

Supplementary material to: Symmetry in the open-system dynamics of quantum correlations

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Bit-flip channel

One-qubit bit-flip channel can be represented by Kraus operators $K'_1 = \sqrt{p}\sigma_1$, $K'_2 = \sqrt{q}I$. The dynamics of an arbitrary X state under one-sided bit-flip channels can be written as

$$\begin{aligned}\Phi_p^{B,U}(X) &= \begin{pmatrix} q\rho_{11} + p\rho_{33} & 0 & 0 & q\rho_{14} + p\rho_{23}^* \\ 0 & q\rho_{22} + \rho_{44}p & q\rho_{23} + p\rho_{14}^* & 0 \\ 0 & q\rho_{23}^* + p\rho_{14} & q\rho_{33} + \rho_{11}p & 0 \\ q\rho_{14}^* + p\rho_{23} & 0 & 0 & q\rho_{44} + \rho_{22}p \end{pmatrix}, \\ \Phi_p^{B,L}(X) &= \begin{pmatrix} q\rho_{11} + p\rho_{22} & 0 & 0 & q\rho_{14} + p\rho_{23} \\ 0 & q\rho_{22} + \rho_{11}p & q\rho_{23} + p\rho_{14} & 0 \\ 0 & q\rho_{23}^* + p\rho_{14}^* & q\rho_{33} + \rho_{44}p & 0 \\ q\rho_{14}^* + p\rho_{23}^* & 0 & 0 & q\rho_{44} + \rho_{33}p \end{pmatrix}.\end{aligned}\tag{S1}$$

By using equation (3) of the main article, we get as the concurrences of the output states

$$\begin{aligned}C(\Phi_p^{B,U}(X)) &= 2\max\left\{0, |\rho_{14}p + \rho_{23}^*q| - \sqrt{(\rho_{22}p + \rho_{44}q)(\rho_{33}p + \rho_{11}q)}, |\rho_{14}^*q + \rho_{23}p| - \sqrt{(\rho_{11}p + \rho_{33}q)(\rho_{44}p + \rho_{22}q)}\right\}, \\ C(\Phi_p^{B,L}(X)) &= 2\max\left\{0, |\rho_{14}^*p + \rho_{23}^*q| - \sqrt{(\rho_{33}p + \rho_{44}q)(\rho_{22}p + \rho_{11}q)}, |\rho_{14}q + \rho_{23}^*p| - \sqrt{(\rho_{11}p + \rho_{22}q)(\rho_{44}p + \rho_{33}q)}\right\}.\end{aligned}\tag{S2}$$

Due to the form of the solutions it is not as simple to solve analytically, when the decay of entanglement is symmetric. Still we can find a set of sufficient conditions for symmetric entanglement decay:

$$(\rho_{14} \in \mathbb{R} \text{ or } \rho_{23} \in \mathbb{R}) \text{ and } (\rho_{22} = \rho_{33} \text{ or } \rho_{11} = \rho_{44})\tag{S3}$$

Bit-phase-flip channel

One-qubit bit-phase-flip channel can be represented by Kraus operators $K'_1 = \sqrt{p}\sigma_z$, $K'_2 = \sqrt{q}I$. The dynamics of an arbitrary X state under one-sided bit-phase-flip channels can be written as

$$\begin{aligned}\Phi_p^{P,U}(X) &= \begin{pmatrix} q\rho_{11} + p\rho_{33} & 0 & 0 & q\rho_{14} - p\rho_{23}^* \\ 0 & q\rho_{22} + \rho_{44}p & q\rho_{23} - p\rho_{14}^* & 0 \\ 0 & q\rho_{23}^* - p\rho_{14} & q\rho_{33} + \rho_{11}p & 0 \\ q\rho_{14}^* - p\rho_{23} & 0 & 0 & q\rho_{44} + p\rho_{22}p \end{pmatrix}, \\ \Phi_p^{P,L}(X) &= \begin{pmatrix} q\rho_{11} + p\rho_{22} & 0 & 0 & q\rho_{14} - p\rho_{23} \\ 0 & q\rho_{22} + \rho_{11}p & q\rho_{23} - p\rho_{14} & 0 \\ 0 & q\rho_{23}^* - p\rho_{14}^* & q\rho_{33} + \rho_{44}p & 0 \\ q\rho_{14}^* - p\rho_{23}^* & 0 & 0 & q\rho_{44} + \rho_{33}p \end{pmatrix}.\end{aligned}\tag{S4}$$

By using equation (3) of the main article, we get as the concurrences of the output states

$$\begin{aligned}C(\Phi_p^{P,U}(X)) &= 2\max\left\{0, |\rho_{23}^*q - \rho_{14}p| - \sqrt{(\rho_{22}p + \rho_{44}q)(\rho_{33}p + \rho_{11}q)}, |\rho_{14}^*q + \rho_{23}p| - \sqrt{(\rho_{11}p + \rho_{33}q)(\rho_{44}p + \rho_{22}q)}\right\}, \\ C(\Phi_p^{P,L}(X)) &= 2\max\left\{0, |\rho_{23}^*q - \rho_{14}^*p| - \sqrt{(\rho_{33}p + \rho_{44}q)(\rho_{22}p + \rho_{11}q)}, |\rho_{14}^*q - \rho_{23}^*p| - \sqrt{(\rho_{11}p + \rho_{22}q)(\rho_{44}p + \rho_{33}q)}\right\}.\end{aligned}\tag{S5}$$