

Supplementary information: Non-autonomous multi-rogue waves for spin-1 coupled nonlinear Gross-Pitaevskii equation and management by external potentials

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Supplementary information

APPENDIX A: The 16 linear equations in Eqs.(17) and (18)

$$r_{11,\eta} - \lambda r_{11} - \phi_1 s_{11} - \phi_0 s_{21} = 0, r_{12,\eta} - \lambda r_{12} - \phi_1 s_{12} - \phi_0 s_{22} = 0, r_{21,\eta} - \lambda r_{21} - \phi_0 s_{11} - \phi_{-1} s_{21} = 0, \\ r_{22,\eta} - \lambda r_{22} - \phi_0 s_{12} - \phi_{-1} s_{22} = 0, s_{11,\eta} + \phi_{11} r_{11} + \phi_{10} r_{21} + \lambda s_{11} = 0, s_{12,\eta} + \phi_{11} r_{12} + \phi_{10} r_{22} + \lambda s_{12} = 0, \\ s_{21,\eta} + \phi_{10} r_{11} + \phi_{-11} r_{21} + \lambda s_{21} = 0, s_{22,\eta} + \phi_{10} r_{12} + \phi_{-11} r_{22} + \lambda s_{22} = 0,$$

and

$$r_{11,\tau} - (2i\lambda^2 + i\phi_1\phi_{11} + i\phi_0\phi_{10})r_{11} - i(\phi_1\phi_{10} + \phi_0\phi_{-11})r_{21} - i(2\lambda\phi_1 + \phi_{1,x})s_{11} - i(2\lambda\phi_0 + \phi_{0,x})s_{21} = 0, \\ r_{12,\tau} - (2i\lambda^2 + i\phi_1\phi_{11} + i\phi_0\phi_{10})r_{12} - i(\phi_1\phi_{10} + \phi_0\phi_{-11})r_{22} - i(2\lambda\phi_1 + \phi_{1,x})s_{12} - i(2\lambda\phi_0 + \phi_{0,x})s_{22} = 0, \\ r_{21,\tau} - i(\phi_0\phi_{11} + \phi_{-1}\phi_{10})r_{11} - (2i\lambda^2 + i\phi_0\phi_{10} + i\phi_{-1}\phi_{-11})r_{21} - i(2\lambda\phi_0 + \phi_{0,x})s_{11} - i(2\lambda\phi_{-1} + \phi_{-1,x})s_{21} = 0, \\ r_{22,\tau} - i(\phi_0\phi_{11} + \phi_{-1}\phi_{10})r_{12} - (2i\lambda^2 + i\phi_0\phi_{10} + i\phi_{-1}\phi_{-11})r_{22} - i(2\lambda\phi_0 + \phi_{0,x})s_{12} - i(2\lambda\phi_{-1} + \phi_{-1,x})s_{22} = 0, \\ s_{11,\tau} - i(-2\lambda\phi_{11} + \phi_{11,x})r_{11} - i(-2\lambda\phi_{10} + \phi_{10,x})r_{21} - (-2i\lambda^2 - i\phi_1\phi_{11} - i\phi_0\phi_{10})s_{11} + i(\phi_0\phi_{11} + \phi_{-1}\phi_{10})s_{21} = 0, \\ s_{12,\tau} - i(-2\lambda\phi_{11} + \phi_{11,x})r_{12} - i(-2\lambda\phi_{10} + \phi_{10,x})r_{22} - (-2i\lambda^2 - i\phi_1\phi_{11} - i\phi_0\phi_{10})s_{12} + i(\phi_0\phi_{11} + \phi_{-1}\phi_{10})s_{22} = 0, \\ s_{21,\tau} - i(-2\lambda\phi_{10} + \phi_{10,x})r_{11} - i(-2\lambda\phi_{-11} + \phi_{-11,x})r_{21} + i(\phi_1\phi_{10} + \phi_0\phi_{-11})s_{11} - (-2i\lambda^2 - i\phi_0\phi_{10} - i\phi_{-1}\phi_{-11})s_{21} = 0, \\ s_{22,\tau} - i(-2\lambda\phi_{10} + \phi_{10,x})r_{12} - i(-2\lambda\phi_{-11} + \phi_{-11,x})r_{22} + i(\phi_1\phi_{10} + \phi_0\phi_{-11})s_{12} - (-2i\lambda^2 - i\phi_0\phi_{10} - i\phi_{-1}\phi_{-11})s_{22} = 0.$$

APPENDIX B: The functions D_1 , G_1 , G_0 and G_{-1} in Eqs.(22)-(24)

$$D_1 = \overline{s_{11}s_{22}}s_{11}s_{22} - \overline{s_{11}s_{22}}s_{12}^2 + \overline{s_{11}r_{21}}r_{21}s_{22} - \overline{s_{11}r_{21}}r_{22}s_{12} - \overline{s_{11}r_{22}}r_{21}s_{12} + \overline{s_{11}r_{22}}r_{22}s_{11} + \\ \overline{s_{22}r_{12}}r_{12}s_{11} + \overline{s_{22}r_{11}}r_{11}s_{22} - \overline{s_{22}r_{11}}r_{12}s_{12} - \overline{s_{22}r_{12}}r_{11}s_{12} - \overline{r_{12}r_{21}}r_{11}r_{22} + \overline{r_{12}r_{21}}r_{12}r_{21} + \overline{r_{22}r_{11}}r_{11}r_{22} -$$

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$$\begin{aligned}
& \overline{r_{22}r_{11}r_{12}r_{21}} - (\overline{s_{12}})^2 s_{11}s_{22} + \overline{s_{12}r_{12}r_{21}s_{12}} - \overline{s_{12}r_{12}r_{22}s_{11}} + \overline{s_{12}r_{22}r_{11}s_{12}} - \overline{s_{12}r_{22}r_{12}s_{11}} - \overline{s_{12}r_{11}r_{21}s_{22}} + \\
& \overline{s_{12}r_{11}r_{22}s_{12}} - \overline{s_{12}r_{21}r_{11}s_{22}} + \overline{s_{12}r_{21}r_{12}s_{12}} + (\overline{s_{12}})^2 s_{12}^2, \\
& G_1 = -s_{12}r_{12}\overline{s_{11}s_{22}} + s_{12}r_{12}(\overline{s_{12}})^2 + s_{22}r_{11}\overline{s_{11}s_{22}} - s_{22}r_{11}(\overline{s_{12}})^2 - \overline{r_{12}s_{12}r_{11}r_{22}} + \overline{r_{12}s_{12}r_{12}r_{21}} + \\
& \overline{r_{22}s_{11}r_{11}r_{22}} - \overline{r_{22}s_{11}r_{12}r_{21}}, \\
& G_0 = s_{11}r_{12}\overline{s_{11}s_{22}} - s_{11}r_{12}(\overline{s_{12}})^2 - s_{12}r_{11}\overline{s_{11}s_{22}} + s_{12}r_{11}(\overline{s_{12}})^2 + \overline{r_{11}s_{12}r_{11}r_{22}} - \overline{r_{11}s_{12}r_{12}r_{21}} - \\
& \overline{r_{21}s_{11}r_{11}r_{22}} + \overline{r_{21}s_{11}r_{12}r_{21}}, \\
& G_{-1} = s_{11}r_{22}\overline{s_{11}s_{22}} - s_{11}r_{22}(\overline{s_{12}})^2 - s_{12}r_{21}\overline{s_{11}s_{22}} + s_{12}r_{21}(\overline{s_{12}})^2 + \overline{r_{11}s_{22}r_{11}r_{22}} - \overline{r_{11}s_{22}r_{12}r_{21}} - \\
& \overline{r_{21}s_{12}r_{11}r_{22}} + \overline{r_{21}s_{12}r_{12}r_{21}}.
\end{aligned}$$

APPENDIX C: The functions D_1 , G_1 , G_0 and G_{-1} in Eqs.(25)-(27)

$$\begin{aligned}
D_1 &= 57 - 40\sqrt{2} - 118\sqrt{2}\eta - 134\sqrt{2}\eta^2 - 64\sqrt{2}\eta^3 + 20\sqrt{2}\eta^5 + 8\eta^6\sqrt{2} + 4\eta^4\sqrt{2} - 176\sqrt{2}\tau^2 + \\
& 256\tau^4\sqrt{2} + 4096\tau^6\sqrt{2} + 800\tau^2\eta^2 + 2304\tau^4\eta^2 + 704\eta\tau^2 + 640\tau^2\eta^3 + 2560\tau^4\eta + 288\tau^2\eta^4 - \\
& 256\tau^2\sqrt{2}\eta + 1280\tau^4\sqrt{2}\eta + 192\tau^2\sqrt{2}\eta^4 + 64\tau^2\sqrt{2}\eta^2 + 1536\tau^4\sqrt{2}\eta^2 + 320\tau^2\sqrt{2}\eta^3 + 172\eta + 214\eta^2 + \\
& 82\eta^4 + 152\eta^3 + 40\eta^5 + 12\eta^6 + 304\tau^2 + 1152\tau^4 + 6144\tau^6, \\
G_1 &= -133 - 110i\sqrt{2}\tau + 512i\tau^5 + 928i\tau^3 + 376i\eta\tau + 94\sqrt{2} + 375\sqrt{2}\eta + 618\sqrt{2}\eta^2 + 541\sqrt{2}\eta^3 + \\
& 78\sqrt{2}\eta^5 + 12\eta^6\sqrt{2} + 270\eta^4\sqrt{2} + 992\sqrt{2}\tau^2 + 3456\tau^4\sqrt{2} + 6144\tau^6\sqrt{2} - 3520\tau^2\eta^2 - 768\tau^4\eta^2 - \\
& 3680\eta\tau^2 - 1344\tau^2\eta^3 - 5376\tau^4\eta - 96\tau^2\eta^4 + 2600\tau^2\sqrt{2}\eta + 4992\tau^4\sqrt{2}\eta + 288\tau^2\sqrt{2}\eta^4 + 2592\tau^2\sqrt{2}\eta^2 + \\
& 2304\tau^4\sqrt{2}\eta^2 + 1248\tau^2\sqrt{2}\eta^3 - 531\eta - 876\eta^2 - 366\eta^4 - 764\eta^3 - 84\eta^5 - 4\eta^6 - 1408\tau^2 - 4736\tau^4 - \\
& 2048\tau^6 + 340i\tau\eta^2 + 8i\eta^4\tau + 128i\tau^3\eta^2 + 128i\eta^3\tau + 1024i\eta\tau^3 - 240i\tau\sqrt{2}\eta^2 - 24i\eta^4\sqrt{2}\tau - \\
& 264i\sqrt{2}\eta\tau - 832i\tau^3\sqrt{2}\eta - 104i\sqrt{2}\eta^3\tau - 384i\tau^3\sqrt{2}\eta^2 + 156i\tau - 640i\sqrt{2}\tau^3 - 1536i\tau^5\sqrt{2}, \\
G_0 &= 780 + 16644\eta^3 - 3252\sqrt{2}\eta - 8296\sqrt{2}\eta^2 - 12000\sqrt{2}\eta^3 - 10840\eta^4\sqrt{2} - 6372\sqrt{2}\eta^5 - \\
& 2472\eta^6\sqrt{2} - 616\eta^7\sqrt{2} - 80\eta^8\sqrt{2} - 32768\tau^6\eta^2 + 139264\tau^6\eta + 6528\eta^5\tau^2 - 512\eta^6\tau^2 + 52224\tau^4\eta^3 - \\
& 6144\tau^4\eta^4 - 9920\sqrt{2}\tau^2 - 62976\tau^4\sqrt{2} - 192512\tau^6\sqrt{2} - 327680\tau^8\sqrt{2} + 272\eta^7 - 16\eta^8 - 65536\tau^8 + \\
& 167936\tau^6 + 5000i\sqrt{2}\eta^2\tau + 4800i\sqrt{2}\eta^3\tau + 2720i\eta^4\sqrt{2}\tau + 928i\sqrt{2}\eta^5\tau + 160i\eta^6\sqrt{2}\tau + 14848i\tau^3\eta^3\sqrt{2} + \\
& 24064i\sqrt{2}\eta\tau^3 + 26624i\tau^3\eta^2\sqrt{2} + 59392i\tau^5\eta\sqrt{2} + 30720i\tau^5\eta^2\sqrt{2} + 3840i\tau^3\eta^4\sqrt{2} + 2800i\sqrt{2}\eta\tau + \\
& 768i\tau^3\eta^4 + 6144i\tau^5\eta^2 + 32i\eta^6\tau + 648i\sqrt{2}\tau + 8768i\sqrt{2}\tau^3 + 38912i\tau^5\sqrt{2} + 81920i\tau^7\sqrt{2} - 552\sqrt{2} + \\
& 7760\eta^5 + 83456\tau^4 + 16384i\tau^7 - 29696i\tau^3\eta - 2704i\eta^4\tau - 6016i\eta^3\tau - 3904i\eta\tau - 448i\eta^5\tau - \\
& 7168i\tau^3\eta^3 - 6784i\eta^2\tau - 25856i\tau^3\eta^2 - 28672i\tau^5\eta - 912i\tau - 11776i\tau^3 - 33792i\tau^5 - 14784\tau^2\eta^5\sqrt{2} - \\
& 206336\tau^4\eta^2\sqrt{2} - 30720\tau^4\eta^4\sqrt{2} - 180480\tau^4\eta\sqrt{2} - 2560\eta^6\sqrt{2}\tau^2 - 315392\tau^6\sqrt{2}\eta - 163840\tau^6\sqrt{2}\eta^2 - \\
& 75648\tau^2\eta^2\sqrt{2} - 42560\tau^2\eta^4\sqrt{2} - 73536\tau^2\eta^3\sqrt{2} - 118272\tau^4\eta^3\sqrt{2} - 42368\sqrt{2}\eta\tau^2 + 4588\eta + 11652\eta^2 + \\
& 100992\tau^2\eta^2 + 190976\tau^4\eta^2 + 58656\eta\tau^2 + 89088\tau^2\eta^3 + 216064\tau^4\eta + 39872\tau^2\eta^4 + 14520\eta^4 + 2328\eta^6 + \\
& 13920\tau^2, \\
G_{-1} &= -124 + 88\sqrt{2} - 32i\eta^6\tau\sqrt{2} - 10240i\eta\tau^5\sqrt{2} - 160i\eta^5\tau\sqrt{2} - 6144i\eta^2\tau^5\sqrt{2} - 768i\eta^4\tau^3\sqrt{2} - \\
& 2560i\eta^3\tau^3\sqrt{2} - 4096i\eta^2\tau^3\sqrt{2} - 416i\eta^4\tau\sqrt{2} - 3584i\tau^3\sqrt{2}\eta - 432i\eta\tau\sqrt{2} - 704i\eta^3\tau\sqrt{2} - 744i\eta^2\tau\sqrt{2} + \\
& 30208\eta^2\tau^4\sqrt{2} - 104i\sqrt{2}\tau - 1024i\eta^3\tau^3 + 512\eta^6\tau^2\sqrt{2} - 1344i\tau^3\sqrt{2} - 6144i\sqrt{2}\tau^5 - 16384i\tau^7\sqrt{2} + \\
& 10048\eta^3\tau^2\sqrt{2} + 3072i\eta\tau^3 + 24832\tau^4\sqrt{2}\eta + 6208\eta^4\tau^2\sqrt{2} + 144i\eta^4\tau - 32i\eta^6\tau - 4096i\eta\tau^5 - \\
& 6144i\eta^2\tau^5 - 768i\eta^4\tau^3 + 6016\tau^2\sqrt{2}\eta + 576i\eta\tau + 32768\eta^2\tau^6\sqrt{2} + 2496\eta^5\tau^2\sqrt{2} + 640i\eta^3\tau + \\
& 19968\eta^3\tau^4\sqrt{2} + 53248\eta\tau^6\sqrt{2} + 1280i\tau^3\eta^2 + 6144\eta^4\tau^4\sqrt{2} + 896i\eta^2\tau - 64i\eta^5\tau + 10368\eta^2\tau^2\sqrt{2} + \\
& 500\sqrt{2}\eta + 1224\sqrt{2}\eta^2 + 1696\eta^3\sqrt{2} + 868\eta^5\sqrt{2} + 1480\eta^4\sqrt{2} + 104\eta^7\sqrt{2} + 360\eta^6\sqrt{2} + 16\eta^8\sqrt{2} + \\
& 1472\sqrt{2}\tau^2 + 8704\tau^4\sqrt{2} + 28672\tau^6\sqrt{2} + 1024i\tau^5 + 65536\tau^8\sqrt{2} - 16384i\tau^7 + 144i\tau + 1536i\tau^3 - \\
& 11904\eta^2\tau^2 - 7840\eta\tau^2 - 6656\eta^2\tau^4 - 1472\eta^4\tau^2 - 8192\eta^3\tau^2 - 19456\eta\tau^4 + 6144\eta^4\tau^4 + 32768\eta^2\tau^6 + \\
& 512\eta^6\tau^2 + 1152\eta^5\tau^2 + 9216\eta^3\tau^4 + 24576\eta\tau^6 - 700\eta - 1684\eta^2 - 1720\eta^4 - 2228\eta^3 - 88\eta^6 - \\
& 720\eta^5 + 16\eta^8 + 48\eta^7 - 2016\tau^2 - 9728\tau^4 - 4096\tau^6 + 65536\tau^8.
\end{aligned}$$