

Supplementary Information

Role of helical edge modes in the chiral quantum anomalous Hall state

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I. RELATION BETWEEN EQUILIBRATED POTENTIALS AND PROBE POTENTIALS

A. Relation between V'_i and V_i for Four terminal QAH bar with chiral(trivial) QAH edge mode and trivial QSH edge modes

See Section 2.3.2 and Fig. 4(b) of the manuscript for the details of the set-up. See Eq. 16 of the manuscript for relations between currents and voltages.

$$\begin{aligned}
 (2-f-f_0)V_1 + \frac{[1-(f+f_0)][1+(f+f_0)R_2]T_2}{[1-R_2^2(f^2+f_0^2)]}V_2 + \frac{R_2(1-f-f_0)^2}{[1-R_2^2(f^2+f_0^2)]}V'_2 &= [(2-f-f_0) + \frac{[1-(f+f_0)][1+(f+f_0)R_2]T_2}{[1-R_2^2(f^2+f_0^2)]} + \frac{R_2(1-f-f_0)^2}{[1-R_2^2(f^2+f_0^2)]}]V'_1, \\
 \frac{[2-(f+f_0)+((1-f)f+(1-f_0)f_0)R_2-(f-f_0)^2R_2^2]T_2}{[1-R_2^2(f^2+f_0^2)]}V_2 + [1-(f+f_0)]V_3 + \frac{(2+(-2+f)f+(-2+f_0)f_0)R_2+(f-f_0)^2R_2^3}{[1-R_2^2(f^2+f_0^2)]}V'_1 &= \\
 \left[\frac{[2-(f+f_0)+((1-f)f+(1-f_0)f_0)R_2-(f-f_0)^2R_2^2]T_2}{[1-R_2^2(f^2+f_0^2)]} + [1-(f+f_0)] + \frac{(-2+f)f+(-2+f_0)f_0)R_2+(f-f_0)^2R_2^3}{[1-R_2^2(f^2+f_0^2)]} \right]V'_2, \\
 (2-f-f_0)V_3 + \frac{[1-(f+f_0)][1+(f+f_0)R_4]T_4}{[1-R_4^2(f^2+f_0^2)]}V_4 + \frac{R_4(1-f-f_0)^2}{[1-R_4^2(f^2+f_0^2)]}V'_4 &= [(2-f-f_0) + \frac{[1-(f+f_0)][1+(f+f_0)R_4]T_4}{[1-R_4^2(f^2+f_0^2)]} + \frac{R_4(1-f-f_0)^2}{[1-R_4^2(f^2+f_0^2)]}]V'_3, \\
 \frac{[2-(f+f_0)+((1-f)f+(1-f_0)f_0)R_4-(f-f_0)^2R_4^2]T_4}{[1-R_4^2(f^2+f_0^2)]}V_4 + [1-(f+f_0)]V_1 + \frac{(2+(-2+f)f+(-2+f_0)f_0)R_4+(f-f_0)^2R_4^3}{[1-R_4^2(f^2+f_0^2)]}V'_3 &= \\
 \left[\frac{[2-(f+f_0)+((1-f)f+(1-f_0)f_0)R_4-(f-f_0)^2R_4^2]T_4}{[1-R_4^2(f^2+f_0^2)]} + [1-(f+f_0)] + \frac{(-2+f)f+(-2+f_0)f_0)R_4+(f-f_0)^2R_4^3}{[1-R_4^2(f^2+f_0^2)]} \right]V'_4.
 \end{aligned} \tag{1}$$

B. Relation between V'_i and V_i for six terminal QAH bar with chiral(topological) QAH edge mode and trivial QSH edge modes

See Section 3.2.2 and Fig. 7(c) of the manuscript for the details of the set-up. See Eq. 22 of the manuscript for relations between currents and voltages.

$$\begin{aligned}
 (1-f)(V_2+V_3) &= 2(1-f)V'_2, \quad (1-f)(V_5+V_6) = 2(1-f)V'_5, \\
 (1-f)V_3 + \left(\frac{T_4(1-f)}{(1-R_4^2f^2)} + \frac{T_4R_4f(1-f)}{1-R_4^2f^2} \right)V_4 + \frac{R_4(1-f)^2}{(1-R_4^2f^2)}V'_4 &= ((1-f) + \left(\frac{T_4(1-f)}{(1-R_4^2f^2)} + \frac{T_4R_4f(1-f)}{1-R_4^2f^2} \right) + \frac{R_4(1-f)^2}{(1-R_4^2f^2)})V'_3, \\
 (1-f)V_6 + \left(\frac{T_1(1-f)}{(1-R_1^2f^2)} + \frac{T_1R_1f(1-f)}{1-R_1^2f^2} \right)V_1 + \frac{R_1(1-f)^2}{(1-R_1^2f^2)}V'_1 &= ((1-f) + \left(\frac{T_1(1-f)}{(1-R_1^2f^2)} + \frac{T_1R_1f(1-f)}{1-R_1^2f^2} \right) + \frac{R_1(1-f)^2}{(1-R_1^2f^2)})V'_3, \\
 (1-f)V_5 + \left(\frac{T_4(1-f)}{(1-R_4^2f^2)} + \frac{T_4R_4f(1-f)}{1-R_4^2f^2} \right)V_4 + \frac{R_4(1-f)^2}{(1-R_4^2f^2)}V'_3 &= ((1-f) + \left(\frac{T_4(1-f)}{(1-R_4^2f^2)} + \frac{T_4R_4f(1-f)}{1-R_4^2f^2} \right) + \frac{R_4(1-f)^2}{(1-R_4^2f^2)})V'_3, \\
 (1-f)V_2 + \left(\frac{T_1(1-f)}{(1-R_1^2f^2)} + \frac{T_1R_1f(1-f)}{1-R_1^2f^2} \right)V_1 + \frac{R_1(1-f)^2}{(1-R_1^2f^2)}V'_6 &= ((1-f) + \left(\frac{T_1(1-f)}{(1-R_1^2f^2)} + \frac{T_1R_1f(1-f)}{1-R_1^2f^2} \right) + \frac{R_1(1-f)^2}{(1-R_1^2f^2)})V'_3.
 \end{aligned} \tag{2}$$

with $a = 1 - R_2^2f^2, c = 1 - R_4^2f^2$.

C. Relation between V'_i and V_i for six terminal QAH bar with chiral(trivial) QAH edge mode and trivial QSH edge modes

See Section 3.3.2 and Fig. 7(d) of the manuscript for the details of the set-up. See Eq. 26 of the manuscript for relations between currents and voltages.

$$\begin{aligned}
& (T_1(2-f-f_0) + T_1R_1(f(1-f) + f_0(1-f_0))/a + T_1R_1^2(f+f_0)(f(1-f) + f_0(1-f_0))/a)V_1 + (1-f-f_0)V_2 \\
& + (R_1((1-f)^2 + (1-f_0)^2) + R_1^3((1-f)f + f_0(1-f_0))^2/a)V_6' = (T_1(2-f-f_0) + T_1R_1(f(1-f) + f_0(1-f_0))/a \\
& + T_1R_1^2(f+f_0)(f(1-f) + f_0(1-f_0))/a + (1-f-f_0) + R_1((1-f)^2 + (1-f_0)^2) + R_1^3((1-f)f + f_0(1-f_0))^2/a)V_1', \\
& (2-f-f_0)V_6 + (T_1(1-f-f_0)/a + T_1R_1(f+f_0)(1-f-f_0)/a)V_1 + R_1(1-f-f_0)^2/aV_1' = ((2-f-f_0) \\
& + (T_1(1-f-f_0)/a + T_1R_1(f+f_0)(1-f-f_0)/a) + R_1(1-f-f_0)^2/a)V_6', \\
& (2-f-f_0)V_2 + (1-f-f_0)V_3 = (3-2(f+f_0))V_2', \\
& (2-f-f_0)V_5 + (1-f-f_0)V_6 = (3-2(f+f_0))V_5', \\
& (T_4(2-f-f_0) + T_4R_4(f(1-f) + f_0(1-f_0))/c + T_4R_4^2(f+f_0)(f(1-f) + f_0(1-f_0))/c)V_4 + (1-f-f_0)V_5 \\
& + (R_4((1-f)^2 + (1-f_0)^2) + R_4^3((1-f)f + f_0(1-f_0))^2/c)V_3' = (T_4(2-f-f_0) + T_4R_4(f(1-f) + f_0(1-f_0))/c \\
& + T_4R_4^2(f+f_0)(f(1-f) + f_0(1-f_0))/c + (1-f-f_0) + R_4((1-f)^2 + (1-f_0)^2) + R_4^3((1-f)f + f_0(1-f_0))^2/c)V_3', \\
& (2-f-f_0)V_3 + (T_4(1-f-f_0)/c + T_4R_4(f+f_0)(1-f-f_0)/c)V_4 + R_4(1-f-f_0)^2/cV_4' = ((2-f-f_0) \\
& + (T_4(1-f-f_0)/c + T_4R_4(f+f_0)(1-f-f_0)/c) + R_4(1-f-f_0)^2/c)V_3'
\end{aligned}$$