

Spin-textures of medium-body boson systems with trapped spin-f cold atoms

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Appendix: Diagonalization of the Hamiltonian based on the Fock-states

For spin-f boson systems, we introduce the Fock-states $|\alpha\rangle \equiv |N_f^\alpha, N_{f-1}^\alpha, \dots, N_{-f}^\alpha\rangle$, where N_μ^α is the number of particles in magnetic component μ ($-f \leq \mu \leq f$), $\sum_\mu N_\mu^\alpha = N$ and $\sum_\mu \mu N_\mu^\alpha = M$, the total magnetization. They are adopted as basis-states for the diagonalization of H_{spin} . The matrix element is

$$\begin{aligned} \langle \alpha' | H_{\text{spin}} | \alpha \rangle = & \frac{1}{2} \sum_{\mu' \nu' \mu \nu} \delta_{\mu' + \nu', \mu + \nu} \sum_{\lambda} g_{\lambda} C_{f\mu'; f\nu'}^{\lambda, \mu' + \nu'} C_{f\mu; f\nu}^{\lambda, \mu + \nu} \\ & (\bar{\delta}_{\mu' \nu'} \bar{\delta}_{\mu \nu} \sqrt{N_{\mu'}^{\alpha'} N_{\nu'}^{\alpha'} N_{\mu}^{\alpha} N_{\nu}^{\alpha}} \delta_{[\alpha']_{\mu' \nu'}; [\alpha]_{\mu \nu}} \\ & + \bar{\delta}_{\mu' \nu'} \delta_{\mu \nu} \sqrt{N_{\mu'}^{\alpha'} N_{\nu'}^{\alpha'} N_{\mu}^{\alpha} (N_{\mu}^{\alpha} - 1)} \delta_{[\alpha']_{\mu' \nu'}; [\alpha]_{\mu \mu}} \\ & + \delta_{\mu' \nu'} \bar{\delta}_{\mu \nu} \sqrt{N_{\mu'}^{\alpha'} (N_{\mu'}^{\alpha'} - 1) N_{\mu}^{\alpha} N_{\nu}^{\alpha}} \delta_{[\alpha']_{\mu' \mu'}; [\alpha]_{\mu \nu}} \\ & + \delta_{\mu' \nu'} \delta_{\mu \nu} \sqrt{N_{\mu'}^{\alpha'} (N_{\mu'}^{\alpha'} - 1) N_{\mu}^{\alpha} (N_{\mu}^{\alpha} - 1)} \delta_{[\alpha']_{\mu' \mu'}; [\alpha]_{\mu \mu}}), \end{aligned}$$

where $|\alpha'\rangle \equiv |N_f^{\alpha'}, N_{f-1}^{\alpha'}, \dots\rangle$, $\delta_{\mu \nu} = 1$ or 0 (if $\mu = \nu$ or $\neq \nu$), $\bar{\delta}_{\mu \nu} = 1 - \delta_{\mu \nu}$, $[\alpha]$ denotes the set of $2f + 1$ numbers $N_f^\alpha, \dots, N_{-f}^\alpha$, $[\alpha]_{\mu \nu}$ ($\mu \neq \nu$) denotes the set $[\alpha]$ except that N_μ^α is changed to $N_\mu^\alpha - 1$ and N_ν^α is changed to $N_\nu^\alpha - 1$, $[\alpha]_{\mu \mu}$ denotes the set $[\alpha]$ except that N_μ^α is changed to $N_\mu^\alpha - 2$, $\delta_{[\beta]; [\alpha]} = 1$ (if all the $2f + 1$ numbers in $[\beta]$ are one-to-one identical to those in $[\alpha]$) or 0 (otherwise), the Clebsch-Gordan coefficients have been introduced. In the first (left) summation, each index runs from $-f$ to f . In the second summation, λ should be even and runs from 0 to $2f$. With these matrix elements, the diagonalization can be carried out numerically. Incidentally, 151 (920) basis-states are used when $f = 3$ and $N = 8$ (13).