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Peer Review File

Gravitational torque drives multidecadal variations in length of day

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Attachments originally included by the reviewers as part of their assessment can be found at the end of this file.

Version 0:

Reviewer comments:

Referee #1

(Remarks to the Author)
Summary of the key results

Zhang and Dumberry rely on the recent estimation of the evolution of the angular position of the Earth's inner core with respect to the mantle, from 1964 to 2019, to investigate the mechanism responsible for the exchanges of angular momentum between the Earth's core and mantle.
This remains very much an open question. They consider the electromagnetic and topographic torque between the fluid core and the mantle on the one hand and the gravitational torque between the solid inner core and the mantle on the other hand. The electromagnetic and topographic torques are estimated from a published model of the fluid velocity at the core surface. The authors infer the gravitational torque from the position of the inner core relative to the mantle and find that only this torque has the right sign (at low frequency) to account for the time evolution of the mantle angular momentum (length-of-day data).

Originality and significance

The study is original and puts together, for the first time, the state-of-the-art knowledge on topographic torques, geomagnetic core-flow models and recent seismic data to build a comprehensive torque budget and explain changes in the length of the day.

To put the study into perspective, we find it noteworthy that exchanges of angular momentum between the core and the mantle dominate the changes in the angular momentum of the solid Earth arising from climate change. As a result, the current incomplete modelling of core-mantle angular momentum exchanges has prevented identifying a fingerprint of climate change in LOD data (see e.g. Agnew, Nature, 628, 333-336, 2024).

Data and methodology: validity of approach, quality of data, quality of presentation

The manuscript is well written with high-quality figures. The supplementary material file gives a wealth of information that answers many of our questions.

The study is restricted to low-frequency variations, which correspond to a (60-70)-year period, since the available seismic data (angular position of the inner core) has limited time resolution. This necessarily leaves out the main difficulty affecting the discussion of LOD (Length-of-Day) changes arising from core-mantle interactions: accounting simultaneously for the angular momentum and torque budgets across various frequency ranges.
The problem can be illustrated with the example of electromagnetic (EM) coupling.
Gillet et al. (120, 3991–4013, 2015) found two different values of the conductance (4×10^8 S and 3×10^9 S) to make the EM torque have an amplitude similar to the observed torque at, respectively, decadal and 6-year periods. Their finding can be

explained by noting that core angular momentum is linearly related to the core surface flow **u**. Therefore, the axial torque that is responsible for the time changes of the core angular momentum is a function of the time derivative of surface core flow **u**. However, most of the torques that have been discussed in this context are also a function of the velocity **u**. Therefore, the torques are related simultaneously to the surface velocity and its time derivative. As a result, the parameters that ensure a good fit at one frequency do not yield a good match to the data at another frequency (or for the more general multi-frequency signal).
The solution to this puzzle has been to assume that at least two torques cancel each other out, particularly at low frequencies. Aubert (2026, https://hal.science/hal-05560846v1, submitted to the open-access journal jSEDI) has just discussed geodynamo simulations calculated with both gravitational and EM torque.
He finds that the two torques cancel each other out at low frequency, as also envisioned by Zhang and Dumberry in their manuscript.
For short periods, however, the EM torque acts alone (see his figure 11c).
This contrasts with the scenario of Zhang and Dumberry, who argue that the gravitational torque is already twice as large as the EM torque at a 70-year period and acts alone at short periods.

To some extent, a study restricted to one frequency range eschews the main difficulty. We propose below an exercise to partially address this issue. Also, we find that it would be useful to discuss why the ordering of the torques is different in the study of Aubert (and in the previous work of Schwaiger, 2024, reference 52 of the main text), based on geodynamo simulations and in the work reported in the submitted manuscript.

There are many intermediary steps from the primary seismic data (temporal changes of seismic waves for repeated earthquakes) to the evolution of the angular position of the inner core. The seismologists had to calculate a model for the lateral variations of the wave speeds in the region sampled by the seismic rays (as explained in Yang and Song, JGR, 125, e2019JB018652, 2020). They also discard events associated with too fast variations that they deem unphysical. Ultimately, it would be beneficial to use the primary data (changes of DF travel times) in order to better control uncertainty propagation.

Appropriate use of statistics and treatment of uncertainties

The authors performed a Bayesian inversion of the different parameters (mantle conductance, inner core viscous relaxation time, ...) that enter the torque values. They account for the uncertainties in the surface flow (as given by the researchers who calculated this flow) and in the angular position of the inner core. Figure S4 compares the model of the authors for the inner core angular position and its associated error to the initial model (by Yang and Song, 2023) on which their study is based. It appears that the model of the authors underestimates the errors, particularly before 1980. It is not clear why they have not simply used the original model (based on a spline fit) as input.

According to Schwaiger et al. (2024), the unresolved part of the radial magnetic field at the core-mantle boundary yields a significant part of the EM torque. Ignoring this contribution may lead to overestimating the mantle conductance. We recommend accounting for this effect when estimating the mean value of the conductance and its uncertainties.

The amplitude of the gravitational torque depends on the density contrast at the inner core boundary. The authors take it to be 570 kg m^{-3} (from the Earth's reference model PREM). It seems, however, that there is no consensus yet on its value. Cao and Romanowicz (GJI, 157, 1146-1151, 2004) obtained density contrast in the range 600-900 kg m^{-3}. Tkalcic et al. (GJI, 179, 425-443, 2009) found that the density contrast cannot exceed 1100-1200 kg m^{-3} and present observations pointing to a contrast as low as 200-300 kg m^{-3}. This clearly gives an additional source of uncertainty on the amplitude of the geoid reported in line 155 of the main text.

Conclusions: robustness, validity, reliability

The robustness of the proposed conclusion is dictated by the quality of the seismic data and by the focus on the low-frequency signal.
These intrinsic limits of the study are clear from the last section of the paper; however, they are less present in the abstract.

Suggested improvements: experiments, data for possible revision

As discussed above, an important test of any model for the axial torques acting at the core-mantle boundary (CMB) is to find agreement between predictions and observations in different frequency ranges. The lack of seismic data with high time resolution makes it difficult to attempt this test for the scenario of Zhang and Dumberry.
Therefore, we recommend instead that the authors calculate a time series of angular positions of the inner core relative to the mantle that would account for the observed torque (the grey curve in Figure 1b rather that only the smooth black curve), using their preferred values for the parameters of their model (relaxation time τ, mantle conductance) and the same core surface flow as in their submitted manuscript. It would indicate whether to expect aliasing in seismic studies. In addition, it needs to be verified that the value of the inner core relaxation time τ estimated in the study is adequate for the gravitational torque to be effective at a 6-year period, where significant core-mantle coupling has been found. Finally, the discussion of the steady part of the torques would give the average value of the misalignment between the inner core and the mantle.

It would be nice to verify that the conclusions are not dependent on the chosen velocity model. For example, the velocity model of B\"arenzung et al. (JGR, 123, 4539-4560, 2018) accounts well also for the time changes in the core angular momentum, although it ends in 2014. Its coefficients (and their variance) are publicly available.

Following other studies, Vidale et al. (Nature Geosciences, 18, 267-272, 2025) argue that both rotation of the inner core and deformation of its surface account for the time variability of seismic waves, from repeated earthquakes, sampling the inner core. The model of Zhang and Dumberry predicts a deformation of the inner core surface. Is it compatible with the analysis of Vidale et al. (and others)?

Finlay et al. (Nature reviews Earth & environment, 4, 377-392, 2023) included in their review paper a comparison between the LOD changes predicted from the flow models used by Zhang and Dumberry and the observed LOD changes (their figure 5). We find it would be useful to point this figure to the readers since it shows the level of agreement reached today between LOD predictions and data.

lines 47-49 'the inner core boundary ... coincides, on a time average, with the gravitational potential': this is not strictly true if a time average gravitational torque cancels out a time average EM torque, see lines 56-58.

lines 129-131 'the inner core differential rotation ... is not observed directly ... rather ... it is based on a model of anisotropy within the inner core.' This gives the original interpretation by Song and Richards (1996). Yang and Song (2023) wrote instead that they were able to track the inner core rotation, using temporal changes from doublets, because of the lateral seismic velocity gradients in the inner core.

Could you give a typical amplitude of the deviation from a sphere of the inner core surface (which is involved in the gravitational torque discussed here)?

It may be helpful to refer to K_{em} as the dimensionless conductance (or conductance in units of 10^8 S), as it has a simple physical interpretation.

There is a typo in the y-axis of Figure S16.

References: appropriate credit to previous work?

The references are appropriate. We have suggested in our report a few additional references.

Gerick and Mandea (GJI, 243, ggaf321, 2025) recently found, using synthetic data extracted from geodynamo models, that it is correct to estimate the surface pressure from the surface velocity using the tangentially geostrophic approximation. It may be helpful to comment on this alternative way to estimate the pressure torque from the velocity (and the CMB topography described in the supplementary file, l 242-257). Oliver et al. (Communications earth and environment, 6, 4, 2025) also support this approach (and linear dependence on topographic amplitude), albeit from a convection model only.

Clarity and context: lucidity of abstract/summary, appropriateness of abstract, introduction and conclusions

The authors speculate at the end of the article on the shorter LOD variations. They could complement this statement by noting that on longer time scales the gravitational torque and the torques operating at the core-mantle boundary almost exactly cancel out.

(Remarks on code availability)
The code is available with a description that allows reproducibility.

Referee #2

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

(Remarks on code availability)

Referee #3

(Remarks to the Author)
Wow.

This is an astonishingly broad-ranging and extensive piece of work discussing the dynamics of the core, and the links between length-of-day, geomagnetic models and Earth structure. I find the paper a little difficult in letter format, because there is so much detail and influence of other work. For example, the topographic torque is discussed, there I find that in order to understand this, I must go into the methods, then the supplementary material, leading to some extensive references that I would have to read in order to understand what is being done. Which is far more a comment on me than on the paper I suspect, but it does mean that I have to take a large amount of background on faith. However, I am fully prepared to make this leap.

I find the conclusions entirely plausible - that oscillations are driven primarily by inner-core gravitational interactions, not least because both electromagnetic and topographic torques lead to results that are 90 degrees out of phase with what is required. (IN passing, I believe that the topographic torque has a strong element of electromatic processes involved, so I am not surprised).

My only suggestion is an issue of presentation. I am fully convinced by the conclusions, but I suspect many of our colleagues have other ideas as so how the system works (particularly coupling from core surface flows rather than inner core). As the authors will know (and is unavoidable), there are a whole range of assumptions that go into this work - down to issues as fundamental as the structure of the core flow depending on ignoring diffusion, and a whole pile of non-uniqueness in its determination. To look at the torque calculation, this depends on strong assumptions (at some points using a steady flow, when past (very old) work suggested that it is the much smaller time-varying part of the flow that is of major importance in time variations in length of day. With all these assumptions then drawing conclusions as to deep-mantle structure and minerology is definitely interesting and appropriate, but also very speculative.

So I am occasionally worried by the strength of the language of the conclusions. Just looking at the abstract, stating that the length of day changes were driven by gravitational torque is for me a very strong statement. I fully agree that this is (for me) the most plausible mechanism, but that isn't a proof. Looking at mantle structure, the constraints are assuming that the forces here are not only dominant, but exclusive.

It is great to see that there is a coherent and consistent framework to explain the observations. So this is clearly a consistent mechanism, but it is not possible to say that it *is* the mechanism (nor will it ever be so possible). So please, just a little care in the presentation of your claims.

(Remarks on code availability)

Version 1:

Reviewer comments:

Referee #1

(Remarks to the Author)
We have appreciated the very detailed rebuttal of our review prepared by the authors. We found it easy to follow. The authors have taken into account all our questions. They have also made all the additional tests that we suggested. Therefore, we are happy with the revised version of the manuscript, and we recommend its publication in Nature.

While Zhang and Dumberry are confident of their explanation of multidecadal (60 to 70 years) length-of-day changes, they reckon that the description of the mechanism acting on shorter time scales needs refining. Their interpretation of these fast changes as the result of a torque, either topographic or electromagnetic, acting at the core surface removes a contradiction between this work and another recent study. Finally, we find it important that the authors have built a new inner core rotation model based on a cubic spline representation as in the original seismological study.

(Remarks on code availability)
Yes. We were able to install and run the code. Detailed instructions are provided in the README file.

Referee #2

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

(Remarks on code availability)
The code can be easily installed and executed, it is well described by README file.

Referee #3

(Remarks to the Author)
I find the responses to my comments completely resolve the issue I had over a possible overclaim to significance - the language now gives a very fair reflection of what I think has been shown. The responses to the other reviewers also seem appropriate - I can't help wishing that this discussion becomes a paper (or even a book....) in its own right. I fully support publication as is. [redacted]

(Remarks on code availability)

Gravitational torque drives multidecadal variations in length of day

Corresponding Authors: Huifeng Zhang and Prof Mathieu Dumberry

This file contains all reviewer reports in order by version.

[redacted]

Response to referees

We are grateful to three anonymous reviewers for their constructive comments [redacted] We have considered all their suggestions and revised our manuscript accordingly. This document provides our point-by-point responses to the questions and comments raised by reviewers. The original comments and questions are shown in **blue**, and our responses are in **black**. Text quoted from the revised manuscript are shown in ***italics***, with line numbers referring to the clean version. Please also refer to the version with changes highlighted in red for a clear record of the modifications.

Reviewers #1 and #2 (Remarks to the Author):

Summary of the key results

Zhang and Dumberry rely on the recent estimation of the evolution of the angular position of the Earth's inner core with respect to the mantle, from 1964 to 2019, to investigate the mechanism responsible for the exchanges of angular momentum between the Earth's core and mantle.
This remains very much an open question. They consider the electromagnetic and topographic torque between the fluid core and the mantle on the one hand and the gravitational torque between the solid inner core and the mantle on the other hand. The electromagnetic and topographic torques are estimated from a published model of the fluid velocity at the core surface.
The authors infer the gravitational torque from the position of the inner core relative to the mantle and find that only this torque has the right sign (at low frequency) to account for the time evolution of the mantle angular momentum (length-of-day data).

Originality and significance

The study is original and puts together, for the first time, the state-of-art knowledge on topographic torques, geomagnetic core-flow models and recent seismic data to build a comprehensive torque budget and explain changes in the length of day.
To put the study into perspective, we find it noteworthy that exchanges of angular momentum between the core and the mantle dominate the changes in the angular momentum of the solid Earth arising from climate change. As a result, the current incomplete modelling of core-mantle angular momentum exchanges has prevented identifying a fingerprint of climate change in LOD data (see e.g. Agnew, Nature, 628, 333–336, 2024).

We thank the reviewers for their kind words, for their general support of our study, for their thorough review, and for the feedback and suggestions they offer. The questions on the differences and similarities of our results with those of J. Aubert (now published in jSEDI 2026, [1]) allowed us to sharpen our manuscript. Likewise, their comments and suggestions on the torque at shorter (<30 yr) periods have been instrumental in our reappraisal of the implications of our results, as we explain below. We believe that our study has been strengthened significantly in light of these changes, and we are grateful for the reviewers for all their comments and suggestions.

Data and methodology: validity of approach, quality of data, quality of presentation

The manuscript is well written with high-quality figures. The supplementary material file gives a wealth of information that answers many of our questions.

We appreciate this positive comment.

The study is restricted to low-frequency variations, which correspond to a (60–70)-year period, since the available seismic data (angular position of the inner core) has limited time resolution. This necessarily leaves out the main difficulty affecting the discussion of LOD (Length-of-Day) changes arising from core-mantle interactions: accounting simultaneously for the angular momentum and torque budgets across various frequency ranges.

We fully agree with the reviewers, ultimately, the goal is to find the proper explanation for the torque on the mantle that explains the LOD changes at all timescales, whether it is for the 6 yr period, for decadal (10-30 yr), multi-decadal (60+ yr) or even at centennial and millennial timescales. However, to us, an even greater difficulty has simply been the lack of a successful prediction of the ΔLOD by a torque (or a combination) at any timescale (other than with core flows designed to match the torque). We feel that it is important to emphasize this. We agree that explaining the torque at different frequencies (for a common set of parameters) remains a challenge, and we give further examples of this challenge in our response below. But we also show that the LOD predictions at shorter timescales remain very poor, and that this poor fit is not simply related to the different values of the torque parameters at different timescales. Given these difficulties, our successful prediction for the multidecadal LOD is a significant step forward.

We want to ensure the reviewers that our focus on the multidecadal torque was not designed as a way to avoid the difficulty of also explaining the torque at other timescales. Rather, our chief objective was to demonstrate that a successful prediction of the torque was possible, at least for the multidecadal timescale, a result that had not been previously achieved.

The problem can be illustrated with the example of electromagnetic (EM) coupling. Gillet et al. (120, 3991–4013, 2015) found two different values of the conductance (4×10^8 S and 3×10^9 S) to make the EM torque have an amplitude similar to the observed torque at, respectively, decadal and 6-year periods. Their finding can be explained by noting that core angular momentum is linearly related to the core surface flow $\mathbf{u}$. Therefore, the axial torque that is responsible for the time changes of the core angular momentum is a function of the time derivative of surface core flow $\mathbf{u}$. However, most of the torques that have been discussed in this context are also a function of the velocity $\mathbf{u}$. Therefore, the torques are related simultaneously to the surface velocity and its time derivative. As a result, the parameters that ensure a good fit at one frequency do not yield a good match to the data at another frequency (or for the more general multi-frequency signal).

Indeed, this has been one of the main challenge, if only one torque mechanism operates. But when two or more mechanisms operate, as we suggest in our study, this challenge can be overcome. Regarding the higher values of the mantle conductance required to explain the 6-yr period, we present examples of such calculations further below. These calculations show that explaining the 6-yr LOD changes is more complicated than simply increasing the amplitude of the torque parameters.

The solution to this puzzle has been to assume that at least two torques cancel each other out, particularly at low frequencies. Aubert (2026, submitted to the open-access journal jSEDI) has just discussed geodynamo simulations calculated with both gravitational and EM torque.
He finds that the two torques cancel each other out at low frequency, also envisioned by Zhang and Dumberry in their manuscript.
For short periods, however, the EM torque acts alone (see his figure 11c).
This contrasts with the scenario of Zhang and Dumberry, who argue that the gravitational torque is already twice large as the EM torque at a 70-year period and acts alone at short periods.

The results of Aubert (jSEDI 2026) are broadly in agreement with the scenario that we propose for the multidecadal timescale. That is, the multidecadal ΔLOD are driven by the gravitational torque, with the EM torque mostly resisting these changes and cancelling a part of the gravitational torque. (The topographic torque was not included in Aubert's study.) In Aubert's results, the gravitational and EM torques tend to cancel each other more exactly than in our best-fit solution. Allowing for the error bars of our retrieved parameters, we find many solutions for which the gravitational and CMB torques also cancel each other more exactly (this is one of the torque regimes that we find, see Figure S7). Given the uncertainty in the torque parameters (as they are sensitive to the inner core and core flow models), it is difficult to compute a precise factor by which the gravitational torque dominates. Therefore, we have revised our text, and no longer argue that the gravitational torque dominates by a factor close to 2. We simply emphasize that our solutions suggest a gravitational and a CMB torque that are close in magnitude.

We have revised our interpretation of the leading order torque at shorter timescale (period < 30 yr). See further below. In this revised interpretation, it is the CMB torque that dominates at short period.

Regarding the relative amplitude of the multidecadal gravitational and EM torques (or CMB torque in general), whether they tend to cancel more exactly or whether one torque dominates more strongly (and which) ultimately depends on the torque parameters. For the parameters chosen by Aubert (2026) ($G = \Delta\sigma_m = 2 \times 10^8$ S and $\Gamma\tau = 10^{21}$ N m yr, either with $\tau \to 0$ (his model U), or with $\tau = 20$ yr (his model G)) the gravitational and EM torques track one another and cancel each other nearly exactly at multidecadal periods. However, he also shows (for example his case GL with $\Delta\sigma_m = 2 \times 10^7$ S, $\Gamma\tau = 10^{21}$ N m yr, $\tau = 20$ yr) that for other parameters one torque can dominate (the gravitational torque in this case).

To further illustrate this (and further support our results), we have carried our own numerical simulations (using the code MagIC) with an array of different EM and gravitational coupling strengths. We are preparing a manuscript describing our results and we wanted to share with the reviewers a case that resembles our best-fit solution. Note that these results are still preliminary and our analyses incomplete; as such we prefer not to include these examples in Supplementary Information, although we would be happy to if the reviewers insist that they should be included.

Fig. R1 shows how the EM (Lorentz) and gravitational torques vary with time for one of our model runs (using $\mathrm{Ra} = 3.5 \times 10^9$, $\mathrm{Ek} = 3 \times 10^{-5}$, $\mathrm{Pm} = 2.5$, stress-free mechanical and fixed-flux thermal boundary conditions.) The EM and gravitational coupling strengths, when scaled to dimensional values, corresponds to a mantle conductance of 10^8 S, and a product $\Gamma\tau = 1.4 \times 10^{20}$ N m yr, matching our best-fit parameters in the EM coupling scenario. This particular dynamo solution is Earth-like, with a dipole to non-dipole ratio of $f_{\mathrm{dip}} = 0.64$, and magnetic Reynolds number $\mathrm{Rm} = 919$, although the ratio of magnetic to kinetic energy $E_{\mathrm{mag}}/E_{\mathrm{kin}} = 1.9$ is much lower than the solution of Aubert (2026). Focusing on specific time windows (see Fig. R1c), we can find times when the torque histories are very similar to those that we retrieved in our study: the gravitational torque is larger than the EM torque, though of similar magnitude, and the two torques are out of phase.

In our study, we are approaching this question based on observations. We do not apriori select torque parameters (except for illustrating cases as in Fig. 1) but seek to find those that provide the best fit to the observed ΔLOD. Our best-fit solution suggests a dominance of the gravitational torque, though broadly of the same magnitude as the CMB torque.

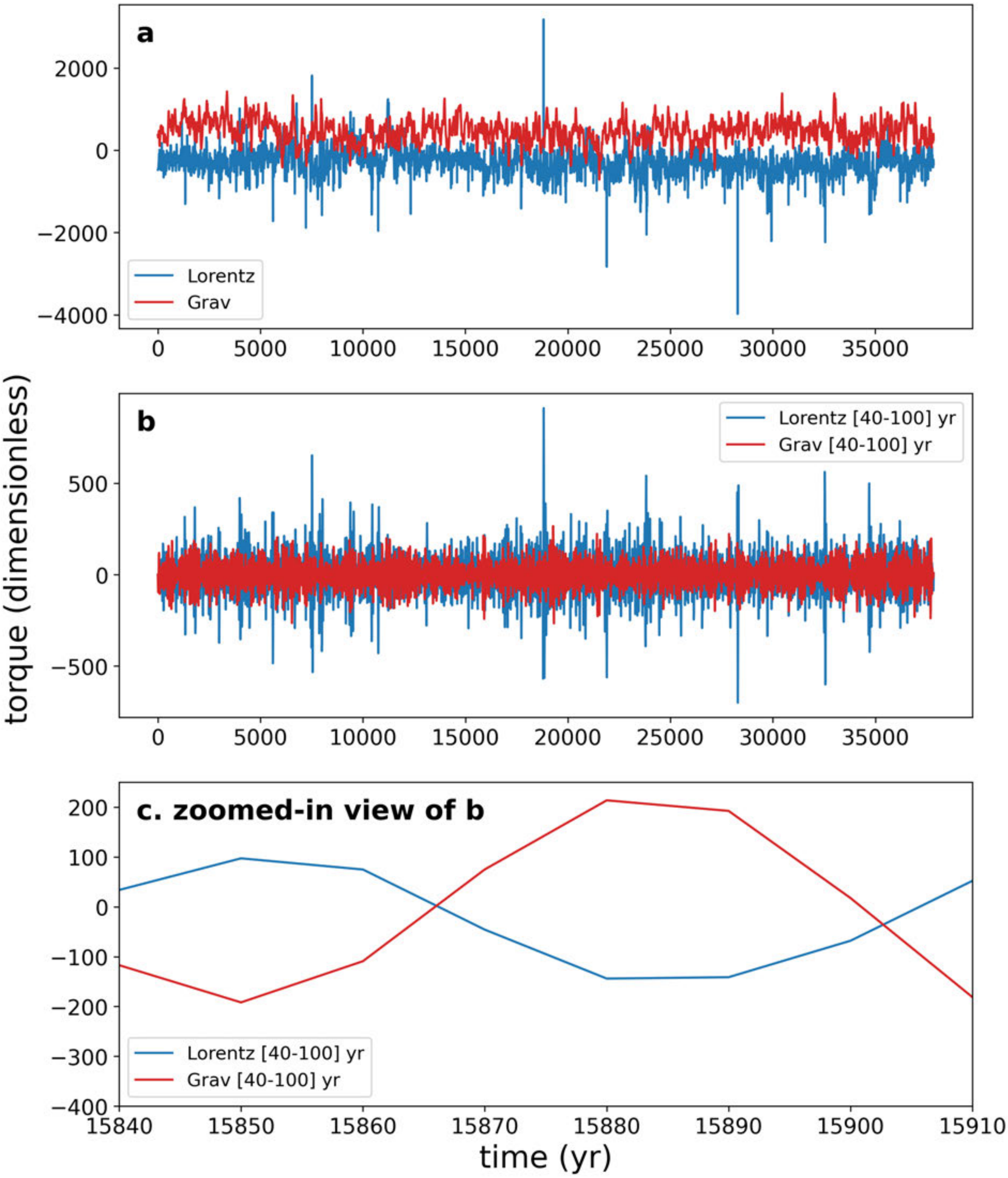

Figure R1: The evolution of the EM (Lorentz, blue) and gravitational torque (red) on the mantle in one numerical model of the dynamo using MagIC. The lower panels (**b** and **c**) show the torques after applying a band-pass filter (40-100 yr).

To some extent, a study restricted to one frequency range eschews the main difficulty. We propose below an exercise to partially address this issue. Also, we find that it would be useful to discuss why the ordering of the torque is different in the study of Aubert (and in the previous work of Schwaiger, 2024, reference 52 of the main text), based on geodynamo simulations and in the work reported in the submitted manuscript.

We thank the reviewers for this suggestion. We have carried out the exercise that they suggest (discussed below) which has allowed us to sharpen our discussion of the torque at shorter timescales.

Regarding the ordering of the torque, Aubert (2026, [1]) explains in his study that the multi-decadal torque is driven by the gravitational torque and resisted by the EM torque at the CMB, exactly in line with our results. We have confirmed this by personal communication with J. Aubert, but this can be verified simply by looking at Figure 11 of his study. When the multidecadal gravitational torque increases (accelerating the mantle, and making the days shorter), the LOD is indeed observed to decrease. This is also the case in the study of Schwaiger et al (2024, [2]), although they only show the time evolution of the EM torque. When the EM torque becomes more negative (see their Fig. 8), implying a more westward torque on the mantle if the EM torque dominates, this should slow down the mantle and increase in LOD. Yet, this corresponds to a *decrease* in LOD (see their Fig. 7). This implies that it is also the gravitational torque that is driving the multidecadal LOD changes in the study of Schwaiger et al (2024) (we had written, incorrectly, in Methods that the EM torque dominated the LOD changes in their study and we have now removed this statement.)

There are many intermediary steps from the primary seismic data (temporal changes of seismic waves for repeated earthquakes) to the evolution of the angular position of the inner core. The seismologists had to calculate a model for the lateral variations of the wave speeds in the region sampled by the seismic rays (as explained in Yang and Song, JGR, 125, e2019JB018652, 2020). They also discard events associated with too fast variations that they deem unphysical.
Ultimately, it would be beneficial to use the primary data (changes of DF travel times) in order to better control uncertainty propagation.

We agree that, ideally, it would be better to use seismic data directly rather than a model based on these data. However, this presents a challenge to us as we are not seismologists. Given a set of available observations (say, DF times), we would not be in a position to fully grasp the limitations of the data that we are using. Hence, we do what is second best: we take a model of inner core rotation fluctuation build upon these seismic observations. This inner core rotation model has been recently confirmed (between 1982–2024) using a different technique (Wu et al, GRL 2026 [3], now included in our list of references). We likewise use a core flow model, not the primary data (magnetic field observations) upon which this model is constructed. These are published models, and from our point of view, these models can be used to test dynamical scenarios. We recognize that both the inner core rotation and core flow models necessarily contain assumptions, and we have tried to be mindful of this in the revised discussion of our results.

Appropriate use of statistics and treatment of uncertainties

The authors performed a Bayesian inversion of the different parameters (mantle conductance, inner core viscous relaxation time, ...) that enter the torque values. They account for the uncertainties in the surface flow (as given by the researchers who calculated this flow) and in the angular position of the inner core. Figure S4 compares the model of the authors for the inner core angular position and its associated error to the initial model (by Yang and Song, 2023) on which their study is based. It appears that the model of the authors underestimates the errors, particularly before 1980. It is not clear why they have not simply used the original model (based on a spline fit) as input.

Indeed, the errors before 1980 were underestimated in our original inner core rotation model (for the angle $\varphi(t)$). This model was based on a simple fit of two sinusoidal curves. The advantage of this strategy was 1) that it was straightforward to determined the initial condition on α (analytical solution of $d\alpha/dt = d\varphi/dt - \alpha/\tau$ based on the first of these sine functions for ϕ_i) and 2) that it allowed for easily testing the sensitivity of the phase and amplitude of the time-history of φ on our torque parameters.

We have built a new inner core rotation model based on a cubic spline representation. We use seven knots with central values of φ taken directly from the Yang & Song (2023) model (specifically, their model based on the SSI-COL path for knots placed at 1964, 1972 and 1984, and their better resolved model based on 6 global paths for knots placed at 2000, 2010, 2015 and 2019). For each sample, a set of random errors $\Delta\phi$ are added at each knot. The errors in the model of Yang & Song based on the 6 paths are small, but we take a conservative approach and use the errors associated with the SSI-COL path model. We now draw time-history models of φ based on this spline representation. Our procedure is explained in detail in the revised version of section 3 of Supplementary Information.

Fig. R2a shows the $\varphi(t)$ model of Yang & Song (2023,[4]) with associated errors. Fig. R2b shows the reference spline curve and the 95% CI based on 10^4 samples from our new inner core rotation model. The error on $\varphi(t)$ is now better reproducing the error prior to 1980 of the model by Yang & Song. Fig. R2c shows the resulting time-history of $\alpha(t)$ from this new model for an assumed inner core relaxation time of $\tau = 6$ yr.

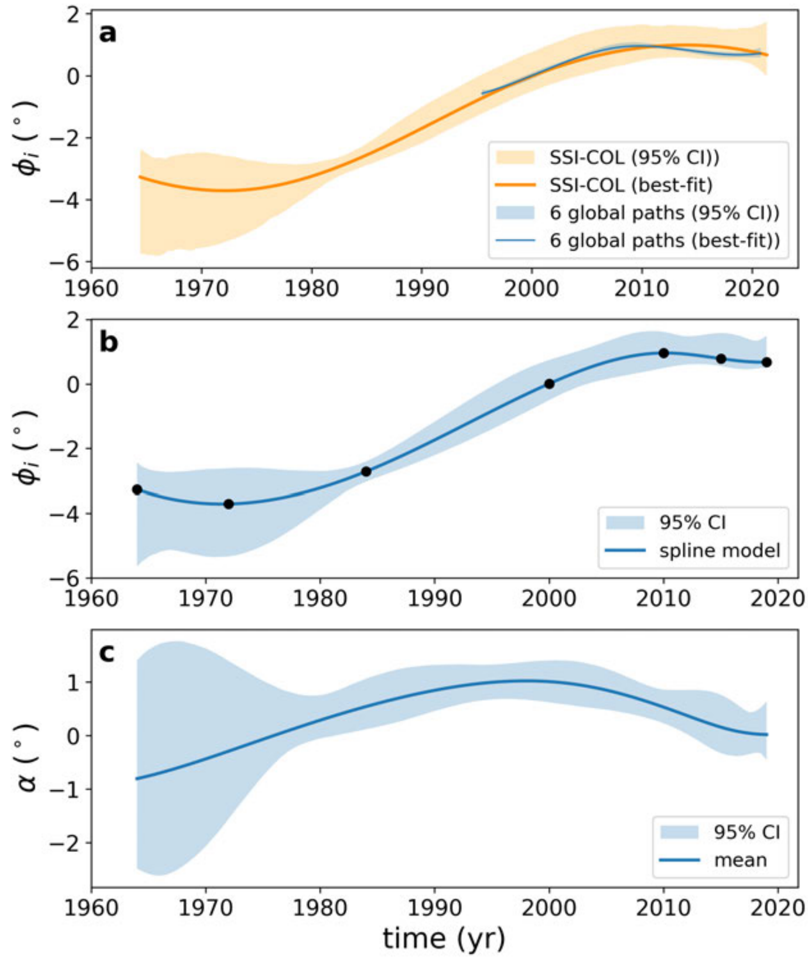

Figure R2: (a) The seismic model of inner core rotation of Yang & Song (2023,[4]). (b) The cubic spline model (blue solid line, knots shown by the black dots) of our model of $\varphi(t)$ and the 95% CI (shaded envelope) based on 10^4 realizations with added errors. (c) The misalignment angle α of the degree 2 order 2 ICB topography, assuming $\tau = 6.0$ yr.

Our results based on this new inner core rotation model are not substantially different compared to our previous results. Tables R1 and R2 summarize our old and new results, respectively. The maximum a posterior estimate (MAP) for Γ has increased slightly for both EM and topographic coupling. The MAP for τ has increased slightly (from 8.8 to 10.2 yr) for EM coupling, and also increased slightly for topographic coupling (from 7.2 to 7.8 yr). The MAP for K_{em} has increased from 0.58 to 1.08, and the MAP for K_{top} has increased from 0.19 to 0.23. The predicted torque histories (revised Fig. 3 of the main text) are not changed significantly (it is difficult to detect a change by eye). The histograms for the torque parameters (revised Fig. 2) are also similar to our previous results.

A consequence of allowing for a larger error in φ prior to 1980 is that the proportion of highly successful models (LOD misfit < 0.2 ms) in the number of accepted samples in our MCMC algorithm is now reduced. This is because, for the same torque parameters, there is now a greater chance that the randomly selected φ model results in a poorer fit to the ΔLOD. Since we base our CI on the highly successful models, and to ensure that the posterior distributions from these successful samples are better highlighted, we have changed Fig. 2 so that it only shows the distribution based on the best samples (the figure showing all samples has been moved to Fig. S5 of Supplementary Information). Likewise, we have modified Figs. 3a and 3d to focus on the samples that provide the best LOD fit.

Table R1: Old Results. The range of the 95% CI, median and maximum a posteriori (MAP) estimates of the torque parameters that provide the best fit to the multidecadal ΔLOD, when the CMB torque is caused by EM coupling (EM, $K_{\text{cmb}} = K_{\text{em}}$) and by topographic coupling (TOP, $K_{\text{cmb}} = K_{\text{top}}$).

	High Prob. Samples		
Case	$\mathbf{\Gamma}$ (10^{19} N m)	$\boldsymbol{\tau}$ (yr)	$\boldsymbol{K}_{\mathbf{cmb}}$
EM	0.9–2.3	3.1–22.4	0.04-1.70
EM (median)	1.2	8.6	0.48
EM (MAP estimate)	1.2	8.8	0.58
TOP	0.9–3.9	1.7–16.3	0.01–0.55
TOP (median)	1.3	6.5	0.15
TOP (MAP estimate)	1.2	7.2	0.19

Table R2: New Results. The range of the 95% CI, median and maximum a posteriori (MAP) estimates of the torque parameters that provide the best fit to the multidecadal ΔLOD, when the CMB torque is caused by EM coupling (EM, $K_{\text{cmb}} = K_{\text{em}}$) and by topographic coupling (TOP, $K_{\text{cmb}} = K_{\text{top}}$).

	High Prob. Samples		
Case	$\mathbf{\Gamma}$ (10^{19} N m)	$\boldsymbol{\tau}$ (yr)	$\boldsymbol{K}_{\mathbf{cmb}}$
EM	0.6–2.2	4.1–30.5	0.05-2.89
EM (median)	1.3	10.5	0.78
EM (MAP estimate)	1.4	10.2	1.08
TOP	0.6–4.2	1.7–21.5	0.01–0.83
TOP (median)	1.5	6.8	0.20
TOP (MAP estimate)	1.5	7.8	0.23

According to Schwaiger et al. (2024), the unresolved part of the radial magnetic field at the core-mantle boundary yields a significant part of the EM torque. Ignoring this contribution may lead to overestimating the mantle conductance. We recommend accounting for this effect when estimating the mean value of the conductance and its uncertainties.

This is a very good comment. We have added a note on this in the discussion (lines 174–176). We have also added a sentence in the Methods (lines 391–394).

Lines 174–176: If the CMB torque is dominated by EM coupling, our best-fit mantle conductance is $G = 1.08 \times 10^8$ *S, although it may be lower if the short-wavelength (and unresolved) CMB magnetic field contributes to a significant part of the EM torque.*

Lines 391–394: Synthetic tests also suggest that the small length scales of the CMB magnetic field ($l > 13$)*, inaccessible from observations, may contribute to about a third of the total EM torque. Our reconstruction of the EM torque may then be underestimated, contributing to an overestimate in our recovered value of* G.

The amplitude of the gravitational torque depends on the density contrast at the inner core boundary. The authors take it to be $\mathbf{570\,kg\,m^{-3}}$ (from the Earth's reference model PREM). It seems, however, that there is no consensus yet on its value. Cao and Romanowicz (GJI, 157, 1146–1151, 2004) obtained density contrast in the range $\mathbf{600 - 900\,kg\,m^{-3}}$. Tkalcic et al. (GJI, 179, 425-443, 2009) found that the density contrast cannot exceed $\mathbf{1100 - 1200\,kg\,m^{-3}}$ and present observations pointing to a contrast as low as $\mathbf{200 - 300\,kg\,m^{-3}}$. This clearly gives an additional source of uncertainty on the amplitude of the geoid reported in line 155 of the main text.

This is a very good point and we thank the reviewers for bringing this to our attention. We have included an added comment in the main text (lines 158–162) and added some text of the uncertainty in the density contrast at the ICB (including the suggested references) in the Methods section (lines 457–460).

Lines 158–162: Our best-fit range of $\Gamma = [0.6 - 4.2] \times 10^{19}\,\mathrm{N\,m}$ *(when combining the results of EM and topographic coupling) is lower than a previous estimate and implies a peak-to-peak* Y_2^2 *CMB geoid topography of* $\sim 31 - 83$ *m and a corresponding* Y_2^2 *ICB topography of* $\sim 11 - 29$ *m (although both estimates depend on the density contrast at the ICB, see Methods).*

Lines 457–460: To convert Γ *to* h_2^2*, we use* ρ_i *= 12730 kg* m^{-3} *and* ρ_f *= 12160 kg* m^{-3} *based on PREM. However, the density contrast at the ICB remains not well constrained, and may be as high as* $600 - 900$ *kg* m^{-3} *or as low as* $200 - 300$ *kg* m^{-3}*. This uncertainty on* ρ_i *and* ρ_f *maps to an uncertainty on the CMB geoid inferred from* Γ.

Conclusions: robustness, validity, reliability

The robustness of the proposed conclusion is dictated by the quality of the seismic data and by the focus on the low-frequency signal.
These intrinsic limits of the study are clear from the last section of the paper; however, they are less present in the abstract.

We have changed a few sentences of the abstract to better reflect that: 1) our study is focused on the multidecadal LOD changes; and 2) that the inferences we draw from our torque-histories rest on the models of inner core rotation and core flow that we have used.

Suggested improvements: experiments, data for possible revision

As discussed above, an important test of any model for the axial torques acting at the core-mantle boundary (CMB) is to find agreement between predictions and observations in different frequency ranges. The lack of seismic data with high time solution makes it difficult to attempt this test for the scenario of Zhang and Dumberry.
Therefore, we recommend instead that the authors calculate a time series of angular positions of the inner core relative to the mantle that would account for the observed torque (the gray curve in Figure 1b rather that only the smooth black curve), using their preferred values for the parameters of their model (relaxation time τ, mantle conductance) and the same core surface flow as in their

submitted manuscript. It would indicate whether to expect aliasing in seismic studies.

We thank the reviewer for this comment and suggestion, and for their questions about the shorter timescale ΔLOD. Based on these, combined with the recent results of J. Aubert (jSEDI 2026 [1]), we have changed our view on the torque that is most likely driving the ΔLOD at shorter (period < 30 yr) timescale. In our original manuscript, we speculated that the shorter 10-30 yr LOD signal was also driven by gravitational coupling. This inference was based on the inability of the predicted EM and topographic torques to account for the required short period torque. However, we had not fully considered the dynamical implications of the torque parameters that we derived. We have changed our view on the torque that dominates the shorter timescale LOD, and now argue that it is the CMB torque that must be primarily driving these variations.

We present the arguments supporting our revised viewpoint in a new section (section 13) in Supplementary Information. One part of the argument rests on the fact that the gravitational torque should be ineffective at periods shorter than τ. We include a new figure (Figure S24, reproduced below as Figure R3) to illustrate this. The top panel shows a synthetic model of the fluctuations in inner core rotation rate Ω_i generated by a Gauss-Markov process. The bottom panel shows a prediction of α (the longitudinal angle of the long equatorial axis of the ICB topography) from

$$\frac{d\alpha}{dt} = \Omega_i - \frac{\alpha}{\tau}\,, \tag{R1}$$

with different choices of τ. For $\tau = 10$ yr, all α fluctuations with a period of 10 yr or shorter are effectively filtered out. Fluctuations in the period range of 10-30 yr remain present but have a weak amplitude. The gravitational torque, which involves the product $\Gamma\alpha$, is thus ineffective at periods of 10 yr and shorter. Consequently, our results imply that it must be the torque at the CMB (EM and/or topographic) that drives the short period LOD changes, including the 6 yr LOD signal. This is in line with the results of Aubert (2026 [1], see his Figure 11), although he uses $\tau = 20$ yr.

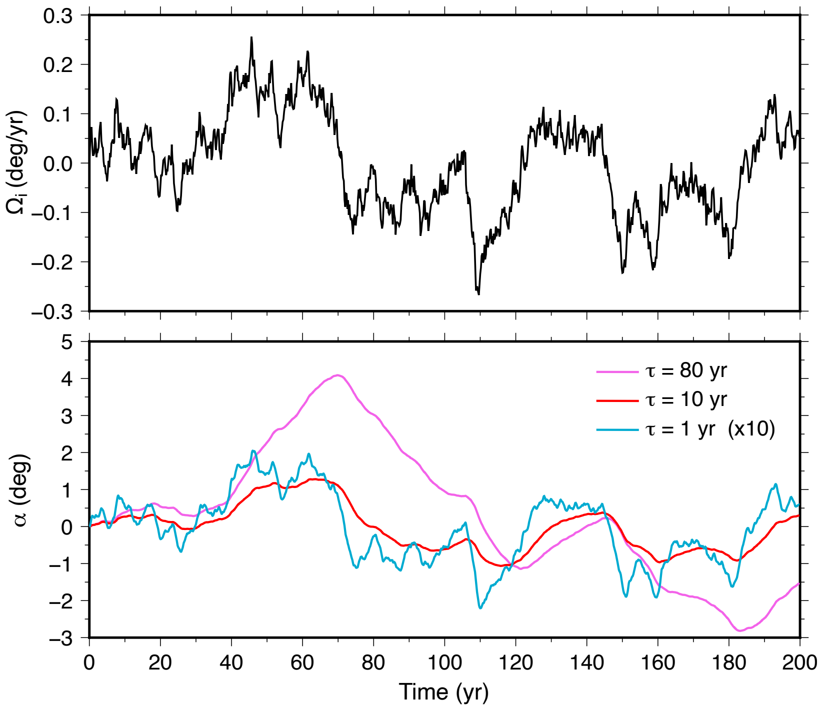

Figure R3: (Top) Synthetic model of fluctuations of Ω_i and (bottom) resulting fluctuations of α for different choices of τ. The results for $\tau = 1$ yr have been multiplied by a factor 10 to improve visualization.

The suggestion of the reviewers – to construct a time series of angular positions of the inner core based on the short period torque caused by gravitational coupling – is a really good idea and we have attempted this. This reconstruction is shown in Figure S26 in Supplementary Information, reproduced below (Figure R4). The peak-to-peak changes in the predicted ϕ_i over a period of 10 yr are approximately 5°. We also plot on Figure R4 the seismically inferred ϕ_i from Yang & Song (2023)[4] which we have used in our study and from the more recent results of Wu et al (2026)[3]. The short period changes in the predicted ϕ_i generally fit within the scatter in the data from this latter study (although barely), and on this ground one could argue that the predicted ϕ_i is not inconsistent with the data. However, the data contain no clear hint of a short period signal that resembles this ϕ_i prediction. Even more problematic is the amplitude of $\Omega_i = d\phi_i/dt$ implied by this prediction of ϕ_i, which is shown in Figure S27 (and Figure R5 below). Changes in Ω_i include fluctuations as large as 2 to 3 °/yr over a few years. This corresponds to rotational speeds at the equator of the inner core of 40 to 60 km/yr, two orders of magnitude larger than the typical changes in zonal core flow at the CMB at subdecadal timescales (e.g. Gillet et al, 2010 [5]). The inefficiency of the gravitational torque at short period, together with the very large Ω_i that it requires, support the view that it is the torque at the CMB that should drive the short period LOD changes.

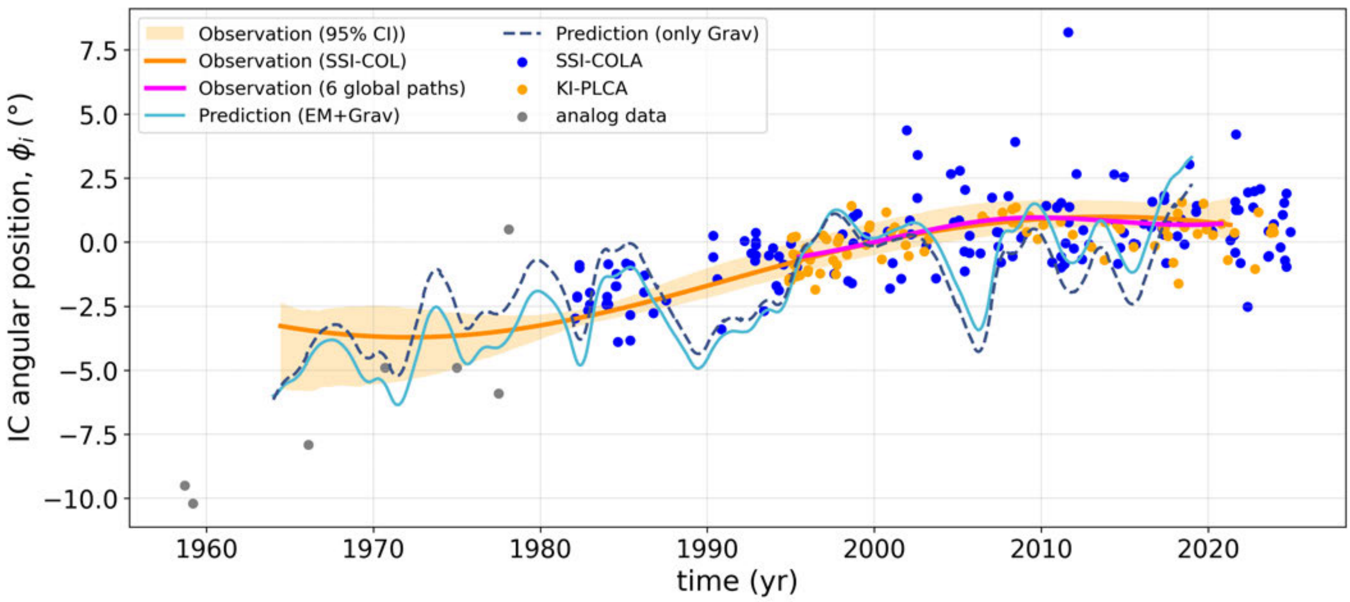

Figure R4: Comparison between the seismic reconstruction of the inner-core angular position from Wu et al. (2026) [3] (orange and magenta dots) and Yang & Song (2023) [4] (orange and magenta curves, with shaded envelopes) and the prediction based on the assumption that the gravitational torque (with our best-fit parameters: $\Gamma = 1.4 \times 10^{19}$ Nm and $\tau = 10.2$ yr) explains the short period (< 30 yr) ΔLOD. The dashed curve assumes no contribution from the CMB torque; the light blue curve is the prediction after removing the EM torque (based on our best-fit conductance $G = 1.08 \times 10^8$ S).

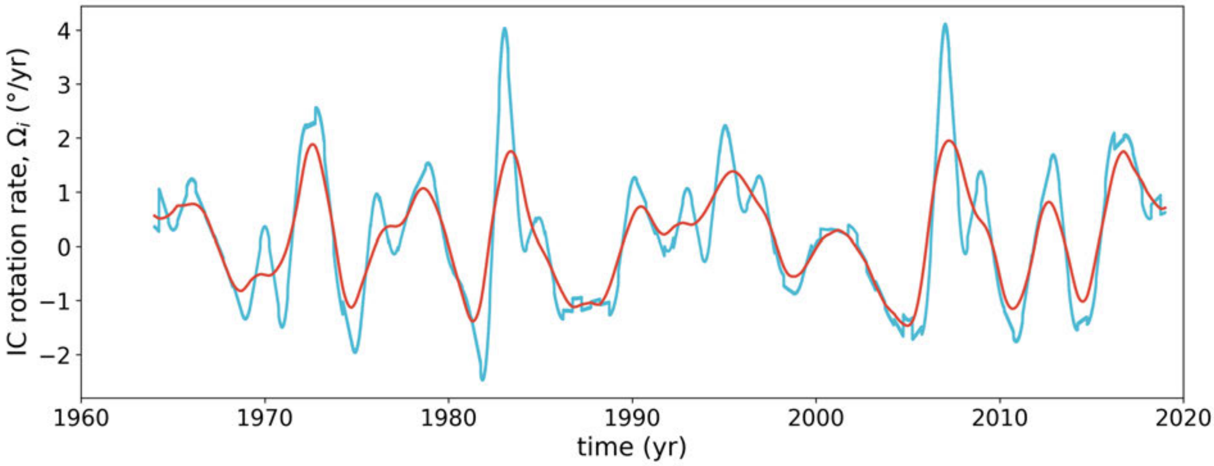

Figure R5: Changes in the inner-core rotation rate Ω_i (blue) implied by the fluctuations in ϕ_i (blue curve in Fig. R4). The orange curve corresponds to a 2-year running average.

However, our study does not provide a complete resolution of the short timescale ΔLOD. This is because the prediction of the EM and topographic torques give a poor fit to the required torque at $10-30$ yr periods. This is illustrated in our new Figure S25 (Figure R6 included below) for the EM prediction (which is equivalent to Fig 1b of the main text except for a slightly different conductance of 1.08×10^8 S, our best-fit value). The reason for this poor fit remains unknown. It could be caused by the limited resolution in the core flow model (the LOD fit in Figure 5 of Finlay et al 2023 [6] is not as good for the 10-30 yr signal as it is for the multidecadal LOD signal). It could also indicate that models of the CMB torque are incomplete.

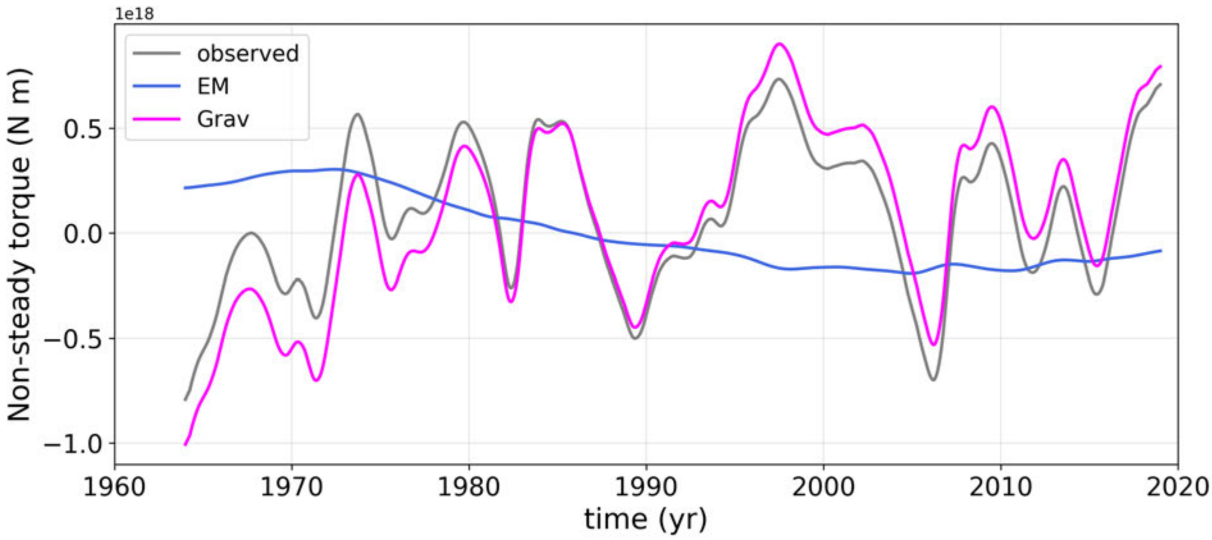

Figure R6: The gravitational torque (magenta) required to explain the torque on the mantle that explains the observed ΔLOD (grey) after removing the EM torque (blue; with conductance $G = 1.08 \times 10^8$ S).

As the reviewers point out above, a higher mantle conductance than at decadal timescale is required for the EM torque to account for the 6 yr LOD signal (Gillet et al 2017 [7]). This is also the case in the dynamo model of J. Aubert (jSEDI 2026 [1]). Figure R7 below shows our reconstruction of the 6 yr LOD signal. In panel **a**, we show a comparison between the observed LOD and that predicted from the core flow model of Istas et al (2023) [8]. Both the LOD signal and the core flow coefficients are band-pass filtered between 4.5 and 7.5 yr. The agreement is very good. In panels

b and **c** we show the prediction of the EM and topographic torques, respectively, also band-pass filtered to focus on the 6 yr signal. A mantle conductance of 2×10^9 S ($K_{\rm em} = 20$) and a topographic coupling factor of $K_{\rm top} = 1.0$ are required to match the amplitude of torque, which are both higher than our best-fit estimates ($K_{\rm em} = 1.08$ and $K_{\rm top} = 0.23$). However, neither of these predictions offer a good match to the required torque, even though panel **a** confirms that the core flow model is sufficiently well resolved to explain the core angular momentum changes. To us, this is indicative that the coupling models at the CMB may be incomplete. And if so, the explanation for the 6 yr torque does not necessarily imply larger coupling factors ($K_{\rm em}$ and $K_{\rm top}$) than those that we find.

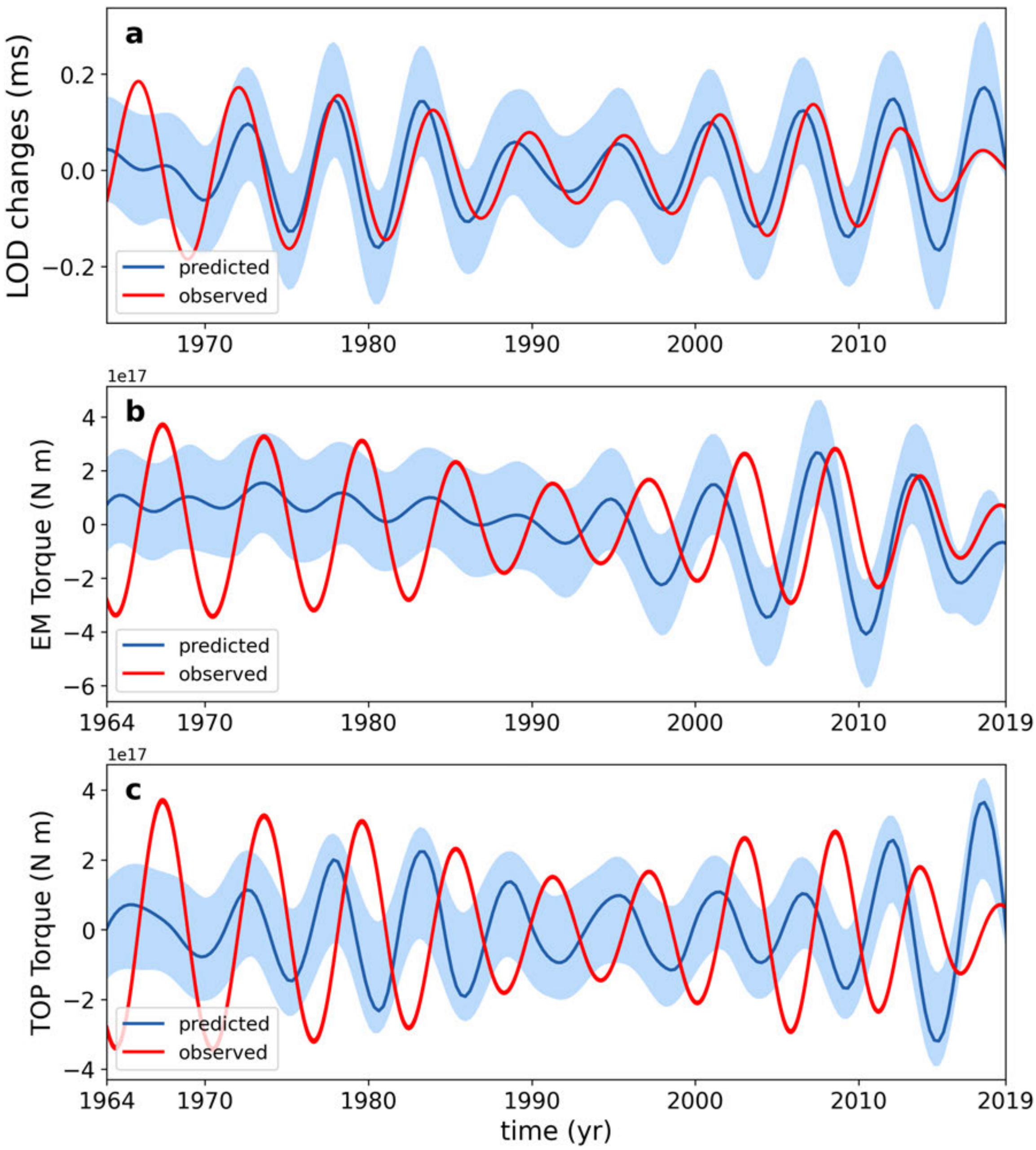

Figure R7: (a) Observed 6 yr LOD signal (red line) and prediction (blue line) from core flows. (b) Comparison between the required torque on the mantle (red line) and the EM torque computed from core flows (blue line) for a mantle conductance of 2×10^9 S. (c) Comparison between the required torque on the mantle (red line) and the topographic torque computed from core flows (blue line) for a coupling factor of $K_{\rm top} = 1.0$. In each panel, the blue shaded envelope denotes the one standard deviation range of the core flow prediction.

We do not have the answer to these questions, but Figures R6 and R7 highlight that the precise nature of the torque at a decadal (10-30 yr) timescale and for the 6 yr signal remains unclear. Our focus is on the multidecadal ΔLOD, so a complete resolution to the torque at shorter period is outside the scope of our study. The question marks that remain regarding the shorter period LOD do not alter our general conclusion, that it is the gravitational torque that drives the multidecadal ΔLOD.

We prefer not to include Figure R7 in our Supplementary Information because we are preparing a separate paper on the core-mantle torque for the 6 yr LOD signal. However, if the reviewers and editor insist that we do, we will comply with this request.

In addition, it needs to be verified that the value of the inner core relaxation time τ estimated in the study is adequate for the gravitational torque to be effective at a 6-year period, where significant core-mantle coupling has been found.

As we explain in our answers above, our view of the dominant torque driving the short ΔLOD has changed. Indeed, for our estimate of $\tau \sim 10$ yr, the gravitational torque becomes ineffective at periods shorter than 10 years.

Finally, the discussion of the steady part of the torques would give the average value of the misalignment between the inner core and the mantle.

This is a good suggestion, and we now include a short discussion in our closing paragraph. The mathematical details of the steady state balance are presented in an added section in Methods. From this steady state balance, and the best-fit values of K_{cmb}, Γ and τ that we retrieve, we calculate a fixed misalignment angle of $\sim 1.2°$ and a steady bulk inner core rotation rate of $\sim 0.12°$/yr. This is very similar to (and consistent with) the rate of $\sim 0.1°$/yr that J. Aubert (jSEDI 2026) finds in his numerical model at Earth-like conditions.

It would be nice to verify that the conclusions are not dependent on the chosen velocity model. For example, the velocity model of Bärenzung et al. (JGR, 123, 4539–4560, 2018) accounts well also for the time changes in the core angular momentum, although it ends in 2014. Its coefficients (and their variance) are publicly available.

This is a good suggestion, and we have now included additional tests using different core flow models in the Supplementary Information (new section 9). Specifically, we use two additional flow models: PNAS2022 (from ref. [9]) and GJI2019 (from ref. [10]). Each of these published models cover a the same timespan as the flow model that we have used for our main results (GJI2023, from ref. [8]), they have the same temporal resolution (1-yr time interval), and we also have access to a large number of model realizations. The difference in the retrieved torque parameters for EM and topographic coupling are shown below in Tables R3 and R4. These comparisons show that the exact values of the MAP and 95% CI of the torque parameters that we retrieve from our MCMC algorithm are sensitive to the choice of core flow models. Nevertheless, the torque parameters that we retrieve are robust in the sense that the MAP estimates do not generally change by more than 30% and the posterior distributions for different flow models overlap to a large degree (see Fig. S14). We have included in our main text a sentence (line 153) about these tests. We have also adapted our text to reflect that the exact values of the torque parameters remain uncertain, even though their scales is robust (line 155–156). Overall, the tests with these additional flow models do not change our main conclusion, that it is the gravitational torque that drives the multidecadal variations in length of day.

Table R3: The range of the 95% CI and maximum a posteriori (MAP) estimates of the torque parameters from different core flow models that provide the best fit to the multidecadal ΔLOD, when the CMB torque is caused by EM coupling ($K_{\text{cmb}} = K_{\text{em}}$).

	High-probability samples		
core flow model	$\boldsymbol{\Gamma}$ (10^{19} N m)	$\boldsymbol{\tau}$ (yr)	$\boldsymbol{K_{\text{em}}}$
GJI2019	1.2 (0.5 ∼ 3.0)	12.3 (5.0 ∼ 60.3)	1.11 (0.08 ∼ 3.42)
PNAS2022	1.3 (0.6 ∼ 2.1)	10.0 (4.2 ∼ 34.3)	0.91 (0.05 ∼ 2.68)
this study (GJI2023)	1.4 (0.6 ∼ 2.2)	10.2 (4.1 ∼ 30.5)	1.08 (0.05 ∼ 2.89)

Table R4: The range of the 95% CI and maximum a posteriori (MAP) estimates of the torque parameters from different core flow models that provide the best fit to the multidecadal ΔLOD, when the CMB torque is caused by topographic coupling ($K_{\text{cmb}} = K_{\text{top}}$).

	High-probability samples		
core flow model	$\boldsymbol{\Gamma}$ (10^{19} N m)	$\boldsymbol{\tau}$ (yr)	$\boldsymbol{K_{\text{top}}}$
GJI2019	1.1 (0.5 ∼ 2.0)	10.6 (3.7 ∼ 33.3)	0.40 (0.03 ∼ 0.96)
PNAS2022	1.7 (0.6 ∼ 4.3)	5.0 (1.6 ∼ 18.9)	0.24 (0.01 ∼ 0.81)
this study (GJI2023)	1.5 (0.6 ∼ 4.2)	7.8 (1.7 ∼ 21.5)	0.23 (0.01 ∼ 0.83)

We did not include tests with the model of Bärenzung et al (2018, [11]) because of its limited temporal coverage, which ends in 2014, and its relatively sparse temporal resolution, with a 2-yr time interval. Nevertheless, we can show that the predictions of the EM and topographic torque from this flow model (which we refer to as JGR2018) have a similar multidecadal variation as the flow model GJI2023 that we use. The JGR2018 model provides the mean flow coefficients, $\bar{\Psi}_l^m(t)$ and $\bar{\Phi}_l^m(t)$, and its standard deviation over 1900–2014 with time interval of 2 years. We construct 400 realizations of the non-steady EM and topographic torques (see Figure R8) based on normally distributed flow coefficients, $\Psi_l^m \sim \mathcal{N}(\bar{\Psi}_l^m, \sigma_\Psi^2)$ and $\Phi_l^m \sim \mathcal{N}(\bar{\Phi}_l^m, \sigma_\Phi^2)$, and the ensemble solution is from the mean value of these 400 realizations. For comparison, we also include the torque predictions based on GJI2023 in Fig. R9 (our Fig. S3).

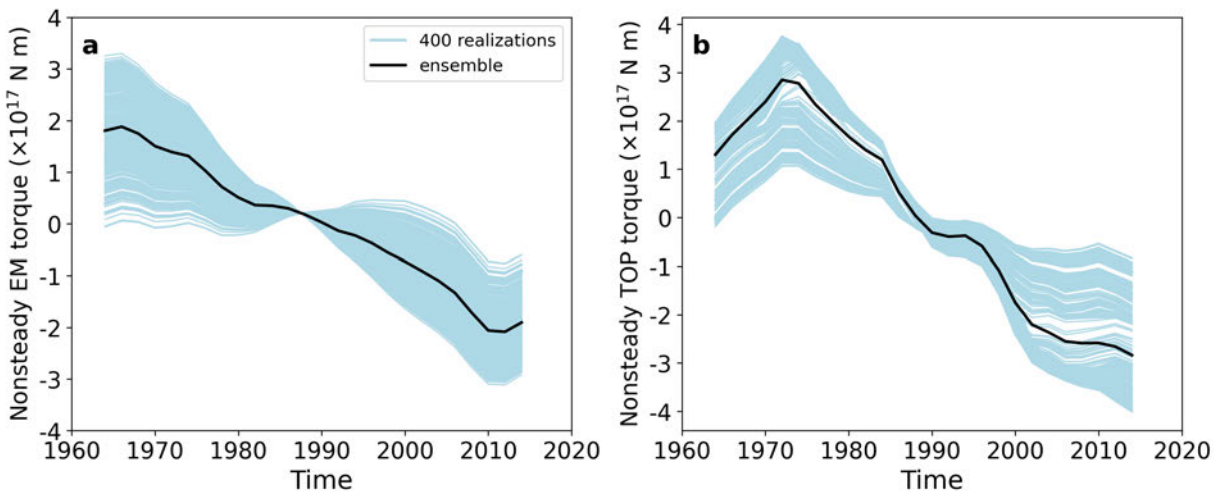

Figure R8: Non-steady (a) EM and (b) topographic torques between 1964-2014 for 400 realizations of the JGR2018 flow model, for $K_{\text{em}} = 1$ and $K_{\text{top}} = 1$, respectively.

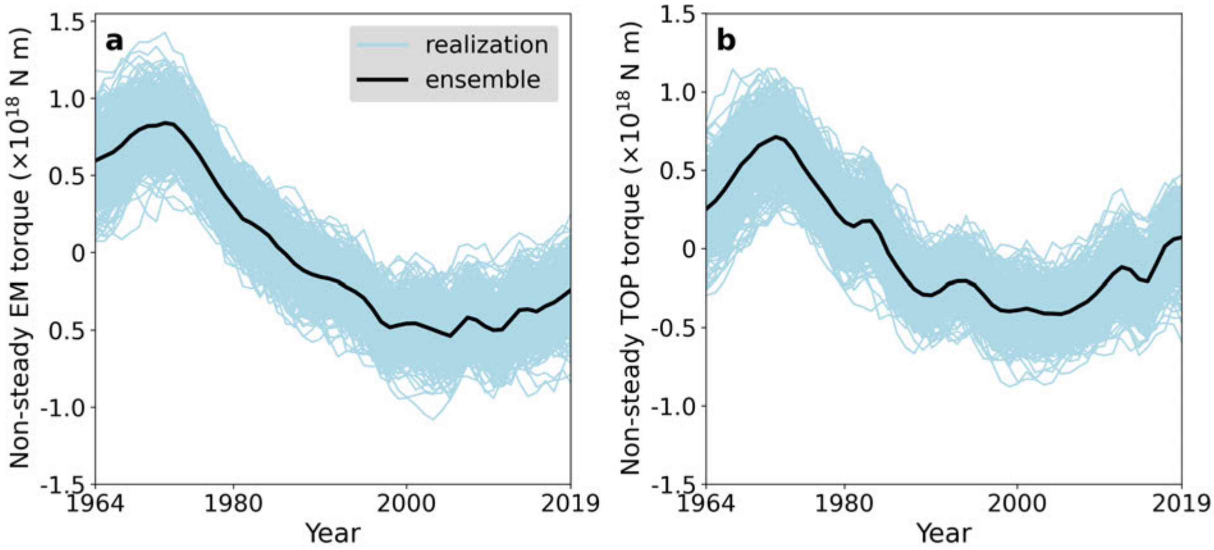

Figure R9: Non-steady (a) EM and (b) topographic torques between 1964-2019 for the 400 realizations of the GJI2023 flow model, for $K_{\rm em} = 3$ and $K_{\rm top} = 1$, respectively.

The general multidecadal torque predictions from JGR2018 differ from those of GJI2023, although they suggest a general downward trend, just as the GJI2023 model does. This implies that the multidecadal ΔLOD phase that these models predict is still generally opposite to that observed. Hence, our conclusion that it is the gravitational torque that drives the ΔLOD is not affected. Clearly, given the difference in the torque predictions, the best fit torque parameters would be different based on the JGR2018 flow model. However, because it ends in 2014, model JGR2018 is missing a few crucial years of data and does not recover well the change in the trend of the EM and topographic torques after 2010. Hence, we feel that it is important to use a flow model that extends to at least 2019 to capture this change of trend and to retrieve the best estimates of the torque parameters. By adding caveats statements in our text, we think that we have made clear to the reader that the accuracy of the torque parameters depends on the accuracy of the flow model.

Following other studies, Vidale et al. (Nature Geosciences, 18, 267–272, 2025) argue that both rotation of the inner core and deformation of its surface account for the time variability of seismic waves, from repeated earthquakes, sampling the inner core. The model of Zhang and Dumberry predicts a deformation of the inner core surface. Is it compatible with the analysis of Vidale et al. (and others)?

The answer to this question ultimately depends on the length scale of the inner core heterogeneities. Our best-fit value for Γ suggests a peak-to-peak degree 2 order 2 geoid at the CMB of 31 to 83 m (based on PREM), which implies a peak-to-peak degree 2 order 2 geoid at the ICB (1/3 of that at the CMB) of 11 to 29 m. In our best-fit scenario, the ICB deforms viscously by about half as much, which amounts to a radial displacement of the order of only about 5-15 m.

Vidale et al. (2025) do not provide any quantitative models (e.g. of heterogeneities at the top of the core) to explain their results. They simply speculate on the mechanisms that could lead to the temporal changes that they observed. In this sense, we cannot confirm or infirm whether the inferred temporal changes in the ICB from viscous deformation that we predict are consistent with seismic observations. We have removed this reference from the main text. However, we have added a short discussion on this point in Methods, in the section on the gravitational torque (lines 460–469).

Lines 460–469: Using PREM, our range of Γ *(*$[0.6 - 4.2] \times 10^{19}$ *N m) corresponds to a range of* h_2^2 *at the CMB of 31 to 83 m. The gravitational potential anomaly of degree 2 varies approximately linearly with radius inside the core so the peak-to-peak topography along the equator of the ICB (as-*

sumed to coincide with an equipotential surface) is $(r_i/r_c)\, h_2^2 \sim 0.35\, h_2^2$, *corresponding to a range of peak-to-peak ICB topography of 11 to 29 m. Although small, an axial rotation of this topography by about* $1°$ *(Fig. S6) is sufficient to provide the necessary gravitational torque on the mantle. Our results suggest that viscous relaxation of this ICB topography occurs over a few decades (Fig. S6), amounting to radial motion of the order of 10 m near the top of the inner core. It is unclear whether such a small amplitude (and low wavelength) displacement can be detected seismically or whether seismic inferences of viscous deformation near the top of the inner core capture instead a more regional deformation.*

Finlay et al. (Nature reviews Earth & environment, 4, 377–392, 2023) included in their review paper a comparison between the LOD changes predicted from the flow models used by Zhang and Dumberry and the observed LOD changes (their Figure 5). We find it would be useful to point this figure to the readers since it shows the level of agreement reached today between LOD predictions and data.

We followed this suggestion. We have added a reference to this figure on lines 75–76 when we discuss the accuracy of the retrieved phase of the EM torque.

Lines 75–76: The latter is determined by the time history of core flows, mostly its large-scale part, and since the latter reproduces well the multidecadal ΔLOD *(see Fig. 5 of ref. [6]), it is well resolved by the SV.*

lines 47–49 'the inner core boundary ... coincides, on a time average, with the gravitational potential': this is not strictly true if a time average gravitational torque cancels out a time average EM torque, see lines 56–58.

We have changed our sentence to:
Lines 47–48: On a time average, the topography of the inner core boundary (ICB) should coincide (save for a small eastward offset) with the gravitational potential imposed by mantle density anomalies.

Although this is a terse way to discuss this permanent offset, it is difficult to include more details here given the constraints on the word limit in the text.

As our response to a similar point above, the steady eastward offset that our best-fit results imply is of the order of $\alpha = 1.2°$. This is small compared to the angle of $90°$ between the peaks and troughs of the degree 2 order 2 geoid.

lines 129–131 'the inner core differential rotation ... is not observed directly ... rather ... it is based on a model of anisotropy within the inner core.' This gives the original interpretation by Song and Richards (1996). Yang and Song (2023) wrote instead that they were able to track the inner core rotation, using temporal changes from doublets, because of the lateral seismic velocity gradients in the inner core.

We thank the reviewers for this correction. We have changed our text to reflect that both the inner core rotation model and the core flow models influence the torque parameters that we retrieve. In this revised text, we no longer refer directly to the way the inner core rotation model is constructed. See lines 147–153. We have moved this information to the start of section 8 of Supplementary Information.

Lines 147–153: Our reconstructed multidecadal torque scenario is supported by the seismically inferred inner core rotation changes, by zonal core flows derived from the SV, and by dynamo simulations.

However, our estimates of the best-fit and CI of Γ*,* τ *and* K_{cmb} *are only as accurate as the models of inner core rotation and core flow that we used. These models contain a number of assumptions, which map into an uncertainty on our retrieved torque parameters. Sensitivity tests (Supplementary Information) show that an increase (decrease) in the amplitude of the inner core differential rotation leads to a proportional decrease (increase) in* Γ*, and that the best-fit estimate of each torque parameter can vary by up to 30% for different core flow models.*

Could you give a typical amplitude of the deviation from a sphere of the inner core surface (which is involved in the gravitational torque discussed here)?

We have added a sentence in the main text (line 161) and also one in the Methods section (lines 460–464) where we add this information.

Line 161: ... and a corresponding Y_2^2 *ICB topography of* $\sim 11 - 29$ *m ...*

Lines 460–463: Using PREM, our range of Γ *(*$[0.6 - 4.2] \times 10^{19}$ *N m) corresponds to a range of* h_2^2 *at the CMB of 31 to 83 m. The gravitational potential anomaly of degree 2 varies approximately linearly with radius inside the core so the peak-to-peak topography along the equator of the ICB (assumed to coincide with an equipotential surface) is* $(r_i/r_c)\, h_2^2 \sim 0.35\, h_2^2$*, corresponding to a range of peak-to-peak ICB topography of 11 to 29 m.*

It may be helpful to refer to K_{em} as the dimensionless conductance (or conductance in units of $\mathbf{10^8}$ S), as it has a simple physical interpretation.

This is a good suggestion. We now refer to K_{em} as the 'dimensionless conductance' (lines 68, 380).

Line 68: ... which we express in terms of a dimensionless conductance K_{em} *(based on a reference value of* 10^8 *S, Methods).*

Line 380: ... the strength of the EM torque scales with the dimensionless conductance K_{em}*.*

There is a typo in the y-axis of Figure S16.

We thank the reviewer for pointing this out. The typo "hight" has been corrected to "height" in Figure S20.

References: appropriate credit to previous work?
The references are appropriate. We have suggested in our report a few additional references. Gerick and Mandea (GJI, 243, ggaf321, 2025) recently found, using synthetic data extracted from geodynamo models, that it is correct to estimate the surface pressure from the surface velocity using the tangentially geostrophic approximation. It may be helpful to comment on this alternative way to estimate the pressure torque from the velocity (and the CMB topography described in the supplementary file, line 242–257). Oliver et al. (Communications earth and environment, 6, 4, 2025) also support this approach (and linear dependence on topographic amplitude), albeit from a convection model only.

We disagree with this suggestion. The geostrophic part of the flow should produce no axial torque. This was demonstrated by Kuang & Bloxham (1997 [12], in our reference list). Hence, the topographic torque is not tied to the accuracy of geostrophic approximation at the CMB. In the paper by Oliver et al (2025, [13]), this is fully recognized and they write explicitly that their model of the torque is instead based on the *non-geostrophic* part of the pressure acting on the CMB topography, specifically, the part of the pressure associated with the $\mathbf{v} \cdot \nabla \mathbf{v}$ acceleration term. While the torque from this non-geostrophic pressure may depend linearly on the topography, a simple order of magnitude suggests that it is too weak to account for the observed LOD changes. The topographic torque from the pressure p acting on CMB topographic bumps (of low spherical harmonic degree) of height h is $\Gamma_{top} \sim 4\pi R_c^2 ph$, where $R_c = 3580$ km is the radius of the core. Taking $h \sim 1$ km, and a typical geostrophic pressure of $p \sim 100$ Pa, this gives $\Gamma_{top} \sim 1.5 \times 10^{19}$ N m. This simple example captures the problem of all studies that attempted to compute the topographic torque based on geostrophic pressure (e.g. Jault & LeMouel, 1989; 1991; Hide et al, 1993, all included in our reference list): the predicted torque is typically too high by at least an order of magnitude. A simple scaling of the pressure p associated with the $\mathbf{v} \cdot \nabla \mathbf{v}$ term is $p \sim \rho_o U^2$, where $\rho_o = 10^4$ kg/m^3 is the density and $U \sim 10^{-4}$ m/s is a typical speed for the non-steady flow at the CMB. This gives $p \sim 10^{-4}$ Pa, and a typical topographic torque of $\Gamma_{top} \sim 4\pi R_c^2 h \rho_o U^2 \sim 1.5 \times 10^{13}$ N m, too low by 5 orders of magnitude.

Oliver et al (2025) argue that the scaling of their non-geostrophic torque is $\Gamma_{top} \sim Ek^{-1/3} R_c^2 h \rho_o U^2$, where $Ek = 10^{-15}$ is the Ekman number. Since $Ek^{-1/3} \sim 10^5$, they get a torque magnitude compatible with that required to explain the LOD changes. But one can be suspicious of this result. First, why should the pressure torque scaling depend on the viscosity (through Ek)? Second, the large pre-factor (10^5) essentially implies a $\mathbf{v} \cdot \nabla \mathbf{v}$ term that is increased by 5 orders of magnitude as a result of the interaction with CMB bumps. This is difficult to reconcile with geodynamo results which (although in the absence of CMB topography) suggest that the $\mathbf{v} \cdot \nabla \mathbf{v}$ term remains orders of magnitude smaller than the mean geostrophic (or QG-MAC) balance. In summary, we find that at this stage, it remains too speculative to argue that the $\mathbf{v} \cdot \nabla \mathbf{v}$ term may participate in the torque balance.

Clarity and context: lucidity of abstract/summary, appropriateness of abstract, introduction and conclusions

The authors speculate at the end of the article on the shorter LOD variations. They could complement this statement by noting that on longer time scales the gravitational torque and the torques operating at the core-mantle boundary almost exactly cancel out.

As we explain in our responses to earlier points, we have changed our view on the torque responsible for the shorter (< 30 yr) LOD changes. We have changed our concluding paragraph to reflect this new inference. We have also taken the suggestion of the reviewers into account by adding a short discussion on the implication of our results for the steady state balance, in particular, the steady rate of inner core rotation that is implied by our best-fit values of G, Γ and τ. The mathematical details of this balance are presented in an added section in Methods.

Code availability

The code is available with a description that allows reproducibility.

We have updated our codes to reflect the changes in our revised manuscript. The original codes used for the IC model sensitivity analysis have been moved to the folder "IC sinusoidal". The updated codes using the spline fit method for the main text IC model, together with the additional tests for different core flow models, have been added to the folder "IC splinefit".

Reviewer #3 (Remarks to the Author):

Wow.

This is an astonishingly broad-ranging and extensive piece of work discussing the dynamics of the core, and the links between length-of-day, geomagnetic models and Earth structure. I find the paper a little difficult in letter format, because there is so much detail and influence of other work. For example, the topographic torque is discussed, there I find that in order to understand this, I must go into the methods, then the supplementary material, leading to some extensive references that I would have to read in order to understand what is being done. Which is far more a comment on me than on the paper I suspect, but it does mean that I have to take a large amount of background on faith. However, I am fully prepared to make this leap.

I find the conclusions entirely plausible - that oscillations are driven primarily by inner-core gravitational interactions, not least because both electromagnetic and topographic torques lead to results that are 90 degrees out of phase with what is required. (IN passing, I believe that the topographic torque has a strong element of electromagnetic processes involved, so I am not surprised.)

We thank the reviewer for their kind words about our work and their general support of our results.

My only suggestion is an issue of presentation. I am fully convinced by the conclusions, but I suspect many of our colleagues have other ideas as how the system works (particularly coupling from core surface flows rather than inner core). As the authors will know (and is unavoidable), there are a whole range of assumptions that go into this work - down to issues as fundamental as the structure of the core flow depending on ignoring diffusion, and a whole pile of non-uniqueness in its determination. To look at the torque calculation, this depends on strong assumptions (at some points using a steady flow, when past (very old) work suggested that it is the much smaller time-varying part of the flow that is of major importance in time variations in length of day). With all these assumptions then drawing conclusions as to deep-mantle structure and mineralogy is definitely interesting and appropriate, but also very speculative.

This is a fair criticism. Indeed, in our original manuscript, the inferences that we drew on deep Earth structures did not properly convey how the best-fitting torque parameters that we retrieve remain uncertain and tied to our adopted models of inner core rotation and core flow at the CMB. We have made changes to the text in an effort to properly convey that, ultimately, our results depend on the accuracy of the inner core rotation and core flow models from which they are derived.

So I am occasionally worried by the strength of the language of the conclusions. Just looking at the abstract, starting that the length of day changes were driven by gravitational torque is for me a very strong statement. I fully agree that this is (for me) the most plausible mechanism, but that isn't a proof. Looking at mantle structure, the constraints are assuming that the forces here are not only dominant, but exclusive.

Indeed, ultimately, our conclusions are based on a model of the inner core rotation, a model of core flows at the CMB, and models of the gravitational and CMB torque. All these models contain a number of assumptions. In this sense, we cannot "prove" that the multidecadal LOD changes are driven by the gravitational torque, we can only make the best inference based on the latest models. It is reassuring, however, that results of dynamo simulations approaching Earth conditions [1] suggest a scenario which is consistent to the one that we infer.

In our revised version, we have made an effort to tone down the strength of our conclusions. We have acknowledged that the best-fit parameters that we retrieve remain uncertain and tied to the accuracy of the inner core rotation and core flow models. The inference that we make on deep Earth structures are still based on our best-fit values and 95% CI that we find; we can only discuss the consequence of the results that we obtain. However, we have changed the first sentence of the Discussion (lines 155–156) to remind the reader that we are more confident about the scales of the parameters than their precise values.

It is great to see there is a coherent and consistent framework to explain the observations. So this is clearly a consistent mechanism, but it is not possible to say that it *is* the mechanism (nor will it ever be so possible). So please, just a little care in the presentation of your claims.

We have altered our abstract to better reflect that our conclusions are based on models of inner core rotation and core flows, and that it is these models that provide the support for the scenario that we find. Other places where we have included caveat statements include:

Lines 91–94: While the torque histories on Fig. 1 rest entirely on the accuracy of the models of inner core rotation and core flow, they are in line with results from dynamo simulations approaching Earth-like conditions which suggest a similar scenario of gravitationally driven multidecadal ΔLOD opposed by a CMB torque.

Lines 147–153: Our reconstructed multidecadal torque scenario is supported by the seismically inferred inner core rotation changes, by zonal core flows derived from the SV, and by dynamo simulations. However, our estimates of the best-fit and CI of Γ, τ and K_{cmb} are only as accurate as the models of inner core rotation and core flow that we used. These models contain a number of assumptions, which map into an uncertainty on our retrieved torque parameters. Sensitivity tests (Supplementary Information) show that an increase (decrease) in the amplitude of the inner core differential rotation leads to a proportional decrease (increase) in Γ, and that the best-fit estimate of each torque parameter can vary by up to 30% for different core flow models.

Lines 155–156: Although the specific numerical values of the torque parameters that we retrieve remain uncertain, their scales have important geophysical implications for the deep mantle and inner core.

References

[1] J. Aubert. “Geodynamo simulations spanning millennia in the physical conditions of Earth’s core”. In: *J. Stud. Earth Deep Inter.* 2 (2026), p. 3.

[2] T. Schwaiger et al. “Wave-like motions and torques in Earth’s core as inferred from geomagnetic data: A synthetic study”. In: *Physics of the Earth and Planetary Interiors* 346 (2024), p. 107104.

[3] K. Wu, X. Song, and Y. Yang. “Pattern of inner-core differential rotation from long-term earthquake sequences and USArray network”. In: *Geophysical Research Letters* 53 (2026), e2025GL119295.

[4] Y. Yang and X. Song. “Multidecadal variation of the Earth’s inner-core rotaion”. In: *Nature Geoscience* (2023).

[5] N. Gillet et al. “Fast torsional waves and strong magnetic field within the Earth’s core”. In: *Nature* 465 (2010), pp. 74–77.

[6] C. C. Finlay et al. “Gyres, jets and waves in the Earth’s core”. In: *Nat. Rev. Earth Environ.* 4 (2023), pp. 377–392.

[7] N. Gillet, D. Jault, and E. Canet. “Excitation of traveling torsional normal modes in an Earth’s core model”. In: *Geophys. J. Int.* 210 (2017), pp. 1503–1516.

[8] M. Istas et al. “Transient core surface dynamics from ground and satellite geomagnetic data”. In: *Geophys. J. Int.* 233.3 (2023), pp. 1890–1915.

[9] N. Gillet et al. “Satellite magnetic data reveal interannual waves in Earth’s core”. In: *Proc. Nat. Acad. Sci.* 119.13 (2022), e2115258119.

[10] N. Gillet, L. Huder, and J. Aubert. “A reduced stochastic model of core surface dynamics based on geodynamo simulations”. In: *Geophts. J. Int.* 219.1 (2019), pp. 522–539.

[11] J. Bärenzung et al. “Modeling and predicting the short-term evolution of the geomagnetic field”. In: *Journal of Geophysical Research: Solid Earth* 123 (2018), pp. 4539–4560.

[12] W. Kuang and J. Bloxham. “On the dynamics of topographical core-mantle coupling”. In: *Phys. Earth planet. Inter.* 99 (1997), pp. 289–294.

[13] T. G. Oliver et al. “Turbulence in Earth’s core generates large topographic torques on the mantle”. In: *Commun. Earth Environ.* 6 (2025), p. 484.

Gravitational torque drives multidecadal variations in length of day

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[redacted]

Response to referees

We are grateful to the two anonymous reviewers [redacted] for their constructive comments, [redacted] This document provides our point-by-point responses to the reviewers' comments. The original comments are shown in **blue**, and our responses are in **black**.

Reviewers #1 and #2 (Remarks to the Author):

We have appreciated the very detailed rebuttal of our review prepared by the authors. We found it easy to follow. The authors have taken into account all our questions. They have also made all the additional tests that we suggested. Therefore, we are happy with the revised version of the manuscript, and we recommend its publication in Nature.
While Zhang and Dumberry are confident of their explanation of multidecadal (60 to 70 years) length-of-day changes, they reckon that the description of the mechanism acting on shorter time scales needs refining. Their interpretation of these fast changes as the result of a torque, either topographic or electromagnetic, acting at the core surface removes a contradiction between this work and another recent study. Finally, we find it important that the authors have built a new inner core rotation model based on a cubic spline representation as in the original seismological study.
Yes. We were able to install and run the code. Detailed instructions are provided in the README file.

We thank the reviewers for their kind words about our work and for their general support of our results. Much remains to be explored to fully understand length-of-day variations across a broad range of timescales. We also thank the reviewers for their thoughtful and important comments, which have significantly improved the clarity and presentation of the manuscript.

Reviewer #2 (Remarks to the Author):

I co-reviewed this manuscript with one of the reviewers who provided the listed reports.
The code can be easily installed and executed, it is well described by README file.

We thank the reviewer for the kind words about our work and for the general support of our results.

Reviewer #3 (Remarks to the Author):

I find the responses to my comments completely resolve the issue I had over a possible overclaim to significance – the language now gives a very fair refection of what I think has been shown. The responses to the other reviewers also seem appropriate – I can't help whishing that this discussion becomes a paper (or even a book....) in its own right. I fully support publication as is. [redacted]

We thank [redacted] for his kind words about our work and for his general support of our results. We will continue to explore this topic and develop it further. Indeed, our plan is to present some of the results that we included in our rebuttal in separate scientific publications.