

Simulating quantum chaos on a quantum computer

Amit Anand^{1,3,4,*}, Sanchit Srivastava^{2,3,4,+}, Sayan Gangopadhyay^{3,4}, and Shohini Ghose^{4,5}

¹Department of Mechanical Engineering, Indian Institute of Engineering Science And Technology, Shibpur, Howrah-711103, West Bengal, India.

²School of Physics, Indian Institute of Science Education and Research, Thiruvananthapuram - 695551, Kerala, India

³Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

⁴Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

⁵Department of Physics and Computer Science, Wilfrid Laurier University, Waterloo, Ontario, Canada N2L 3C5

*Corresponding author: a63anand@uwaterloo.ca

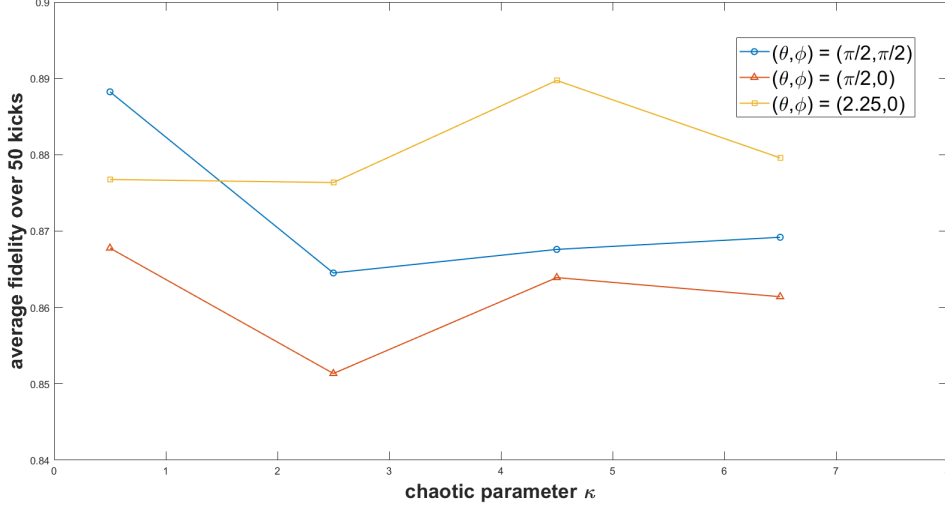
+Corresponding author: s49sriva@uwaterloo.ca

Supplementary Note 1

Errors in IBMQ

We have used IBM Q Experience processors - IBM *vigo* and IBM *quito* - to obtain experimental results presented in this paper. Both processors have 5-qubits and a Quantum Volume (QV) of 16. These processors were chosen for their low single-qubit and CNOT errors (Error values available on IBM Q experience website). The error in the experimental values also depends on the specific qubits chosen for the computation. For each processor, the computational qubits were chosen so as to minimize the error. The sources of error can be both statistical fluctuations and systematic errors in the hardware implementation [1]. Relaxation and decoherence of the qubits in a noisy environment is a major source of systematic errors. The CNOT gate, which involves two-qubit operations is ten times as noisy as the single qubit rotations. This is because of higher errors introduced by unwanted qubit interactions during the implementation of multi-qubit operations. There are also read-out errors introduced by the final quantum measurement process. These errors could be theoretically modeled and the experimental data could be filtered to account for them. Quantum error correcting circuits could also be used to

mitigate the systematic errors. This could lead to better agreement between theoretical and experimental values of the physical quantities like concurrence. However, we do not investigate error mitigation in this paper.



Supplementary Figure 1: Average fidelity for 3 different initial conditions and 4 values of κ . We see that irrespective of the initial points or value of chaoticity parameter, the average fidelity is within the expected range. We conclude that the initial states or chaoticity parameter values do not affect the average fidelity.

Since IBM Q Experience devices are universal quantum computers, one can implement any unitary transformation using quantum circuits which are composed of basic quantum logic gates (single qubit rotations and the CNOT operation). The 2-qubit quantum kicked top circuit was composed in IBM's Qiskit - a python based programming language for the IBM Q Experience. Each circuit was run for 5000 shots to obtain the measurement statistics. State tomography was performed using the tools provided by Qiskit for the reconstruction of the density matrix. The reconstructed density matrix was used to calculate the relevant physical quantities such as fidelity and concurrence.

Supplementary Note 2

Concurrence as a Measure of Entanglement

Entanglement of formation is the most widely used measure of entanglement [2]. For a general state ρ , it is the minimum average von Neumann entropy overall pure-state decomposition of ρ . While this minimum is in general challenging to compute efficiently, for two qubits, the concurrence is more popular for quantifying entanglement in bipartite qubit systems. It is efficient to compute and is a monotonically increasing function of entanglement of formation. Concurrence is computed as follows [3]. For a two-qubit density matrix ρ , first the spin flipped state $\tilde{\rho} = \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$ (where σ_y is Pauli matrix and ρ^* is complex conjugate of ρ in the standard basis) is computed. Then concurrence is defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}) \quad (1)$$

where λ_i are eigenvalues of $\rho\tilde{\rho}$ such that $\lambda_4 \leq \lambda_3 \leq \lambda_2 \leq \lambda_1$ and $0 \leq C \leq 1$. It is 0 for separable states and unity for the Bell states. The relationship between concurrence and chaos has been investigated in previous works [4][5].

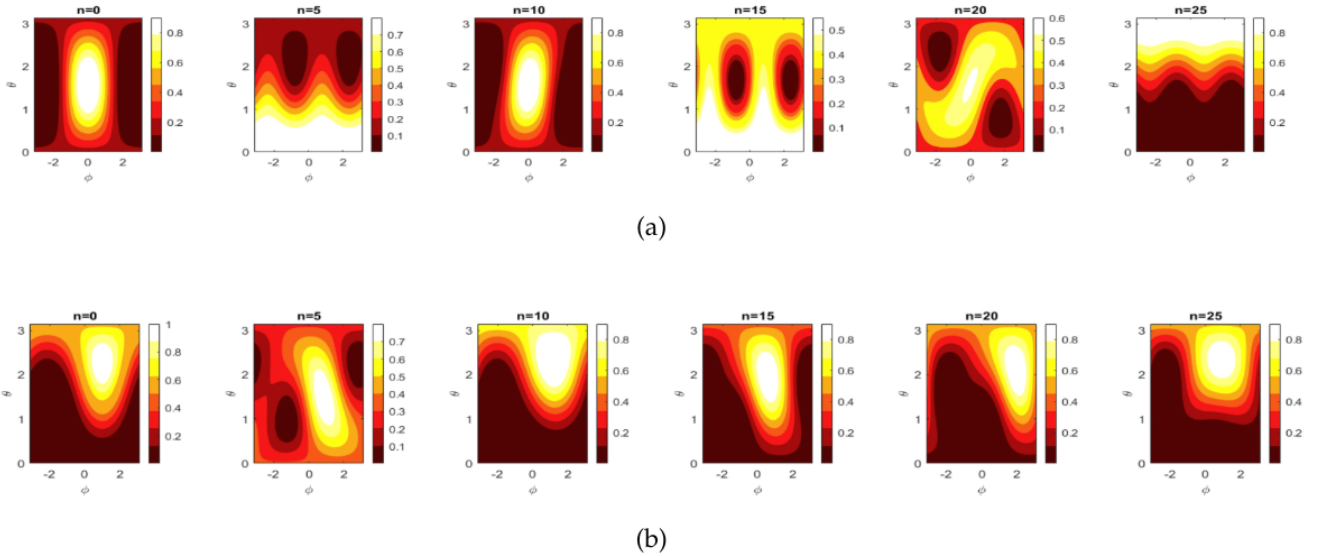
Supplementary note 3

Husimi phase space distribution and O_{SCS}

To study quantum-classical correspondence in the quantum kicked top, its Husimi distribution is often compared with the classical phase space distribution [6]. Husimi distribution has also been used as a visual aid to study dynamical tunneling in the same model [7]. It is a positive valued quasi-probability distribution given by

$$Q(\theta, \phi) = \frac{2j+1}{4\pi} \langle \theta, \phi | \rho | \theta, \phi \rangle \quad (2)$$

which is equal to $\frac{2j+1}{4\pi} |\langle \theta, \phi | \psi \rangle|^2$ for pure states; the overlap of a pure angular momentum state $|\psi\rangle$ and spin coherent state $|\theta, \phi\rangle$. We drop the normalisation constant of $\frac{2j+1}{4\pi}$ from this expression as j remains fixed. For a given state ρ , the maxima of it's Husimi distribution corresponds to the measure O_{SCS} . The Husimi distributions for points $(\theta, \phi) = (\frac{\pi}{2}, 0)$ and $(2.25, 1.0)$ for $\kappa = 2.5$ and various values of N are given in Fig.2. We see that the Husimi distribution for $(\frac{\pi}{2}, 0)$ is more delocalized, which corresponds to a lower O_{SCS} value in Figure 7 of the main article. On the other hand, the Husimi Distribution remains more localized, corresponding to a higher O_{SCS} value.

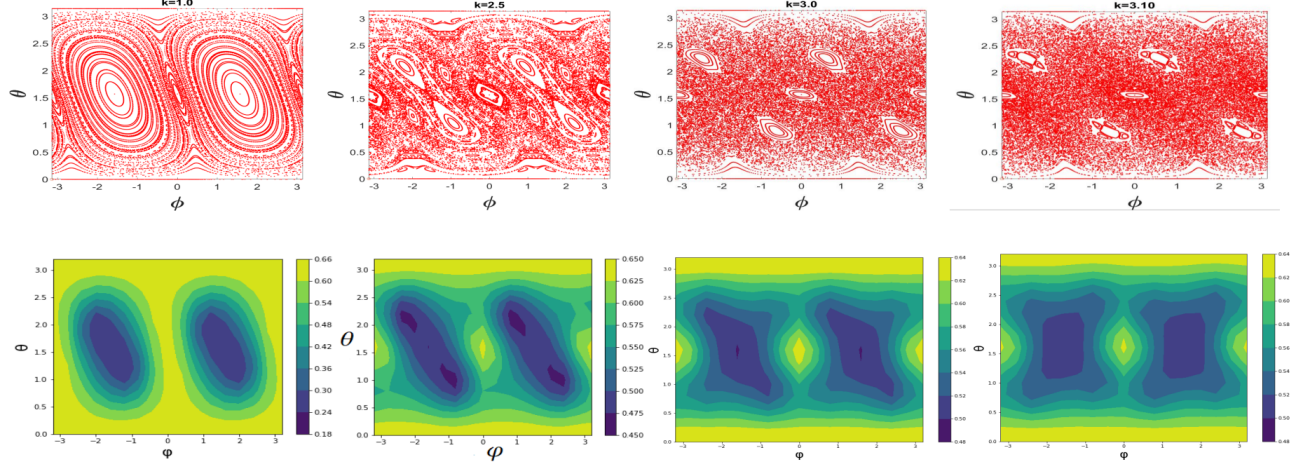


Supplementary Figure 2: Husimi distribution to visualize delocalization of two different initial states. $(\theta, \phi) = (\frac{\pi}{2}, 0)$ (top), $(\theta, \phi) = (2.25, 1.0)$ for $\kappa = 2.5$ (bottom)

Supplementary Note 4

Average concurrence contour plots for different values of κ

In this section we show the correspondence between the classical phase space and the average concurrence contour plots for different values of κ in the range 0 to π . For low values of the chaoticity parameter (say, $\kappa = 1$), the classical phase space is dominated by regular regions.



Supplementary Figure 3: Classical phase space and the corresponding average concurrence contour plots for $\kappa = 1.0, 2.5, 3.0$ and 3.10 . The contour plots have been obtained by averaging over 200 kicks on the simulator backend of IBMQ.

In this case, the corresponding contour plot of concurrence reflects this regular structure of the phase space which is consistent with the behaviour as mentioned in the results in the main text, i.e., the lowest average concurrence values correspond to the fixed points, intermediate values to chaotic regions and highest values appear for the points of period-4 orbits. For higher values of kappa, the features in the quantum dynamics become too localized to be exactly resolvable by the contour plots of 2-qubit average concurrence. However, we note that the contour plots (for $\kappa = 3.0$ and $\kappa = 3.10$) still accurately reflect the behaviour of the phase space regions as stated in results in the main text. We have considered the range $\kappa = (0, \pi)$ as for the 2-qubit case the concurrence behaviour is symmetric about $k = \pi$ (refer Figure 4 of the main article) with 2π being the periodicity.

References

1. Garcíea-Pérez, G., Rossi, M. A. & Maniscalco, S. IBM Q Experience as a versatile experimental testbed for simulating open quantum systems. *npj Quantum Information* **6**, 1–10. <https://www.nature.com/articles/s41534-019-0235-y> (2020).
2. Fan, H., Matsumoto, K. & Imai, H. Quantify entanglement by concurrence hierarchy. *Journal of Physics A: Mathematical and General* **36**, 4151–4158. <https://iopscience.iop.org/article/10.1088/0305-4470/36/14/316> (2021) (Mar. 2003).

3. Wootters, W. K. Entanglement of Formation of an Arbitrary State of Two Qubits. *Phys. Rev. Lett.* **80**, 2245–2248. <https://link.aps.org/doi/10.1103/PhysRevLett.80.2245> (10 Mar. 1998).
4. Bettelli, S. & Shepelyansky, D. L. Entanglement versus relaxation and decoherence in a quantum algorithm for quantum chaos. *Phys. Rev. A* **67**, 054303. <https://link.aps.org/doi/10.1103/PhysRevA.67.054303> (5 May 2003).
5. Wang, X., Ghose, S., Sanders, B. C. & Hu, B. Entanglement as a signature of quantum chaos. *Phys. Rev. E* **70**, 016217. <https://link.aps.org/doi/10.1103/PhysRevE.70.016217> (1 July 2004).
6. Kumari, M. & Ghose, S. Quantum-classical correspondence in the vicinity of periodic orbits. *Phys. Rev. E* **97**, 052209. <https://link.aps.org/doi/10.1103/PhysRevE.97.052209> (5 May 2018).
7. Dogra, S., Madhok, V. & Lakshminarayan, A. Quantum signatures of chaos, thermalization, and tunneling in the exactly solvable few-body kicked top. *Phys. Rev. E* **99**, 062217. <https://link.aps.org/doi/10.1103/PhysRevE.99.062217> (6 June 2019).