

Supplementary Materials

This Supplementary Material details the flip-flop dual-component model and its relations to the Klein-Gordon and **Schrödinger equation**. First, we offer a simple dual-component model to describe quantum behavior of relativistic particles in 2D space-time. Let us consider a pair of real functions to describe this binary system with dual components. The most elementary equation is a set of linearly coupled first-order differential equations in time and space as given by

$$\begin{aligned}\frac{\partial}{\partial t} f(x,t) &= \omega g(x,t) \\ \frac{\partial}{\partial t} g(x,t) &= -\omega f(x,t) \\ \frac{\partial}{\partial x} f(x,t) &= -k g(x,t) \\ \frac{\partial}{\partial x} g(x,t) &= k f(x,t)\end{aligned}\tag{S1}$$

The above equation describes flip-flop dynamics in time and space between two components $f(x,t)$ and $g(x,t)$. The above rate equation in 2D space-time has equal footing in space and time, analogous to Dirac's treatment of a relativistic electron, except that he considered 4D space-time and introduced gamma matrices or spinor matrices that lead to the electron spin. With two fundamental parameters frequency ω . And wave vector k . These coupled equations describe oscillations in time and space with a solution and, represent a wave traveling along the x-axis, where A is an amplitude and ϕ a phase. By changing either or the solution represents a wave traveling in an opposite direction. A photon's most fundamental physical properties are frequency and light velocity $c \omega = k c$. For an acoustic wave, the velocity c is replaced by sound velocity. For a relativistic particle with a rest mass; the most fundamental physical parameters are its rest mass and momentum. Based on de Broglie's hypothesis of matter-wave duality and Einstein's relativity theory of mass-energy equivalence

$$\begin{aligned}\omega &= c \sqrt{k^2 + m_0^2 c^2} / \hbar \\ k &= p / \hbar\end{aligned}\tag{S2}$$

one could replace the frequency and wave vector in eq. (1) by the above relations. For a photon without a rest mass, the solution of Eq. (S1) becomes

$$\begin{aligned}f(x,t) &= A \cos(k(x - ct) + \phi) \\ g(x,t) &= A \sin(k(x - ct) + \phi)\end{aligned}\tag{S3}$$

By defining a complex function then, one has $\Psi(x,t) = e^{i(k(x-ct)+\phi)}$ which represents the complex electric or magnetic field solution for a light wave. For a relativistic particle, one can derive from Eq. (S1)

$$\frac{\partial^2}{\partial t^2} \Psi(x,t) = -\frac{m_0^2 c^4}{\hbar^2} \Psi(x,t) + c^2 \frac{\partial^2}{\partial x^2} \Psi(x,t) \quad (\text{S4})$$

Using the dual-component model, we could re-derive the above Klein-Gordon equation for a scalar particle with a rest mass m_0 . Unlike this K-G equation which involves 2nd derivatives in space and time of a complex-value wave function, our dual-component model in Eq. (S1) involves only the 1st order derivative with a real-value function.

For a free non-relativistic particle one has $\omega \approx m_0 c^2 / \hbar + \hbar k^2 / 2m_0$, one can reduce eq. (1) to

$$\begin{aligned} \frac{\partial}{\partial t} f(x,t) &= \left(\frac{m_0 c^2}{\hbar} + \frac{\hbar k^2}{2m_0} \right) g(x,t) \\ \frac{\partial}{\partial t} g(x,t) &= -\left(\frac{m_0 c^2}{\hbar} + \frac{\hbar k^2}{2m_0} \right) f(x,t) \\ \frac{\partial}{\partial x} f(x,t) &= -k g(x,t) \\ \frac{\partial}{\partial x} g(x,t) &= k f(x,t) \end{aligned} \quad (\text{S5})$$

The fast oscillating dependence in the above rate equation can be transformed away by redefining a new set of functions $F(x,t) \equiv \cos(m_0 c^2 t / \hbar) f(x,t) + \sin(m_0 c^2 t / \hbar) g(x,t)$ and $G(x,t) \equiv -\sin(m_0 c^2 t / \hbar) f(x,t) + \cos(m_0 c^2 t / \hbar) g(x,t)$, one has

$$\begin{aligned} \frac{\partial}{\partial t} F(x,t) &= \frac{\hbar k^2}{2m_0} G(x,t) \\ \frac{\partial}{\partial t} G(x,t) &= -\frac{\hbar k^2}{2m_0} F(x,t) \\ \frac{\partial}{\partial x} F(x,t) &= -k G(x,t) \\ \frac{\partial}{\partial x} G(x,t) &= k F(x,t) \end{aligned} \quad (\text{S6})$$

with $\Psi(x,t) \equiv F(x,t) + i G(x,t)$ one can obtain

$$i\hbar \frac{\partial}{\partial t} \Psi(x,t) = -\frac{\hbar^2}{2m_0} \frac{\partial^2}{\partial x^2} \Psi(x,t) \quad (\text{S7})$$

Here we pedagogically re-derive the **Schrödinger** equation. We use a simple two-component model involving elementary linearly coupled differential equations and Planck-Einstein-de Broglie relations.

The primary purpose of proposing the flip-flop dual-component model in 2D space-time is to treat mass oscillations due to Heisenberg's uncertainty principle for a relativistic neutrino or electron. This dual-component model with linearized time and space derivatives could be extended to 4D space-time to treat other elementary particles, such as leptons, quarks, and gauge bosons. Further development and new results will be published elsewhere in the future.