

Appendix

The coefficients of Eq. (25-27) may be listed as follows:

$$\begin{aligned} g(l_1, l_2) &= I_1(kl_1)K_1(kl_2) - I_1(kl_2)K_1(kl_1), \\ h(l_1, l_2) &= I_0(kl_1)K_1(kl_2) + I_1(kl_2)K_0(kl_1), \\ \Lambda &= (k\varepsilon_m^{(2)}h(r_1, R)g(r_2, R) - k\varepsilon_m^{(1)}h(r_2, R)g(r_1, R)) + ik(\varepsilon_m^{(2)} - \varepsilon_m^{(1)})(h(r_1, R)h(r_2, R) + g(r_1, R)g(r_2, R))\zeta_z \\ &\quad + k(h(r_1, R)g(r_2, R)\varepsilon_m^{(1)} - h(r_2, R)g(r_1, R)\varepsilon_m^{(2)})\zeta_z^2. \end{aligned}$$

The coefficients of Eq. (29) may be listed as follows:

$$\begin{aligned} A_1 &= ((\rho_2 + \mu'_2)h(r_2, R)g(r_1, R) - (\rho_1 + \mu'_1)h(r_1, R)g(r_2, R))/(kg(r_1, R)g(r_2, R)), \\ A_2 &= (U_{02}(\rho_2 + \mu'_2)h(r_2, R)g(r_1, R) - U_{01}(\rho_1 + \mu'_1)h(r_1, R)g(r_2, R))/(kg(r_1, R)g(r_2, R)) \\ A_3 &= \mu_2(2K_2N_2 + \mu_2)h(r_2, R)/(k(K_2N_2 + \mu_2)g(r_2, R)) - \mu_1(2K_1N_1 + \mu_1)h(r_1, R)/(k(K_1N_1 + \mu_1)g(r_1, R)) \\ &\quad + \alpha_1(\rho_1\mu'_2h(r_2, R)g(r_1, R) - \rho_2\mu'_1h(r_1, R)g(r_2, R) + g(r_1, R)h(r_2, R)\rho_1\rho_2)/(k\rho_1\rho_2g(r_1, R)g(r_2, R)) \\ &\quad + i((\rho_2U_{02}h(r_2, R)/g(r_2, R)) - (\rho_1U_{01}h(r_1, R)/g(r_1, R))), \\ A_4 &= (iH_0^2(\varepsilon_m^{(1)} - \varepsilon_m^{(2)})^2h(r_1, R)h(r_2, R))/(h(r_1, R)g(r_2, R)\varepsilon_m^{(2)} - h(r_2, R)g(r_1, R)\varepsilon_m^{(1)}) \\ &\quad - U_{01}h(r_1, R)(\mu_1(2K_1N_1 + \mu_1) + ik\rho_1U_{01}(K_1N_1 + \mu_1))/(k(K_1N_1 + \mu_1)g(r_1, R)) \\ &\quad + U_{02}h(r_2, R)(\mu_2(2K_2N_2 + \mu_2) + ik\rho_2U_{02}(K_2N_2 + \mu_2))/(k(K_2N_2 + \mu_2)g(r_2, R)), \\ A_5 &= \frac{T}{R^2} - K_1N_1\alpha_1\mu_1h(r_1, R)/k\rho_1(K_1N_1 + \mu_1)g(r_1, R) + K_2N_2\alpha_1\mu_2h(r_2, R)/k\rho_2(K_2N_2 + \mu_2)g(r_2, R) - \alpha_1\mu_1h(r_1, R)/k\rho_1g(r_1, R) \\ &\quad + \alpha_1\mu_2h(r_2, R)/k\rho_2g(r_2, R) - i(\alpha_1U_{01}h(r_1, R)/g(r_1, R) - \alpha_1U_{02}h(r_2, R)/g(r_2, R)). \end{aligned}$$

The coefficients of Eq. (32) may be listed as follows:

$$\begin{aligned} a_0 &= (h(r_1, R)/k g(r_1, R))(\rho_1 + \mu'_1/k_1) - (h(r_2, R)/k g(r_2, R))(\rho_2 + \mu'_2/k_1), \\ a_1 &= (h(r_2, R)/g(r_2, R))(2\rho_2 + \mu'_2/k_1)U_{02} - (h(r_1, R)/g(r_1, R))(2\rho_1 + \mu'_1/k_1)U_{01}, \\ b_1 &= (h(r_1, R)/k \rho_1g(r_1, R))(K_1N_1 + \mu_1/k_1)((\mu_1\rho_1/k_1)(2K_1N_1 + \mu_1/k_1) + \alpha_1(K_1N_1 + \mu_1/k_1)(\rho_1 + \mu'_1/k_1)) \\ &\quad - (h(r_2, R)/k \rho_2g(r_2, R))(K_2N_2 + \mu_2/k_1)((\mu_2\rho_2/k_1)(2K_2N_2 + \mu_2/k_1) + \alpha_1(K_2N_2 + \mu_2/k_1)(\rho_2 + \mu'_2/k_1)), \\ a_2 &= (1/R^2 - k^2)T - (h(r_1, R)/k \rho_1g(r_1, R))((\alpha_1\mu_1(2K_1N_1 + \mu_1/k_1)/k_1(K_1N_1 + \mu_1/k_1)) - k^2\rho_1^2U_{01}^2) \\ &\quad + (h(r_2, R)/k \rho_2g(r_2, R))((\alpha_1\mu_2(2K_2N_2 + \mu_2/k_1)/k_1(K_2N_2 + \mu_2/k_1)) - k^2\rho_2^2U_{02}^2) \\ &\quad + ((kH_0^2(\varepsilon_m^{(1)} - \varepsilon_m^{(2)})^2h(r_1, R)h(r_2, R))/(h(r_2, R)g(r_1, R)\varepsilon_m^{(1)} - h(r_1, R)g(r_2, R)\varepsilon_m^{(2)})), \\ b_2 &= (\mu_2U_{02}h(r_2, R)(2K_2N_2 + \mu_2/k_1)/k_1g(r_2, R)(K_2N_2 + \mu_2/k_1)) - (\mu_1U_{01}h(r_1, R)(2K_1N_1 + \mu_1/k_1)/k_1g(r_1, R)(K_1N_1 + \mu_1/k_1)) \\ &\quad - \alpha_1U_{01}(h(r_1, R)/g(r_1, R)) + \alpha_1U_{02}(h(r_2, R)/g(r_2, R)). \end{aligned}$$