

# **Additive Quantile Mixed Effects Modelling with Application to Longitudinal CD4 Count Data**

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**Supplementary information 1: R package additive quantile mixed model, *aqmm()*, sample outputs using CAPRISA 002 Acute Infection Study data across various quantile levels.**

Quantile 0.05

Fixed effects:

|                                   | Estimate    | Std. Error | t value | Pr(> t )  |     |
|-----------------------------------|-------------|------------|---------|-----------|-----|
| (Intercept)                       | 1.6004e+01  | 6.6336e-01 | 24.1254 | < 2.2e-16 | *** |
| Age                               | 3.9817e-02  | 1.3558e-02 | 2.9369  | 0.003706  | **  |
| Educational_levelSecondary school | -4.4909e-01 | 5.7315e-01 | -0.7835 | 0.434240  |     |
| ARTpost ART initiation            | 7.4305e-01  | 8.7964e-02 | 8.4472  | 6.219e-15 | *** |
| Baseline_VL1                      | -3.8346e-06 | 8.4209e-07 | -4.5537 | 9.174e-06 | *** |
| Residence_mod2                    | -5.0002e-01 | 1.6685e-01 | -2.9967 | 0.003076  | **  |
| Marital_statusStable              | 6.1352e-01  | 1.6551e-01 | 3.7068  | 0.000272  | *** |
| Marital_statusMany                | -2.2771e+00 | 2.7072e-01 | -8.4112 | 7.802e-15 | *** |
| s(Time_in_Months)Fx1              | -2.5075e+00 | 5.4263e-01 | -4.6210 | 6.854e-06 | *** |
| s(Baseline_BMI1)Fx1               | 5.4382e+00  | 1.0786e+00 | 5.0419  | 1.034e-06 | *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Covariance matrix of the random effects:

|                | (Intercept) | Time_in_Months |
|----------------|-------------|----------------|
| (Intercept)    | 2.748e-02   | 0.000e+00      |
| Time_in_Months | 0.000e+00   | 8.104e-18      |

Variances of the (random) smooth terms:

| s(Time_in_Months) | s(Baseline_BMI1) |
|-------------------|------------------|
| 8.796             | 1876.501         |

Residual scale parameter: 0.4223

Log-likelihood: -24280

Tuning parameter: 0.0001923

Number of observations: 7019

Number of groups: 235

Quantile 0.25

Fixed effects:

|                                   | Estimate    | Std. Error | t value | Pr(> t )  |     |
|-----------------------------------|-------------|------------|---------|-----------|-----|
| (Intercept)                       | 1.9647e+01  | 4.7491e-01 | 41.3708 | < 2.2e-16 | *** |
| Age                               | 2.0960e-02  | 1.1454e-02 | 1.8299  | 0.068760  | .   |
| Educational_levelSecondary school | -4.7338e-01 | 4.1009e-01 | -1.1543 | 0.249745  |     |
| ARTpost ART initiation            | 1.5296e+00  | 5.9779e-02 | 25.5868 | < 2.2e-16 | *** |
| Baseline_VL1                      | -2.0864e-06 | 2.6911e-07 | -7.7530 | 4.538e-13 | *** |
| Residence_mod2                    | 2.4989e-01  | 5.4473e-02 | 4.5875  | 7.927e-06 | *** |
| Marital_statusStable              | 3.0457e-01  | 1.5489e-01 | 1.9664  | 0.050647  | .   |
| Marital_statusMany                | -7.8584e-01 | 2.5890e-01 | -3.0353 | 0.002724  | **  |
| s(Time_in_Months)Fx1              | -2.3766e+00 | 5.5497e-01 | -4.2824 | 2.875e-05 | *** |
| s(Baseline_BMI1)Fx1               | 5.6868e+00  | 1.1094e+00 | 5.1261  | 6.982e-07 | *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Covariance matrix of the random effects:

|                | (Intercept) | Time_in_Months |
|----------------|-------------|----------------|
| (Intercept)    | 8.687e-01   | 0.000e+00      |
| Time_in_Months | 0.000e+00   | 1.929e-16      |

Variances of the (random) smooth terms:

| s(Time_in_Months) | s(Baseline_BMI1) |
|-------------------|------------------|
| 28.94             | 6463.83          |

Residual scale parameter: 1.335

Log-likelihood: -21031

Tuning parameter: 0.0003077

Number of observations: 7019

Number of groups: 235

Quantile 0.5

Fixed effects:

|                                   | Estimate    | Std. Error | t value | Pr(> t )  |     |
|-----------------------------------|-------------|------------|---------|-----------|-----|
| (Intercept)                       | 2.1204e+01  | 5.3400e-01 | 39.7075 | < 2.2e-16 | *** |
| Age                               | 4.1792e-02  | 5.2561e-03 | 7.9511  | 1.358e-13 | *** |
| Educational_levelSecondary school | -1.6529e-02 | 6.6199e-01 | -0.0250 | 0.980105  |     |
| ARTpost ART initiation            | 1.5968e+00  | 4.0196e-02 | 39.7245 | < 2.2e-16 | *** |
| Baseline_VL1                      | -1.7910e-06 | 2.4096e-07 | -7.4325 | 3.088e-12 | *** |
| Residence_mod2                    | 9.9844e-02  | 3.3382e-02 | 2.9910  | 0.003132  | **  |
| Marital_statusStable              | 5.4241e-01  | 1.1403e-01 | 4.7568  | 3.772e-06 | *** |
| Marital_statusMany                | -8.4317e-01 | 1.0906e-01 | -7.7310 | 5.181e-13 | *** |
| s(Time_in_Months)Fx1              | -2.1985e+00 | 4.7348e-01 | -4.6432 | 6.222e-06 | *** |
| s(Baseline_BMI1)Fx1               | 5.5767e+00  | 1.3014e+00 | 4.2851  | 2.844e-05 | *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Covariance matrix of the random effects:

|                | (Intercept) | Time_in_Months |
|----------------|-------------|----------------|
| (Intercept)    | 3.543e-02   | 0.000e+00      |
| Time_in_Months | 0.000e+00   | 3.328e-17      |

Variances of the (random) smooth terms:

| s(Time_in_Months) | s(Baseline_BMI1) |
|-------------------|------------------|
| 36.74             | 7823.81          |

Residual scale parameter: 1.735

Log-likelihood: -20671

Tuning parameter: 0.0003077

Number of observations: 7019

Number of groups: 235

Quantile 0.75

Fixed effects:

|                                   | Estimate    | Std. Error | t value | Pr(> t )  |     |
|-----------------------------------|-------------|------------|---------|-----------|-----|
| (Intercept)                       | 2.4167e+01  | 1.0536e+00 | 22.9374 | < 2.2e-16 | *** |
| Age                               | 3.3143e-02  | 7.7590e-03 | 4.2716  | 3.006e-05 | *** |
| Educational_levelSecondary school | 3.8519e-01  | 1.0677e+00 | 0.3608  | 0.718663  |     |
| ARTpost ART initiation            | 1.5292e+00  | 5.4576e-02 | 28.0196 | < 2.2e-16 | *** |
| Baseline_VL1                      | -1.5700e-06 | 1.6001e-07 | -9.8123 | < 2.2e-16 | *** |
| Residence_mod2                    | 1.2747e-01  | 1.4362e-01 | 0.8876  | 0.375851  |     |
| Marital_statusStable              | 4.9069e-01  | 1.5944e-01 | 3.0776  | 0.002381  | **  |
| Marital_statusMany                | -1.1719e+00 | 2.5698e-01 | -4.5603 | 8.916e-06 | *** |
| s(Time_in_Months)Fx1              | -2.2829e+00 | 4.9993e-01 | -4.5665 | 8.680e-06 | *** |
| s(Baseline_BMI1)Fx1               | 5.7904e+00  | 1.2077e+00 | 4.7945  | 3.188e-06 | *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Covariance matrix of the random effects:

|                | (Intercept) | Time_in_Months |
|----------------|-------------|----------------|
| (Intercept)    | 2.453e-01   | 0.000e+00      |
| Time_in_Months | 0.000e+00   | 5.451e-17      |

Variances of the (random) smooth terms:

| s(Time_in_Months) | s(Baseline_BMI1) |
|-------------------|------------------|
| 30.28             | 6290.32          |

Residual scale parameter: 1.43

Log-likelihood: -21680

Tuning parameter: 0.0003077

Number of observations: 7019

Number of groups: 235

Quantile 0.85

Fixed effects:

|                                   | Estimate    | Std. Error | t value  | Pr(> t )  |     |
|-----------------------------------|-------------|------------|----------|-----------|-----|
| (Intercept)                       | 2.5845e+01  | 8.8147e-01 | 29.3207  | < 2.2e-16 | *** |
| Age                               | 3.1118e-02  | 1.2061e-02 | 2.5802   | 0.0105958 | *   |
| Educational_levelSecondary school | 5.0743e-01  | 9.6045e-01 | 0.5283   | 0.5978665 |     |
| ARTpost ART initiation            | 1.4546e+00  | 1.2778e-01 | 11.3838  | < 2.2e-16 | *** |
| Baseline_VL1                      | -1.7132e-06 | 1.5806e-07 | -10.8391 | < 2.2e-16 | *** |
| Residence_mod2                    | -6.0903e-02 | 1.9103e-01 | -0.3188  | 0.7501999 |     |
| Marital_statusStable              | 6.2981e-01  | 1.3699e-01 | 4.5974   | 7.594e-06 | *** |
| Marital_statusMany                | -1.5711e+00 | 4.2531e-01 | -3.6940  | 0.0002852 | *** |
| s(Time_in_Months)Fx1              | -2.2924e+00 | 3.8936e-01 | -5.8877  | 1.642e-08 | *** |
| s(Baseline_BMI1)Fx1               | 5.6964e+00  | 1.1277e+00 | 5.0513   | 9.898e-07 | *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Covariance matrix of the random effects:

|                | (Intercept) | Time_in_Months |
|----------------|-------------|----------------|
| (Intercept)    | 3.454e-01   | 0.000e+00      |
| Time_in_Months | 0.000e+00   | 7.671e-17      |

Variances of the (random) smooth terms:

| s(Time_in_Months) | s(Baseline_BMI1) |
|-------------------|------------------|
| 21.92             | 4979.39          |

Residual scale parameter: 1.068

Log-likelihood: -22972

Tuning parameter: 0.0002564

Number of observations: 7019

Number of groups: 235

Quantile 0.95

Fixed effects:

|                                   | Estimate    | Std. Error | t value | Pr(> t )  |     |
|-----------------------------------|-------------|------------|---------|-----------|-----|
| (Intercept)                       | 2.9379e+01  | 6.3243e-01 | 46.4537 | < 2.2e-16 | *** |
| Age                               | 2.0274e-02  | 1.7881e-02 | 1.1338  | 0.25823   |     |
| Educational_levelSecondary school | 8.3235e-01  | 5.5746e-01 | 1.4931  | 0.13699   |     |
| ARTpost ART initiation            | 1.7007e+00  | 1.3216e-01 | 12.8690 | < 2.2e-16 | *** |
| Baseline_VL1                      | -1.7020e-06 | 2.2139e-07 | -7.6881 | 6.714e-13 | *** |
| Residence_mod2                    | -8.8465e-01 | 2.2161e-01 | -3.9920 | 9.212e-05 | *** |
| Marital_statusStable              | 6.3398e-01  | 2.9600e-01 | 2.1418  | 0.03342   | *   |
| Marital_statusMany                | -3.6497e+00 | 4.4508e-01 | -8.2002 | 2.920e-14 | *** |
| s(Time_in_Months)Fx1              | -2.3324e+00 | 4.3732e-01 | -5.3335 | 2.605e-07 | *** |
| s(Baseline_BMI1)Fx1               | 5.2604e+00  | 1.0753e+00 | 4.8920  | 2.056e-06 | *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Covariance matrix of the random effects:

|                | (Intercept) | Time_in_Months |
|----------------|-------------|----------------|
| (Intercept)    | 4.675e-02   | 0.000e+00      |
| Time_in_Months | 0.000e+00   | 1.044e-17      |

Variances of the (random) smooth terms:

| s(Time_in_Months) | s(Baseline_BMI1) |
|-------------------|------------------|
| 10.13             | 2183.69          |

Residual scale parameter: 0.4828

Log-likelihood: -26193

Tuning parameter: 0.0003846

Number of observations: 7019

Number of groups: 235

Quantile 0.99

Fixed effects:

|                                   | Estimate    | Std. Error | t value  | Pr(> t )  |     |
|-----------------------------------|-------------|------------|----------|-----------|-----|
| (Intercept)                       | 2.9479e+01  | 7.2187e-01 | 40.8373  | < 2.2e-16 | *** |
| Age                               | 3.5241e-02  | 3.2735e-02 | 1.0765   | 0.2829879 |     |
| Educational_levelSecondary school | 2.0311e+00  | 9.3341e-01 | 2.1760   | 0.0307302 | *   |
| ARTpost ART initiation            | 2.8819e+00  | 2.1028e-01 | 13.7050  | < 2.2e-16 | *** |
| Baseline_VL1                      | -1.6168e-06 | 2.1655e-07 | -7.4661  | 2.531e-12 | *** |
| Residence_mod2                    | -1.4498e+00 | 3.0687e-01 | -4.7243  | 4.356e-06 | *** |
| Marital_statusStable              | 2.8311e+00  | 1.8621e-01 | 15.2040  | < 2.2e-16 | *** |
| Marital_statusMany                | -4.2155e+00 | 1.6546e-01 | -25.4775 | < 2.2e-16 | *** |
| s(Time_in_Months)Fx1              | -1.6920e+00 | 4.6815e-01 | -3.6143  | 0.0003815 | *** |
| s(Baseline_BMI1)Fx1               | 5.2328e+00  | 1.1256e+00 | 4.6487   | 6.075e-06 | *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Covariance matrix of the random effects:

|                | (Intercept) | Time_in_Months |
|----------------|-------------|----------------|
| (Intercept)    | 3.326e-03   | 0.000e+00      |
| Time_in_Months | 0.000e+00   | 2.963e-18      |

Variances of the (random) smooth terms:

| s(Time_in_Months) | s(Baseline_BMI1) |
|-------------------|------------------|
| 2.669             | 576.902          |

Residual scale parameter: 0.1276

Log-likelihood: -30865

Tuning parameter: 0.0002404

Number of observations: 7019

Number of groups: 235