

On the Contribution of Work or Heat in Exchanged Energy via Interaction in Open bipartite Quantum Systems

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Supplementary Note 1

The infinitesimal change in the eigenvectors of the state of the system $\rho_A(t)$ in Eq. (13) is achieved as

$$\begin{aligned} d(|\psi_i(t)\rangle\langle\psi_i(t)|) &= |\psi_i(t+dt)\rangle\langle\psi_i(t+dt)| - |\psi_i(t)\rangle\langle\psi_i(t)| \\ &= U(dt)|\psi_i(t)\rangle\langle\psi_i(t)|U^\dagger(dt) - |\psi_i(t)\rangle\langle\psi_i(t)|, \end{aligned} \quad (1)$$

where $U(dt)$ is the unitary operator transforming the orthogonal basis $\{|\psi_i(t)\rangle\}_{i=1}^d$ to another orthogonal basis $\{|\psi_i(t+dt)\rangle\}_{i=1}^d$ ^{S2}. Using Eq. (1) and Eq. (6) of the main text we have

$$\sum_{i=1}^d p_i(t) d(|\psi_i(t)\rangle\langle\psi_i(t)|) = U(dt)\rho_A(t)U^\dagger(dt) - \rho_A(t). \quad (2)$$

Eq. (2) is the unitary part of the total change in the state of the system $\rho_A(t)$. Now substituting Eq. (2) into Eq. (13) of the main text the second term on the right hand side of Eq. (13) reads

$$\text{tr}\left\{\sum_{i=1}^d p_i(t) d(|\psi_i(t)\rangle\langle\psi_i(t)|) H_A(t)\right\} = \text{tr}\{d\rho_A^U(t) H_A(t)\}, \quad (3)$$

in which $d\rho_A^U(t) \equiv U(dt)\rho_A(t)U^\dagger(dt) - \rho_A(t)$. Thus Eq. (3) is in fact the energy which is unitarily exchanged between the two quantum systems through the interaction.

Supplementary Note 2

The average of the internal energy of system A at time t is defined as ^{S1}

$$\begin{aligned} \langle E_A(t) \rangle &= \text{tr}\{\rho_A(t) H_A(t)\} \\ &= \text{tr}\left\{\sum_{i=1}^d p_i(t) |\psi_i(t)\rangle\langle\psi_i(t)| H_A(t)\right\} \\ &= \sum_{i=1}^d p_i(t) \langle\psi_i(t)| H_A(t) |\psi_i(t)\rangle. \end{aligned} \quad (4)$$

On the other hand

$$H_A(t) = \sum_j E_j(t) |E_j(t)\rangle\langle E_j(t)|, \quad (5)$$

therefore

$$\begin{aligned}\langle E_A(t) \rangle &= \sum_{i=1}^d \sum_j p_i(t) E_j(t) |\langle \psi_i(t) | E_j(t) \rangle|^2 \\ &= \sum_j q_j(t) E_j(t),\end{aligned}\tag{6}$$

where

$$\begin{aligned}q_j(t) &= \sum_{i=1}^d p_i(t) |\langle \psi_i(t) | E_j(t) \rangle|^2, \\ &= \sum_{i=1}^d p_i(t) R_{i \rightarrow j}(t),\end{aligned}\tag{7}$$

with $R_{i \rightarrow j}(t)$ the transition probability from the eigenbasis $|\psi_i(t)\rangle$ to the eigenbasis $|E_j(t)\rangle$. The connection between $p_i(t)$ and $q_i(t)$ is given through Eq. (7). Now from Eqs. (6) and (7) we have

$$\langle E_A(t) \rangle = \sum_{i=1}^d \sum_j p_i(t) R_{i \rightarrow j}(t) E_j(t).\tag{8}$$

Then

$$d\langle E_A(t) \rangle = \underbrace{\sum_{i=1}^d \sum_j dp_i(t) R_{i \rightarrow j}(t) E_j(t)}_{d\langle Q_A(t) \rangle} + \underbrace{\sum_{i=1}^d \sum_j p_i(t) d(R_{i \rightarrow j}(t) E_j(t))}_{d\langle R(t) \rangle}.$$

Hence we have

$$d\langle Q_A(t) \rangle = \sum_{i=1}^d \sum_j dp_i(t) R_{i \rightarrow j}(t) E_j(t),\tag{9}$$

$$d\langle W_A(t) \rangle = \sum_{i=1}^d \sum_j p_i(t) dR_{i \rightarrow j}(t) E_j(t) + \sum_{i=1}^d \sum_j p_i(t) R_{i \rightarrow j}(t) dE_j(t),$$

which means that two terms contribute to $dW(t)$, one is originating from the variation of transition probabilities (the first term) and the other originates from the variations of the energy levels (the second term).

References

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- S2.** Heinosaari, T. & Ziman, M. The mathematical language of quantum theory: from uncertainty to entanglement (Cambridge University Press, 2011).