requires one excitation in the coupler and one in the target qubit, but with a leakage there is only one total excitation between the two qubits. As such the overall process when we suffer leakage before the CZ gate is an overall identity process on the other qubit.

B. Leakage during a CZ gate

We now derive the leakage error model for the CZ gate. For this model to be valid, we require that only two of the four cavities interact during the gate, and the CZ gate is realized using a CPHASE-type interaction, that is, only one of the four two-qubit basis states gradually gains an entangling phase until the CZ unitary is realized. This can be achieved with the following general interaction Hamiltonian

$$\mathcal{H}_{\text{ent}}/\hbar = \chi_{\text{ent}} |1_L 1_L\rangle\langle 1_L 1_L| \equiv \chi_{\text{ent}} \text{CZ} \quad (11)$$

The CZ gate is then implemented by unitary evolution of this Hamiltonian for a time $t_{\text{CZ}} = \pi/\chi_{\text{ent}}$.

Leakage midway through the gate will shut off the interaction at an unknown time, and can be modeled as a CPHASE gate with unknown angle, followed by unconditional reset to the $|00\rangle$ state.

A leakage event occurs at some random time during the gate, which we approximate as uniformly random, resulting in first a CPHASE interaction of some unknown phase between 0 and π , followed by leakage to $|00\rangle$ that turns off the interaction. The average channel *prior to leakage* can be computed by evaluating the following integral

$$\Lambda_{\text{CZ}}(\rho) = \frac{1}{\pi} \int_0^\pi \text{CPHASE}(\phi) \rho \text{CPHASE}(-\phi) d\phi. \quad (12)$$

To evaluate the integral, we make use of the identity $\text{CPHASE}(\phi) = Z_c Z_t(\phi/2) Z_c(-\phi/2) Z_t(-\phi/2)$, where $Z_c Z_t(\theta) := \cos(\theta/2) \mathbb{1} - i \sin(\theta/2) Z_c Z_t$ etc. We also make use of another identity: $\text{CZ} = \frac{1}{2} (I + Z_c + Z_t - Z_c Z_t)$. We rewrite the first identity with trigonometric functions

such that

$$\text{CPHASE}(\phi) = x(\phi)\mathbb{1} + y(\phi)(Z_c + Z_t - Z_c Z_t) \quad (13)$$

$$= (x(\phi) - y(\phi))\mathbb{1} + 2y(\phi)\text{CZ} \quad (14)$$

where

$$x(\phi) = \cos^3(\phi/4) + i\sin^3(\phi/4) \quad (15)$$

$$y(\phi) = ie^{-i\phi/4}\sin(\phi/4)\cos(\phi/4) \quad (16)$$

Our goal is to arrive at a channel of the form

$$\Lambda_{\text{CZ}}(\rho) = a_0\mathbb{1}\rho\mathbb{1} + a_1\text{CZ}\rho\text{CZ} + a_2(\text{CZ}\rho\mathbb{1} - \mathbb{1}\rho\text{CZ}) \quad (17)$$

for which we determine

$$a_0 = \frac{1}{\pi} \int_0^\pi |x(\phi) - y(\phi)|^2 d\phi = \frac{1}{\pi} \int_0^\pi \cos^2(\phi/2) d\phi = \frac{1}{2} \quad (18)$$

$$a_1 = \frac{1}{\pi} \int_0^\pi 4|y(\phi)|^2 d\phi = \frac{1}{\pi} \int_0^\pi \sin^2(\phi/2) d\phi = \frac{1}{2} \quad (19)$$

$$a_2 = \frac{1}{\pi} \int_0^\pi 2(x(\phi) - y(\phi))y^*(\phi) d\phi = \frac{i}{2\pi} \int_0^\pi \sin(\phi) d\phi = \frac{i}{\pi} \quad (20)$$

which finally allows us to write

$$\Lambda_{\text{CZ}}(\rho) = \frac{1}{2}(\mathbb{1}\rho\mathbb{1} + \text{CZ}\rho\text{CZ}) + \frac{i}{\pi}(\text{CZ}\rho\mathbb{1} - \mathbb{1}\rho\text{CZ}). \quad (21)$$

The ‘off-diagonal’ elements of this channel of the form $\mathbb{1}\rho\text{CZ}$ and $\text{CZ}\rho\mathbb{1}$ are measurable, as shown in Fig. 4f in the imaginary part of the χ -error matrix for $N = 1$ repeats. They deviate in magnitude from 1π due to a significant fraction of the leakage occurring during state preparation and measurement, rather than the CZ gate itself. These terms can often be dropped in an error correction context without consequence, since they are annihilated by stabilizer measurements. (In a similar way to how coherent Z rotation errors may be twirled into a pure Pauli channel by the act of measuring X stabilizers.) In this case we define the approximate channel

$$\Lambda'_{\text{CZ}}(\rho) = \frac{1}{2}(\mathbb{1}\rho\mathbb{1} + \text{CZ}\rho\text{CZ}) \quad (22)$$

Under this simplified channel, we can say that leakage during our CZ gate results in a 50-50 mixture of performing the complete CZ gate and performing identity, prior to leakage occurring, which is useful for discretizing the location of leakages within a gate sequence. Formulating a complete error channel for our gate requires us to include the leakage jump operators. Here we assume the cavities in each dual-rail have equal loss rates, and that no-jump backaction can be ignored (this is typically a small effect anyways and can be exactly mitigated with ‘echo’ pulses if needed [3]). For convenience, we also define the ‘unconditional reset’ operator to be

$$\begin{aligned} \Lambda_{r,x}(\rho) &= |00\rangle\langle 01|_x \rho |01\rangle\langle 00|_x \\ &+ |00\rangle\langle 10|_x \rho |10\rangle\langle 00|_x \\ &+ |00\rangle\langle 00|_x \rho |00\rangle\langle 00|_x, \end{aligned}$$

where $x = (c, t)$ may denote the control or target qubit to be the qubit which leaks during the gate. The channel that describes one of the qubits leaking during the CZ gate can now be written as

$$\Lambda_{\text{leakage}}^x(\rho) = (1 - p_e^x)\mathbb{1}\rho\mathbb{1} + p_e^x\Lambda_{r,x}(\Lambda_{\text{CZ}}(\rho)), \quad (23)$$

where p_e^x is the probability of qubit x undergoing leakage during the gate. The final error channel we use to model the CZ gate and its errors consists of cascading four error channels together. Chronologically, these are the error channel for leakage on the control, then the error channel for leakage on the target (these two commute), followed by the ideal CZ unitary channel (which acts as identity on the leakage space), followed by a pure dephasing channel. Finally our complete CZ gate error channel is written as:

$$\varepsilon_{\text{CZ}}(\rho) = \Lambda_{\text{dephase}} \circ \Lambda_{\text{ideal}} \circ \Lambda_{\text{leakage}}^t \circ \Lambda_{\text{leakage}}^c(\rho) \quad (24)$$

where

$$\Lambda_{\text{ideal}}(\rho) = \text{CZ}\rho\text{CZ} \quad (25)$$

$$\Lambda_{\text{dephase}}(\rho) = (1 - p_Z^c - p_Z^t - p_{ZZ})\mathbb{1}\rho\mathbb{1} \quad (26)$$

$$+ p_Z^c Z_c \rho Z_c + p_Z^t Z_t \rho Z_t \quad (27)$$

$$+ p_{ZZ} Z_t Z_c \rho Z_c Z_t. \quad (28)$$

where $p_Z^{c(t)}$ is the probability of a phase flip error on the control (target) qubit, and p_{ZZ} is the probability of a correlated dephasing error that affects both qubits.

C. Analysis of experimental results

Having provided a theoretical treatment of leakage propagation in our CZ gate, in this section we provide additional experimental details. Briefly reviewing the experiment, we initialize the control qubit in either the leakage state $|00\rangle$ or one of the computational states $|00\rangle$, $|01\rangle$. Subsequently we perform quantum process tomography (QPT) on the target qubit after performing N repeated CZ gates. To capture the impact of leakage before or during the CZ gates, we condition the QPT for cases when the control qubit is measured to be in the leakage state $|00\rangle$.

To analyze our process tomography results we apply an error analysis described in [15], which we will briefly sketch here. A quantum process can be written using the χ matrix representation as

$$\rho_{\text{out}} = \sum_{m,n} \chi_{mn} E_m \rho_{\text{in}} E_n^\dagger, \quad (29)$$

and here we choose the Pauli basis, $\{E_m\} = \{I, X, Y, Z\}$. For our analysis, we are interested to study the *deviation* from the ideal or expected process, motivating the use of the $\chi^{(\text{err})}$ matrix. The total process χ can be described as the composition of a target unitary U followed by an

error process, $\chi^{(err)}$. This error process is another valid process and can be described by its own χ matrix as

$$\rho_{out} = \sum_{m,n} \chi_{mn}^{(err)} E_m U \rho_{in} U^\dagger E_n^\dagger, \quad (30)$$

The error process matrix can be computed by

$$\chi^{(err)} = V \chi V^\dagger, V_{mn} = \text{Tr}(E_m^\dagger E_n U) / d, \quad (31)$$

where $d = 2^N$ with N representing the number of qubits.

This error process matrix has a simple and intuitive interpretation. In the ideal case, the error process $\chi^{(err)}$ should equal to the identity process with a single non-zero element, $\chi_{I,I} = 1$. Any other non-zero elements indicate a deviation from the intended process.

VII. NO-JUMP BACKACTION IN THE SWS GATE

No-jump backaction is a form of non-unitary, coherent error which arises when we detect that no excitation loss errors have occurred during our CZ gate. In essence, when we learn no loss occurred, we know it is more likely that the control qubit was in the dual-rail state $|10\rangle$ rather than $|01\rangle$, since the $|10\rangle$ state does not swap into the more lossy coupler. This amounts to gradual weak measurement of the control qubit in the Z -basis.

To see what limit this error places on our CZ gate fidelity, it is useful to define the quantities

$$p_{loss}^{a_1} = \frac{t_{gate}}{T_1^{a_1}}, \quad (32)$$

$$p_{loss}^c = \frac{t_{gate}}{T_1^c}. \quad (33)$$

After applying the gate once, the resulting state after postselection may be approximately written as

$$\rho = \frac{\text{CZ} \rho' \text{CZ}}{\|\rho'\|}, \quad (34)$$

where

$$\rho' = \hat{E} \rho_{init} \hat{E}^\dagger, \quad (35)$$

$$\hat{E} = \sqrt{1 - p_{loss}^{a_1}} |0\rangle_c \langle 0|_c \otimes \mathbb{1}_t + \sqrt{1 - p_{loss}^c} |1\rangle_c \langle 1|_c \otimes \mathbb{1}_t \quad (36)$$

and $\hat{E}$ is the (non-trace preserving) Kraus operator describing no-jump backaction. The subscripts denote control and target qubits respectively.

The states most sensitive to this effect are ones where the control qubit is on the equator of its Bloch sphere. However, we can show that for small $|p_{loss}^{a_1} - p_{loss}^c|$, the error is approximately

$$\varepsilon_{NJ} \approx \frac{(p_{loss}^{a_1} - p_{loss}^c)^2}{4}, \quad (37)$$

which scales quadratically with the imbalance in loss rates. For our hardware parameters, we expect this effect to be small after one CZ gate, contributing with an error $< 10^{-5}$ after one gate. However, after ~ 100 consecutive gates, due to the quadratic scaling of this effect can be noticeable at the few percent level.

In any practical setting, our CZ gate will be interleaved with single qubit rotations and measurements, which has a similar effect to “echoing out” the dynamics associated with no-jump backaction, ensuring that the effective error stays below 10^{-5} .

It can be shown that a single X gate on the control qubit mid-way through a repeated series of CZ gates is sufficient to completely nullify the effects of no-jump backaction. We additionally perform an X gate on the target qubit as well in our quantum state tomography experiment since X gates in general are useful for echoing out any sources of low frequency dephasing noise affecting our qubits. The fact that we still observe non-linear dephasing on the control qubit even with the presence of the echo pulses indicates no-jump backaction cannot be responsible for this effect.

We now briefly describe why a single X gate mid-way through our repeated CZ sequence is sufficient to nullify the effects of no-jump backaction. The intuition as to why this works is that, in the period of time that elapses from the beginning of the sequence to the final measurement, each state in the control qubit will spend the same amount of time occupying the lossy coupler mode. Thus, when we measure that photon loss has not occurred in the control qubit, we (and the environment) gain no additional information about which state the control qubit began in.

We show this mathematically by considering an idling control dual-rail qubit, which undergoes non-unitary (and non-trace preserving) evolution due to the collapse operator

$$\hat{L} = \sqrt{\kappa} |00\rangle \langle 01|_c, \quad (38)$$

where κ models the average loss rate experienced by the $|01\rangle_c$ state as it swaps in and out of the coupler through many repeated CZ gates.

With the Lindblad master equation we can specifically examine the no-jump part of the evolution as

$$\dot{\rho} = -\frac{1}{2} (\hat{L}^\dagger \hat{L} \rho + \rho \hat{L}^\dagger \hat{L}) \quad (39)$$

$$= -\frac{\kappa}{2} (|01\rangle \langle 01|_c \rho + \rho |01\rangle \langle 01|_c) \quad (40)$$

$$= -\frac{\kappa}{2} \rho - \frac{\kappa}{4} (Z_c \rho + \rho Z_c). \quad (41)$$

We can solve this equation for a “qubitized” density matrix that describes the dual-rail codespace of the control qubit as

$$\rho(t) = \begin{pmatrix} \rho_{00}(t) & \rho_{01}(t) \\ \rho_{10}(t) & \rho_{11}(t) \end{pmatrix}. \quad (42)$$

Solving this differential equation gives us

$$\rho(t) = \begin{pmatrix} \rho_{00}e^{-\kappa t} & \rho_{01}e^{-\kappa t/2} \\ \rho_{10}e^{-\kappa t/2} & \rho_{11} \end{pmatrix}. \quad (43)$$

This evolution describes a gradual measurement in the Z basis which gradually polarizes us to the state $|10\rangle_c$, the least lossy basis state.

Applying a single X gate at time $t = \tau/2$ is sufficient to completely undo this polarization effect. After this X pulse, now the state that was initially $|10\rangle$ will now swap into the coupler and the density matrix evolves under the differential equation

$$\dot{\rho} = -\frac{\kappa}{2}\rho + \frac{\kappa}{4}(Z_c\rho + \rho z_c), \quad (44)$$

which has the general solution

$$\rho(t) = \begin{pmatrix} \rho_{00} & \rho_{01}e^{-\kappa t/2} \\ \rho_{10}e^{-\kappa t/2} & \rho_{11}e^{-\kappa t} \end{pmatrix}, \quad (45)$$

describing gradual polarization to the $|01\rangle_c$ state.

Thus, after evolving for time $\tau/2$, applying the X gate and then evolving for time $\tau/2$ (and finally applying another X gate if desired), we are left in the state

$$\rho(t) = \begin{pmatrix} \rho_{00}e^{-\kappa t} & \rho_{01}e^{-\kappa t} \\ \rho_{10}e^{-\kappa t} & \rho_{11}e^{-\kappa t} \end{pmatrix} = e^{-\kappa t}\rho. \quad (46)$$

And so, the density matrix remains the same as our initial density matrix, except for multiplication by the scalar factor $e^{-\kappa t}$ which represents the probability we do not detect a loss error.

Our claim that a single X gate on the control qubit mid-way through the CZ gate sequence is sufficient to completely undo the polarizing effect of no-jump back action is also supported by our QuTiP full master equation simulations.

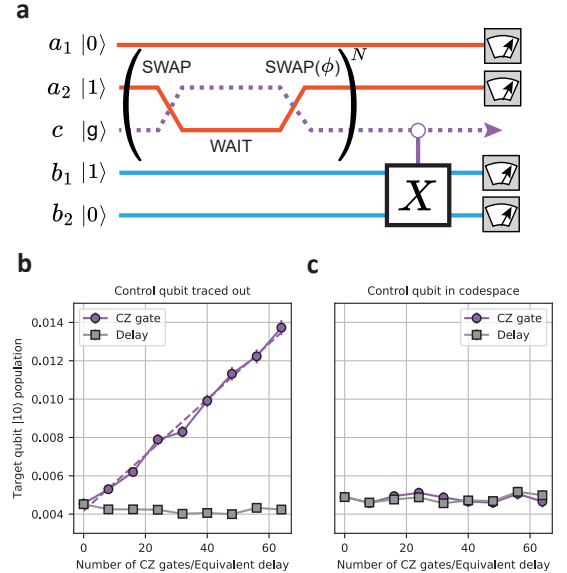
VIII. CZ INDUCED COUPLER HEATING

Since the transmon coupler mode c is temporarily populated during the application of the CZ gate it is important to ensure that all the population is transferred back to the control dual-rail before the gate ends, failing to do so will dephase the target qubit. In this section we describe the experimental protocol to measure the residual coupler excitation at the end of the CZ gate.

We do this by preparing our system in the state $|01g10\rangle$ and repeatedly applying the CZ gate followed by a slow selective X gate on the target qubit. This selective X gate only swaps excitation between cavities b_1 and b_2 when the coupler is in the $|g\rangle$ state and implements an identity on the target qubit when the coupler is in the $|e\rangle$ state. This pulse sequence maps the residual coupler excitation onto the target qubit $|10\rangle$ state population. At the end of the sequence we measure the two dual-rail qubits. We process the data in two ways. First, we

trace out the control qubit and measure the population of the target qubit in the state $|10\rangle$. This encodes the unpostselected residual coupler excitation at the end of N CZ gate applications.

We observe a small but measurable increase in the coupler population of 0.014% per gate application (see Fig S5b). Next, we use the same dataset but postselect on the measurement outcomes such that the control qubit is assigned to be in the codespace. In this case we observe no measurable increase of the target qubit $|10\rangle$ population after 64 applications of the gate. Its important to note here that because of the excitation preserving property of the gate whenever we inadvertently leave the coupler in the excited state we will necessarily observe the control qubit in the vacuum state. This allows us to postselect on this error channel. Lastly, we also compare this measurement to the case when we replace the CZ gate with a delay of equivalent duration and find that we see no measurable difference between the two sequences when the control qubit is postselected to be in the code space.



Supplementary Figure S5. **Residual coupler excitation** **a.** Pulse level protocol used to map the residual coupler population into target qubit. **b.** Target qubit $|01\rangle$ population as a function of number of CZ gates in the sequence or delay when the control qubit measurement outcomes are traced out **c.** Same as **b** but when control qubit measurement outcomes are postselected to be in the codespace.

IX. CORRELATED LEAKAGE AND COUPLER EXCITATION

Although rare, there is a mechanism by which photon loss on a target qubit midway through an SWS gate can result in both the control and target qubit ending in the

vacuum state, and an excitation remaining in the coupler. We refer to this as a correlated error because a single loss mechanism can result in a double erasure. This effect is most pertinent when $g_{ac} \sim \chi_{bc}$, when the control qubit does not fully swap its photon from a_2 into the coupler mode, even for the ideal unitary dynamics. Photon loss from b_1 at unknown time dephases any excitations in the coupler, and in this parameter regime, there is also a chance the coupler excitation is not fully swapped back into the cavity mode a_2 , resulting in the ‘double-erasure’ state $|00e00\rangle$.

For our hardware parameters, we expect to end up in the double-erasure state which occurs with probability 1.3×10^{-5} per CZ gate, according to our QuTiP simulations. Equivalently, if photon loss occurs in the target qubit during the gate, the excitation is expected to remain stuck in the coupler with probability 1.4%. This probability scales quadratically with the ratio $|\chi_{bc}|/g_{ac}$ and is not expected to limit gate performance.

A related error, dephasing in the coupler mode midway through the SWS, can also put us in the detectable leakage states $|00e10\rangle$ or $|00e01\rangle$. This arises because dephasing of the coupler mode no longer guarantees that second SWAP has the correct phase. Like the aforementioned double erasure, this error is also suppressed quadratically with the ratio $|\chi_{bc}|/g_{ac}$, which serves as another reason to increase g_{ac} in future implementations.

With our QuTiP master equations, we predict the system leaks to these two states with probability 6.5×10^{-5} per gate, assuming a coupler dephasing time of 1001 μs which we measure for our system. The total probability the coupler is predicted to end in the excited state is 8.0×10^{-5} per gate, comparable to what we measure in experiment.

X. MEASURING RESIDUAL COUPLER POPULATION

In addition to measuring coupler heating per CZ gate, we also quantify the residual thermal coupler population in our system. Residual population on the inter-dual-rail qubit coupler can induce a correlated dephasing error on both the control and target qubit as the dispersive coupling will cause a frequency shift on both qubits. Such an error can be thought of as a state preparation error as it corresponds to incorrect preparation of the initial state of the system at the beginning of each experimental shot. As such, it does not add to the error-per-gate but instead contributes to the ‘offset’ error at the level of the thermal population of the coupler, e.g. $\bar{n}_{\text{th}}^{(\text{coupler})}$.

Because our system does not have a direct readout for the coupler—and as we will see, the residual population is fairly low—this can be a rather challenging measurement where common procedures like the Rabi population measurement [16] are insufficient to resolve the residual thermal population.

We perform the following protocol to measure residual

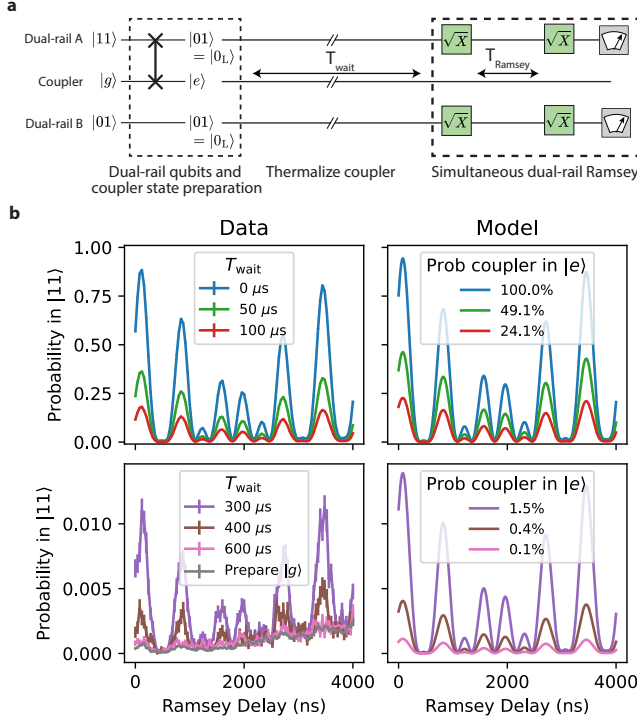
thermal population in the inter dual-rail coupling transmon. Our key insight is that the coupler excitations will induce a frequency shift in both of the neighboring dual-rail qubits and that we can disambiguate from other thermal excitations by correlating results from both control and target qubits. To accomplish this, we perform simultaneous Ramsey experiments on both control and target dual-rail qubits, see Fig. S6a. In the case that the coupler is in the ground state, then both qubits will have zero detuning and we should measure high probability in the $|11\rangle$ joint dual-rail state. If the coupler is in the excited state, then we will observe oscillations in the $|11\rangle$ population at frequencies that correspond to the cross-kerrs between the coupler and dual-rail qubits, see Table II. In our experiment we perform a Ramsey sequence with maximum delay of 4 μs to acquire several periods of oscillations for a snapshot of the coupler thermal population.

In our experiment we can also intentionally vary the population in the coupling transmon using the following protocol. We initialize one of the dual-rail qubits with an excitation in both cavities, e.g. the physical $|11\rangle$ state, see Dual-rail A in Fig. S6a. Then we perform a coupler-cavity SWAP (similar to what is used for the SWS gate) to transfer one excitation into the coupling transmon and to initialize the dual-rail qubit in the codespace state $|01\rangle$. We can then add a variable delay to allow the coupling transmon to partially thermalize before performing the simultaneous Ramsey measurement. The full sequence is shown in Fig. S6a. Our measured results are shown in Fig. S6b at several different delay times (T_{wait}) after initializing the coupler transmon in $|e\rangle$ from zero delay to a delay of 600 μs . We observe oscillations with a magnitude that decreases with decreasing coupler population. The longest delay, $T_{\text{wait}} = 600 \mu\text{s}$, corresponds to $\sim 8.5 T_1$ where the transmon should be completely thermalized and we observe small oscillations at the 10^{-3} level. We also perform the same experiment without initializing the coupling transmon in the excited state (grey data in Fig. S6b, bottom left panel) and observe similar results. Both results allow us to bound the coupler thermal population to about 0.1%. These results are also supported by master equation simulations of the same protocol and are shown in the right column of Fig. S6b.

This thermal population can be reduced in future works by employing dissipation-based leakage reduction units [17–22] or by directly detecting coupler population through a direct readout [23] to convert this leakage into an erasure error.

XI. MEASURING BIT-FLIP ERRORS

Figure S7 presents additional data, obtained with other variations of the initial logical states of the target and control qubits, with the same measurement protocol described in the main text. The fits to the linear slopes (solid lines) are used to extract the quantities plotted in Fig. 3f. With the exception of the state where an exci-

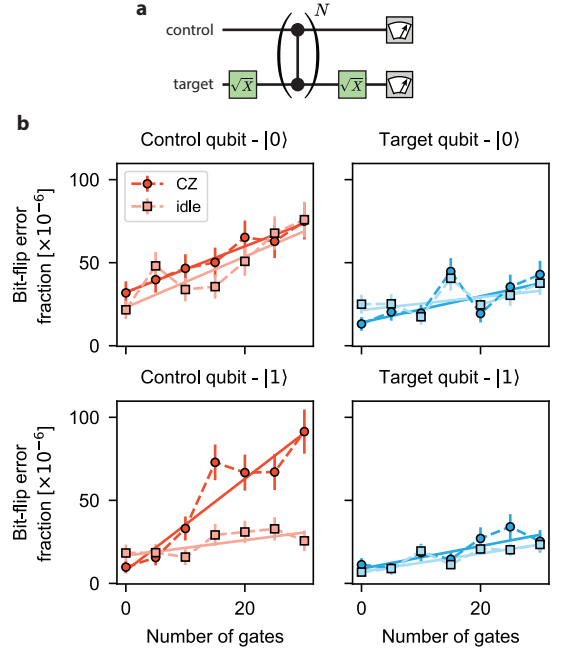


Supplementary Figure S6. **Measuring inter-dual-rail coupling transmon thermal population** **a**. Experimental sequence for quantifying coupler thermal population. The sequence begins with a step initializing the coupling transmon in $|e\rangle$ and a variable delay time to allow the coupling transmon to thermalize. The subsequent simultaneous Ramsey experiment on both dual-rail qubits enables measurement of the residual population in the coupler. **b**. Experimental results (left column) and simulation results (right column) of the simultaneous Ramsey experiment. Each color corresponds to a different delay time for data (left) and coupler excitation probability for the simulation (right). The magnitude of the oscillations corresponds to the probability that the coupler is excited. In particular we use the result from $T_{\text{wait}} = 600 \mu\text{s}$ (Data, pink) where the transmon state is at thermal equilibrium. For this data we extract a bound for $n_{\text{th}} \sim 0.1\%$, particularly comparing with the simulation result setting a 0.1% coupler population (Model, pink). This result is also in good agreement to the case when the coupler is left in the thermal state (grey).

tation is swapped into the coupler, the rates are indistinguishable from the idling bit flip rates. As in [3], the quantities we extract are likely only an apparent rate, due to the finite leakage discrimination fidelity of the end-of-the-line measurements, multiplied by the increasing number of erasures with time.

XII. REPRODUCIBILITY AND PERFORMANCE STABILITY

We present some early results on CZ gate reproducibility and stability over time. In Fig. S8 we show repeated



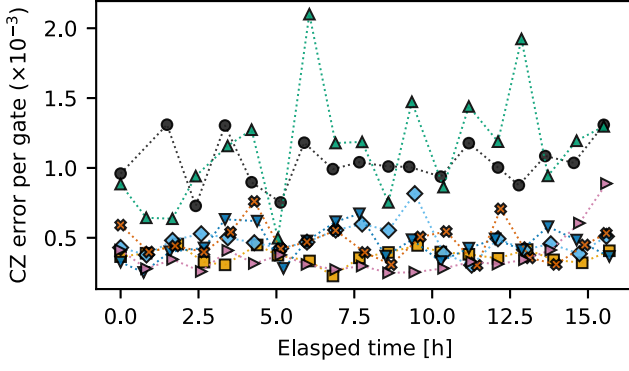
Supplementary Figure S7. **Measuring bit-flip errors during CZ gate** **a**. Circuit used to measure bit-flip errors **b**. Results of bit-flip measurements for the various combinations of initial logical states of the target and control dual-rails, compared to the idling states when no CZ gates are performed. The extracted linear trends (solid lines) are used to produce the quantities presented in Fig. 3f.

measurements of interleaved RB on seven different two-qubit gate systems. For these measurements we calibrate the gates once before the start of these repeated benchmarking measurements. These results reveal similar performance at or below 0.1% error per CZ gate, but also indicate excellent stability for most of the systems with gate error fluctuations at a small fraction of the total gate error. Ongoing work aims to uncover the source of the residual fluctuations seen for two of the seven systems, and we suspect sources of error like environmental temperature fluctuations that affect the output power of our control systems or fluctuating two-level systems that can decrease coherence.

XIII. ANALYTICAL DERIVATION OF GATE PARAMETERS

In our gate, there are three parameters that must be calibrated. These are the duration of each of the cavity-coupler ‘SWAP’ pulses, t_{SWAP} , the delay time between these two pulses, t_{wait} , and the phase of the second SWAP pulse, relative to the first, which we denote as ϕ .

In this section we derive the analytical expressions for these three parameters required to implement a unitary that is locally equivalent to a CZ gate on the dual-rail



Supplementary Figure S8. **CZ error per gate timeseries.** We present CZ gate benchmarking on seven different two-qubit gate systems. These results show high reproducibility of our gate performance in the Main Text and also show good stability of many hours of operation.

subspace, up to local Z rotations. We find these parameters for the assumption that we have square, instantaneous pulses and that our pulse Hamiltonian takes the form

$$\mathcal{H}/\hbar = \frac{1}{2} \left(g_{ac}(t)a_2^\dagger c + g_{ac}^*(t)a_2 c^\dagger \right) + \chi_{bc} b_1^\dagger b_1 c^\dagger c \quad (47)$$

In the actual experiment, these values need to be calibrated and will differ from the theoretical values we are about to derive due to finite pulse ramp times and extra Hamiltonian terms such as cavity-cavity cross Kerr.

Nevertheless, deriving these parameters is useful for numerical simulation, and for understanding the physics of the gate.

One thing to note is that for a given set of hardware parameters, (g_{ac}, χ_{bc}) , there are multiple sets of pulse parameters, $(t_{\text{SWAP}}, t_{\text{wait}}, \phi)$ which can enact a CZ gate. To simplify this, we can choose to fix one of the parameters. In [24], Rosenblum *et al.* choose to fix $\phi = \pi$, and calibrate the two pulse timing parameters. We instead choose to fix $t_{\text{SWAP}} = \pi/g_{ac}$, taking g_{ac} to be real and positive. (Experimentally, this means we only need to calibrate a delay time and a phase, the easiest parameters to sweep).

Given this constraint, the analytical expressions for t_{wait} and ϕ are

$$t_{\text{wait}} = \frac{\pi}{|\chi_{bc}|} - \frac{\pi}{g_{ac}}, \quad (48)$$

$$\phi = \chi_{bc} t_{\text{wait}} \quad (49)$$

$$+ 2 \arctan \left(-\frac{\sqrt{g_{ac}^2 + \chi_{bc}^2}}{\chi_{bc}} \cot \left(\frac{\pi \sqrt{g_{ac}^2 + \chi_{bc}^2}}{2g_{ac}} \right) \right), \quad (50)$$

derived below. With these parameters, the gate can be

modeled through three different piece-wise Hamiltonians:

$$\mathcal{H}_1/\hbar = \frac{g_{ac}}{2} (a_2^\dagger c + a_2 c^\dagger) + \chi_{bc} b_1^\dagger b_1 c^\dagger c \quad (51)$$

$$\mathcal{H}_2/\hbar = \chi_{bc} b_1^\dagger b_1 c^\dagger c \quad (52)$$

$$\mathcal{H}_3/\hbar = \frac{g_{ac}}{2} (e^{i\phi} a_2^\dagger c + e^{-i\phi} a_2 c^\dagger) + \chi_{bc} b_1^\dagger b_1 c^\dagger c \quad (53)$$

acting for times, t_{SWAP} , t_{wait} , t_{SWAP} respectively. It also follows that the total gate duration is given by

$$t_{\text{gate}} = t_{\text{wait}} + 2t_{\text{SWAP}} = \frac{\pi}{|\chi_{bc}|} + \frac{\pi}{g_{ac}} \quad (54)$$

The unitary on the dual-rail space that is implemented is

$$U_{\text{gate}} = Z_1(\pi - \phi) Z_2(\pi) U_{\text{CZ}} \quad (55)$$

where we use the usual convention

$$U_{\text{CZ}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (56)$$

A. Deriving t_{wait}

During the gate, only the dual-rail state $|0110\rangle$ gains an entangling phase, via the dispersive interaction. To find the wait time, we must tally up the entangling phase accumulated during each of the three piece-wise Hamiltonians. The total entangling phase (which must be π for a CZ gate) is then given by

$$\phi_{\text{ent}} = 2\phi_{\text{SWAP}} + \phi_{\text{wait}} = \pi \quad (57)$$

where ϕ_{wait} , the phase accumulation during the delay time is simply $\phi_{\text{wait}} = \chi_{bc} t_{\text{wait}}$. Finding the entangling phase contribution from $\mathcal{H}_1$ and $\mathcal{H}_3$ is a little trickier, but can be found by considering a ‘qubitized’ Hamiltonian of the form

$$\mathcal{H}_q = \frac{g_{ac}}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \chi_{bc} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (58)$$

which describes the dynamics of the dual-rail state $|01g10\rangle$ swapping to the state $|00e10\rangle$ in a two-level manifold. If we were to enact this Hamiltonian for time $\tau = 2\pi/\Omega$, where $\Omega = \sqrt{g_{ac}^2 + \chi_{bc}^2}$ is the detuned Rabi rate, then we would find the unitary this enacts is

$$e^{i\mathcal{H}_q\tau} = e^{i(\pi - \chi_{bc}\tau/2)} \mathbb{1} \quad (59)$$

Usually this phase is ignored as a ‘global phase’, but in our case, because this is a phase on just one of the dual-rail basis states, it is not global, and it contributes to a measurable phase on our dual-rail qubit. The phase contribution due to χ_{bc} from a ‘complete orbit’ of a detuned Rabi oscillation is $\chi_{bc}\tau/2$. (The π offset does not

contribute to the entangling phase). If we instead were to enact $\mathcal{H}_q$ for time $t = t_{\text{SWAP}} = \pi/g_{ac}$, we would instead acquire a smaller phase, scaled by the fraction of the ‘orbit’ we complete:

$$\phi_{\text{SWAP}} = \frac{\chi_{bc}\tau}{2} \frac{t_{\text{SWAP}}}{\tau} = \frac{\chi_{bc}t_{\text{SWAP}}}{2} \quad (60)$$

This is equivalent to the entangling phase we acquire from enacting $\mathcal{H}_1$ for time t_{SWAP} . The contribution to the total entangling phase from $\mathcal{H}_3$ is also identical.

From 60 and 57 we can write

$$2 \frac{\chi_{bc}t_{\text{SWAP}}}{2} + \chi_{bc}t_{\text{wait}} = \pi \quad (61)$$

and is easily rearranged to give

$$t_{\text{wait}} = \frac{\pi}{|\chi_{bc}|} - \frac{\pi}{g_{ac}} \quad (62)$$

which is the first expression we wanted to derive and takes a surprisingly simple form.

B. Deriving ϕ_{SWAP}

As may be expected from the form of the analytical expression for ϕ_{SWAP} in Eq. 48, this derivation will require some geometry and trigonometry. It is helpful to refer to Fig. S9. The angle ϕ_3 is related to ϕ_{SWAP} as $\phi_3 = \pi - \phi_{\text{SWAP}}$. We also know the value of t_{wait} and so $\phi_1 = \chi_{bc}t_{\text{wait}}$. If we can calculate ϕ_2 then we can use the fact $\phi_1 + 2\phi_2 + \phi_3 = 2\pi$ to calculate ϕ_3 and thus ϕ_{SWAP} . We start by noting that the evolution during $\mathcal{H}_1$ (and $\mathcal{H}_3$) is an ellipse in the XY plane. On the Bloch sphere, these amount to detuned Rabi oscillations but when projected onto the XY plane these become elliptical. We define the major axis of this ellipse to be $2y_0$ and the minor axis to be $2x_0$. It can be shown that the ratio $y_0/x_0 = \chi_{bc}/\Omega$.

If we were to center this ellipse at the origin (see Fig. S9), then the (x, y) coordinates can be parameterized by angle $\theta(t)$, where a complete orbit of the detuned Rabi oscillation is completed at $t = 2\pi/\Omega$. The (x, y) coordinates are then given by

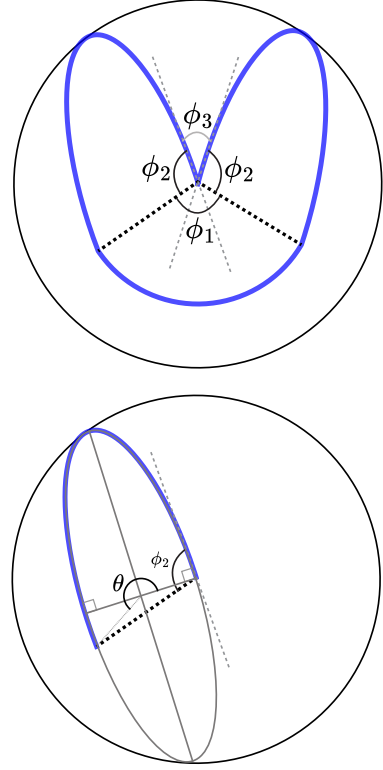
$$x(t) = x_0 \cos(\theta(t)) \quad (63)$$

$$y(t) = y_0 \sin(\theta(t)). \quad (64)$$

However, since we only activate $\mathcal{H}_1$ for time $t = t_{\text{SWAP}}$, our partially complete orbit ends at $\theta(t) = \Omega t_{\text{SWAP}} = \theta'$. We can now simply write down our end point, but will also shift the origin of our coordinate system such that $(0, 0)$ coincides with the beginning of the trajectory (i.e. the origin is the center of the XY plane of our Bloch sphere projection). Now, the endpoints of our trajectory are written as

$$x' = x_0(1 - \cos(\Omega t_{\text{wait}})) \quad (65)$$

$$y' = y_0 \sin(\Omega t_{\text{wait}}). \quad (66)$$



Supplementary Figure S9. (Top) Top-down view of the Bloch sphere trajectory of the dual-rail basis state $|0_L 1_L\rangle$, projected into the XY plane. The Bloch sphere is defined as in the left panel of Fig. S10. Finding the analytical expression for ϕ_{SWAP} amounts to calculating ϕ_3 on this diagram. (Bottom) Top down view of the same trajectory, focusing on the evolution during $\mathcal{H}_1$, with useful angles marked to find ϕ_2 .

We can use this to find

$$\phi_2 = \frac{\pi}{2} + \tan^{-1} \left(\frac{|y'|}{|x'|} \right), \quad (67)$$

and so we may finally arrive at

$$\phi_{\text{SWAP}} = \phi_1 + 2\phi_2 - \pi \quad (68)$$

$$= \chi_{bc}t_{\text{wait}} + 2 \tan^{-1} \left(\frac{|y'|}{|x'|} \right) \quad (69)$$

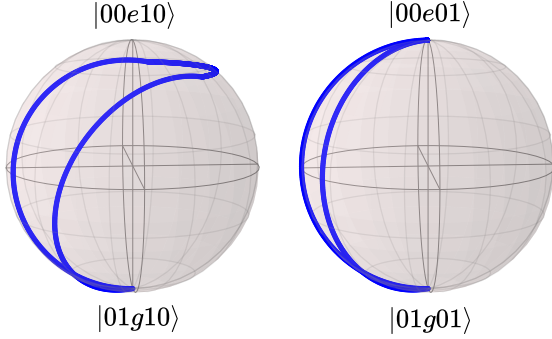
$$= \chi_{bc}t_{\text{wait}} + 2 \tan^{-1} \left(\frac{-y_0 \sin(\Omega t_{\text{SWAP}})}{x_0(1 - \cos(\Omega t_{\text{SWAP}}))} \right) \quad (70)$$

$$= \chi_{bc}t_{\text{wait}} + 2 \tan^{-1} \left(-\frac{\Omega}{\chi_{bc}} \cot \left(\frac{\Omega t_{\text{SWAP}}}{2} \right) \right), \quad (71)$$

where we have used the condition $\Omega t_{\text{SWAP}} > \pi$.

Error channel	Error state(s) after gate	Probability (QuTiP)	Probability (at short gate depth)
Photon loss on control DR	$ 00g01\rangle, 00g10\rangle$	0.337%	0.400(4)%
Photon loss on target DR	$ 10g00\rangle, 01g00\rangle$	0.0982%	0.096(4)%
Photon loss on both DRs	$ 00g00\rangle$	0.0003%	-
Control DR excitation stuck in coupler mode	$ 00e10\rangle, 00e01\rangle$	0.006%	-
Only one excitation remains in coupler mode	$ 00e00\rangle$	0.0014%	-
Control DR dephasing	$Z_c \psi_{\text{ideal}}\rangle$	0.0169%	0.039(1)%
Target DR dephasing	$Z_t \psi_{\text{ideal}}\rangle$	0.0048%	0.0112(9)%
Control and target DR dephasing	$Z_c Z_t \psi_{\text{ideal}}\rangle$	0.0001%	-
No error	$ \psi_{\text{ideal}}\rangle$	99.5351%	-

TABLE IV. Estimated error budget for a single CZ gate, based on QuTiP simulations and measured hardware coherence times. The rightmost column compares to experimentally measured values at short numbers of gate repeats.



Supplementary Figure S10. Evolution of the dual-rail basis states $|0_L 1_L\rangle$ (left) and $|0_L 0_L\rangle$ (right) during the SWS gate, obtained from our QuTiP simulations.

XIV. QUTIP MASTER EQUATION SIMULATIONS AND CZ GATE ERROR BUDGET

We simulate expected gate performance by simulating evolution under the three piecewise Hamiltonians described in 51 (i.e. with the square pulse simplification), modelling all 5 modes a_1, a_2, c, b_1, b_2 and their respective collapse operators, modelling white-noise dephasing and excitation loss. The loss rates for these collapse operators are obtained from our measured hardware coherence times.

These simulations capture the effects of no-jump back-action, expected Z-frame updates, correlated errors, and verify that our expressions for derived gate parameters in 48 are exact.

We can use our QuTiP simulations in the absence of decoherence to examine the intermediate evolution of dual-rail basis states during the SWS gate, as shown in Fig. S10. The $|0_L\rangle|1_L\rangle$ completes a closed trajectory within the $|01g10\rangle - |00e10\rangle$ manifold, as shown on the Bloch sphere, gaining a total phase shift of $\pi + \phi_{\text{SWAP}}$, and population returns exactly to the initial $|01g10\rangle$ state. In contrast, the evolution of the $|0_L 1_L\rangle$ dual-rail state is much simpler, whereby the state $|01g01\rangle$ swaps completely into the coupler, and then is completely swapped back to gain a geometric phase ϕ_{SWAP} . The net result is an entangling phase of π and a deterministic Z-

phase of ϕ_{SWAP} on the control qubit, a unitary that is equivalent to a CZ gate. The evolution of the dual-rail states $|1_L 0_L\rangle$ and $|1_L 1_L\rangle$ is trivially identity since there are no photons in mode a_2 to swap into the coupler, and thus no phase accumulation.

We can use this QuTiP simulation to numerically simulate the expected error budget for our gate, given measured hardware coherence times, and compare it against some of our measured gate errors. For this, we use the coherence times listed in Tab. I and II, taking the longer echo times as our dephasing times. The simulated error budget is shown in Tab. XIV and allows us to estimate the probabilities for different CZ gate error channels that are too small to measure directly in experiment.

XV. FUNDAMENTAL LIMITS TO SWS GATE PERFORMANCE

SWS error rate	Error scaling
p_e^{control}	$\frac{1}{G} \frac{1}{\alpha_c T_c^1}$
p_e^{target}	$\frac{1}{\alpha_c T_c^1}$
p_Z^{control}	$\frac{1}{G} \frac{1}{\alpha_c T_c^\phi}$
p_Z^{target}	$G \times \frac{1}{\alpha_c T_c^\phi}$

TABLE V. Fundamental limits to the SWS if we assume inherited coupler decoherence is responsible for all cavity decoherence. G typically takes a value between 0.01-0.05 in our hardware.

The SWS gate uses the dispersive interaction of a transmon coupler as the source of entanglement. This has many similarities to schemes where dispersively coupled transmon ancillae are used for quantum control of cavity modes. As such, one may suspect that transmon decoherence would set a fundamental limit on our gate fidelity.

To investigate this limit, we consider a hypothetical scenario where only the transmon coupler suffers from loss and dephasing noise, whilst all cavity modes have no intrinsic noise (infinite cavity T_1 and T_ϕ). In this limit, the cavities dispersively coupled will inherit both dephasing noise and loss from the coupler via the inverse

Purcell effect. This can be quantified by defining the hybridization factors

$$G_a = \left(\frac{g_{ac}^0}{\Delta_{ac}} \right)^2 \quad (72)$$

$$G_b = \left(\frac{g_{bc}^0}{\Delta_{bc}} \right)^2, \quad (73)$$

where g_{ac}^0 is the vacuum Rabi coupling and Δ_{ac} is the mode detuning between modes a_2 and c (in the context of a Jaynes-Cummings Hamiltonian) and similarly for g_{bc}^0 Δ_{ac} for modes b_1 and c . G_a and G_b also represent the energy participation of the dressed cavity modes in the coupler mode.

Under an assumed white noise dephasing model, we expect the following decoherence times inherited by the cavity modes from the coupler:

$$T_{a_2}^1 = \frac{T_c^1}{G_a} \quad (74)$$

$$T_{b_1}^1 = \frac{T_c^1}{G_b} \quad (75)$$

$$T_{a_2}^\phi = \frac{T_c^\phi}{(G_a)^2} \quad (76)$$

$$T_{b_1}^\phi = \frac{T_c^\phi}{(G_b)^2} \quad (77)$$

The goal is to calculate the SWS gate errors $p_e^{\text{control}}, p_e^{\text{target}}, p_z^{\text{control}}, p_z^{\text{target}}$ based on these coherence times and also the duration of the SWS gate, which is limited to $\tau = \pi/|\chi_{bc}|$.

It will be insightful to rewrite this χ_{bc} in relation to the anharmonicity of the transmon coupler, which is approximately $\chi_{bc} \approx 2G_b\alpha_c$ where α_c is the anharmonicity of the coupler.

Now, we simply use the ratio of gate time to coherence time to calculate the SWS error limits. We omit prefactors of order unity and assume $G \sim G_a \sim G_b$. For our hardware, G typically varies between 0.01 and 0.05. As an example,

$$p_e^{\text{target}} \sim \frac{\tau}{T_{b_1}^1} \approx \frac{\pi}{|\chi_{bc}|T_{b_1}^1} \approx \frac{G_b\pi}{2G_b\alpha_c T_c^1} \sim \frac{1}{\alpha_c T_c^1}. \quad (78)$$

For the control qubit, we must take into account the fact that the excitation is swapped into the coupler for most of the gate, and so experience coupler decoherence more directly. As an example,

$$p_z^{\text{control}} \sim \frac{\tau}{T_c^\phi} \approx \frac{\pi}{|\chi_{bc}|T_c^\phi} \sim \frac{1}{G} \frac{1}{\alpha_c T_c^\phi} \quad (79)$$

The results of this analysis for the other three error channels is shown in Table V.

Factors of the form $1/\alpha_c T_c^\phi$ are indicative of transmon gate fidelities at the decoherence limit (i.e. typical single qubit gate fidelities on transmons). The fact that p_z^{target} may be several orders of magnitude better than the transmon decoherence limit makes the SWS gate extremely promising for flag-qubit or stabilizer measurement circuits, with the maximum bias between p_z^{target} to p_z^{control} set by $1/G^2$. We should expect p_z^{target} to further improve as we reduce extrinsic sources of cavity dephasing (e.g. dephasing inherited from the ancilla transmons).

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