

A Hierarchical Approach for Evaluating Athlete Performance with an Application in Elite Basketball

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SPECIFICATIONS OF VARIANCE COEFFICIENT MATRICES

We consider the specifications of the general matrices under the model (3) for a given season i (V_i), or for a given team j within season i (V_{ij}), or for a given athlete k within team j within season i (V_{ijk}). First, we defined the variance of $\gamma_{ijkl}^{(S)}$, which is given by

$$\text{Var}\left(\gamma_{ijkl}^{(S)}\right) = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma^2 = R_1,$$

and covariances of the joint variability of $\gamma_{ijkl}^{(S)}$ and $\gamma_{i'j'k'l'}^{(S)}$, with $ijkl \neq i'j'k'l'$, is given by

$$\begin{aligned} \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{ijkl}^{(S)}\right) &= \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = R_2; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{ijk'l'}^{(S)}\right) &= \sigma_1^2 + \sigma_2^2 = R_3; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{ij'kl}^{(S)}\right) &= \sigma_1^2; \\ \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'jkl}^{(S)}\right) &= 0; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{ijk'l'}^{(S)}\right) &= \sigma_1^2 + \sigma_2^2 = R_3; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{ij'kl}^{(S)}\right) &= \sigma_1^2; \\ \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'kl}^{(S)}\right) &= 0; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'k'l}^{(S)}\right) &= \sigma_1^2; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'kl}^{(S)}\right) &= 0; \\ \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'k'l}^{(S)}\right) &= 0; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'k'l'}^{(S)}\right) &= \sigma_1^2; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'k'l'}^{(S)}\right) &= \sigma_1^2; \\ \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'k'l'}^{(S)}\right) &= 0; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'k'l'}^{(S)}\right) &= 0; & \text{Cov}\left(\gamma_{ijkl}^{(S)}, \gamma_{i'j'k'l'}^{(S)}\right) &= 0; \end{aligned}$$

Based on the variance and covariances defined above, the variance-covariance matrices can be defined as follows

$$\mathbf{V}_{ijk} = \begin{bmatrix} R_1 & R_2 & \dots & R_2 \\ R_2 & R_1 & \dots & R_2 \\ \vdots & \vdots & \ddots & \vdots \\ R_2 & R_2 & \dots & R_1 \end{bmatrix}; \quad \mathbf{V}_{ij} = \begin{bmatrix} \mathbf{V}_{ijk} & \mathbf{R}_3 & \dots & \mathbf{R}_3 \\ \mathbf{R}_3 & \mathbf{V}_{ijk} & \dots & \mathbf{R}_3 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_3 & \mathbf{R}_3 & \dots & \mathbf{V}_{ijk} \end{bmatrix}; \quad \mathbf{V}_i = \begin{bmatrix} \mathbf{V}_{ij} & \mathbf{R}_4 & \dots & \mathbf{R}_4 \\ \mathbf{R}_4 & \mathbf{V}_{ij} & \dots & \mathbf{R}_4 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_4 & \mathbf{R}_4 & \dots & \mathbf{V}_{ij} \end{bmatrix},$$

where

$$\mathbf{R}_3 = \begin{bmatrix} R_3 & R_3 & \dots & R_3 \\ R_3 & R_3 & \dots & R_3 \\ \vdots & \vdots & \ddots & \vdots \\ R_3 & R_3 & \dots & R_3 \end{bmatrix}; \text{ and } \mathbf{R}_4 = \begin{bmatrix} \sigma_1^2 & \sigma_1^2 & \dots & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \dots & \sigma_1^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_1^2 & \sigma_1^2 & \dots & \sigma_1^2 \end{bmatrix}.$$