Supplementary information of

Multi-component quantitative magnetic resonance imaging by phasor representation

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When recorded with a low number of points (i.e. echoes or *b*-values), a decay is undersampled and truncated. This means that Equations 1 and 2 are no longer valid. A modified phasor approach [12] can calculate the average decay constant (i.e. T_2 or $1/D$). For temporal decays *f*, which are sampled with total number of sampling of K and total measurement time window of T the signal is recorded in sampling intervals of T/K at integer multiples of *k*:

$$f(k) = e^{-\frac{(k+\frac{1}{2})\frac{T}{K}}{T_2}}$$

Figure S1 shows the schematic diagram for acquisition of data and used nomenclatures.

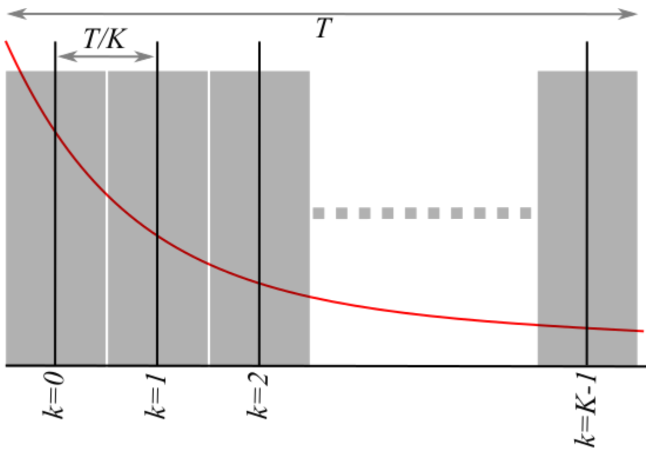

Fig. S1. Schematic diagram of the undersampling of a decay at intervals of T/K.

The normalized Fourier transformation of *f(k)* at first harmonics:

$$F(T_2) = \left[\sum_{k=0}^{K-1} e^{i\frac{2\pi}{T}(k+\frac{1}{2})\frac{T}{K}} \cdot e^{-\frac{(k+\frac{1}{2})\frac{T}{K}}{T_2}}\right] / \left[\sum_{k=0}^{K-1} e^{-\frac{(k+\frac{1}{2})\frac{T}{K}}{T_2}}\right]$$

Analytical solution of equation above, provides the exactly similar equation used to analyze binned fluorescence decay curves [12] :

$$F(T_2) = \frac{1}{\cos(\frac{\pi}{K}) - \sin(\frac{\pi}{K})\coth\left(\frac{T}{2KT_2}\right)i}$$

T_2 can be estimated by using the real and imaginary part of $F(T_2)$:

$$T_2 = \frac{T}{2K\operatorname{arccoth}\left(\frac{\operatorname{Im}(F)}{\operatorname{Re}(F)}\cot(\frac{\pi}{K})\right)}$$

When the sampling intervals is large, $T/K \to 0$ and

$$F(T_2) = \frac{1}{1 - \mathrm{i}\frac{2\pi}{T}T_2}$$

and T_2 can be estimated by:

$$T_2 = \frac{T}{2\pi}\frac{\mathrm{Im}(F)}{\mathrm{Re}(F)}$$

In a similar way, for diffusion MRI the magnetization decays exponentially with an experimental *b*-value ($M = M_0 e^{-bD}$ for mono-exponential decay, with diffusion coefficient *D*). For exponential decays composed of a large number of *b*-values, the average *ADC* can be estimated by:

$$ADC = \frac{2\pi}{B}\frac{\mathrm{Re}_{\mathrm{n}=1}}{\mathrm{Im}_{\mathrm{n}=1}}$$

To standardize and optimize the graphical representation of the phasors, we add ½ to *k*. For T_2, this means that the echo decay time axis is shifted by -½ echo time, while for diffusion MRI the *b*-axis is shifted by +½ '*b*-step'. These shifts only result in phase shifts in the Fourier domain, which is apparent as a rotation in the phasor plot. As shown above, the calculation of MRI parameters from phasor coordinates also takes this phase shift into account.

Supplementary video captions

Supplementary Video 1: T_2 maps of the full quantitative MRI dataset of an in vivo human head (figure 3B).

Supplementary Video 2: RGB contribution maps of the separate components in the quantitative MRI dataset of an in vivo human head (figure 4B).

Supplementary Video 3: Color-coded back-projection of the segmented pixels into the images of the quantitative MRI dataset of an in vivo human head (figure 4E).

Supplementary Video 4: Average apparent diffusion coefficient maps of a multislice 2D quantitative diffusion MRI dataset of an in vivo human head (figure 5B).

Supplementary Video 5: Average apparent diffusion coefficient maps of a multislice 2D quantitative diffusion MRI datasets of an in vivo mouse head (figure 5D).