

Supplementary Material

Besides the existence of long-range Néel order in Néel phase, one question raised naturally is whether the long-range order exists in the nematic or chiral phase in our model. To deep insight into the characteristics of the ground state, we define the following types of two-point correlation to examine our model further. Due to the translational invariance of the Hamiltonian Eq. ??, the two-point correlation that is used to characterize our model can be defined as

$$\begin{aligned} G_r^{xy} &= \langle S_i^x S_j^y \rangle, \\ G_r^{yx} &= \langle S_i^y S_j^x \rangle, \\ G_r^{+-} &= \langle S_i^+ S_j^- \rangle, \\ G_r^{zz} &= \langle S_i^z S_j^z \rangle, \end{aligned} \tag{1}$$

where S^+ and S^- are spin-up and spin-down operators, $r = |i - j|$ denotes the distance between the lattice site i and j . The behavior of the two-point correlation G_r^{xy} and G_r^{yx} presents an oscillating decline as the function of r when systems are interacting with the GI, then long-range spin-nematic order and long-range chiral order can obtain from $G_r^{xy} + G_r^{yx}$ and $G_r^{xy} - G_r^{yx}$ respectively for sufficiently large r .

In Fig. ??, the three types of two-point correlation are plotted as the function of r in three different phases at $\Delta = 0.5$. One can conclude there is no significant difference between phase I and II in terms of the behavior of G_r^{xy} , G_r^{yx} , and G_r^{+-} , while we have $|G_1^{xy}| > |G_1^{yx}|$ in phase I and $|G_1^{xy}| < |G_1^{yx}|$ in phase II. The boundary of the oscillation envelope of the correlation function $|G_r^{xy}|$, $|G_r^{yx}|$, and $|G_r^{+-}|$ is approximately decay as a power function of r .

Fig. ?? illustrates the different types of two-point correlation as the function of r in three different phases at $\Delta = 1.5$. The way to pin out the phase I and phase II can be obtained from the magnitude of G_1^{xy} and G_1^{yx} . Otherwise, phase I and phase II are also known as the existence of long-range Néel order compared with the phase III.

The behavior of long-range correlation is more complicated when we take $\Delta = -1.5$. Based on the result in the main text, five of the different phases are recognized. The phase IV and phase V have saturated value of G_r^{zz} , while in the other three phases this kind of correlation decays very quickly with the distance of r . By contrast, none of G_r^{xy} or G_r^{yx} in phase IV and phase V is witnessed at large distance r but the sign of the G_1^{xy} (or G_1^{yx}) can be a good quantity to define phase IV and phase V. The upper plane of Fig. ?? shows the correlations vary as the distance r in phase I and phase II. The G_r^{xy} , G_r^{yx} , and G_r^{+-} in both phase I and phase II present an incommensurate oscillating with the r . To further differentiate those two phases, we calculate the spin structure factor based on G_r^{+-} in the lower right part of Fig. ?? in which the location of momentum q that corresponds to the peak of $S_{+-}(q)$ is differing.

In summary, in the region $\Delta = 0.5$, only the phase III does exist long-range spin nematic order. For $\Delta = 1.5$, the phase I and phase II express the long-range Néel order, and phase III have the feature of long-range spin nematic order. For $\Delta = -1.5$, the phase I and phase II has the long-range chiral order, the phase IV and phase V have the FM order, while the phase III has the feature of long-range spin nematic order.

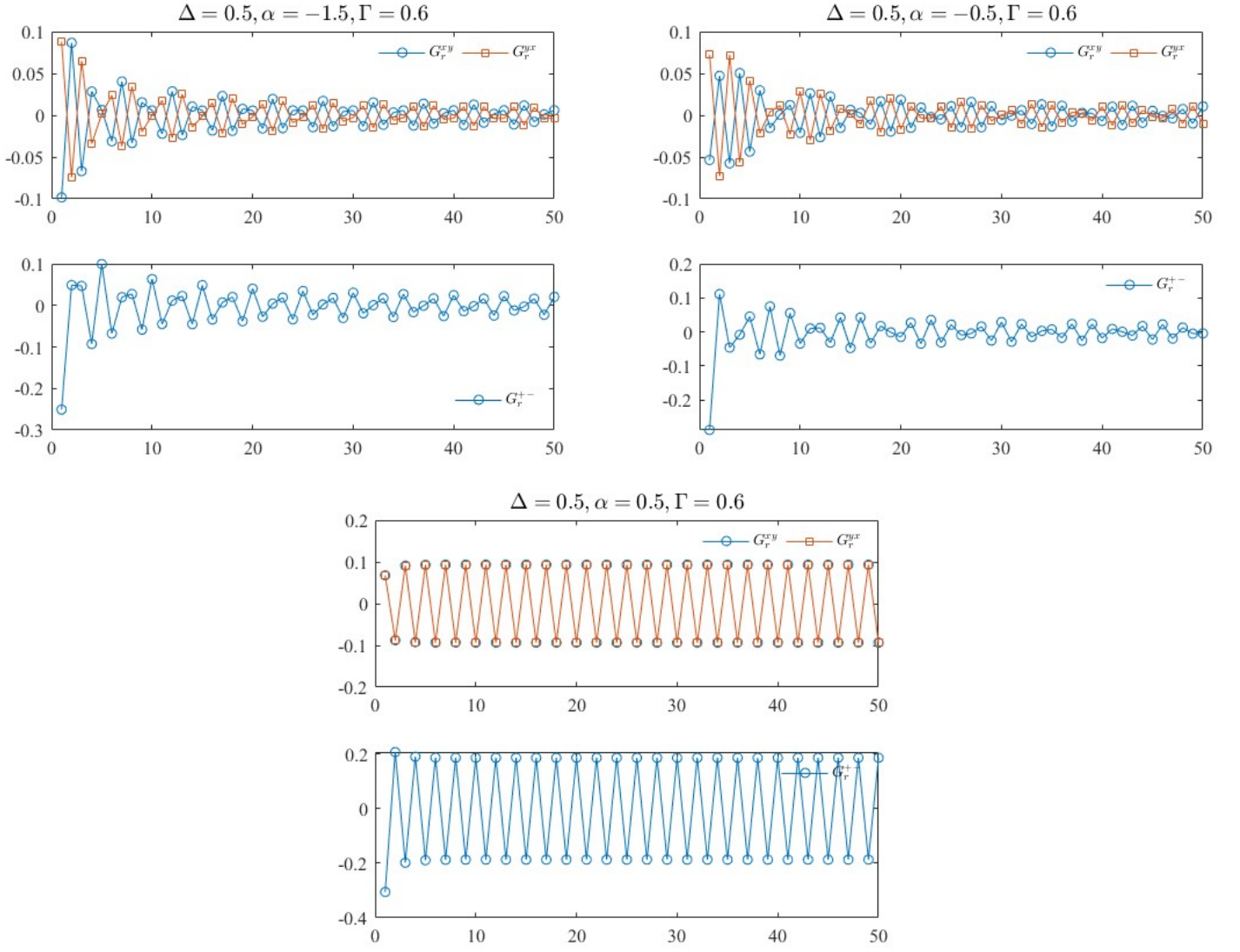


FIG. s1: (color online) Two-point correlation G_r^{xy} , G_r^{yx} , and G_r^{+-} as a function of r in different phase I (upper left), II (upper right), and III (lower) shown on Fig. 6 (a).

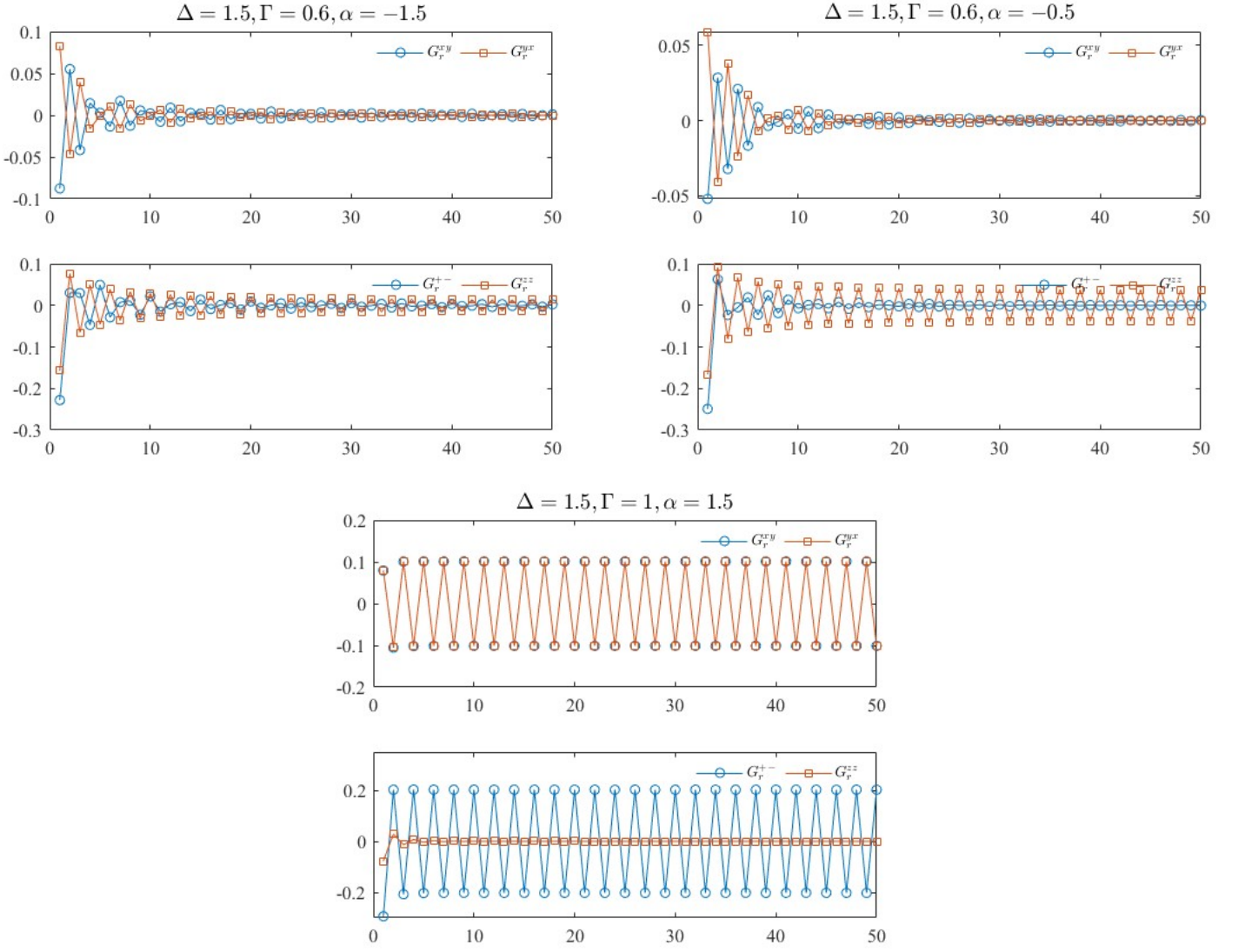


FIG. s2: (color online) Two-point correlation G_r^{xy} , G_r^{yx} , G_r^{+-} , and G_r^{zz} as a function of r in different phase I (upper left), II (upper right), and III (lower) shown on Fig. 6 (b).

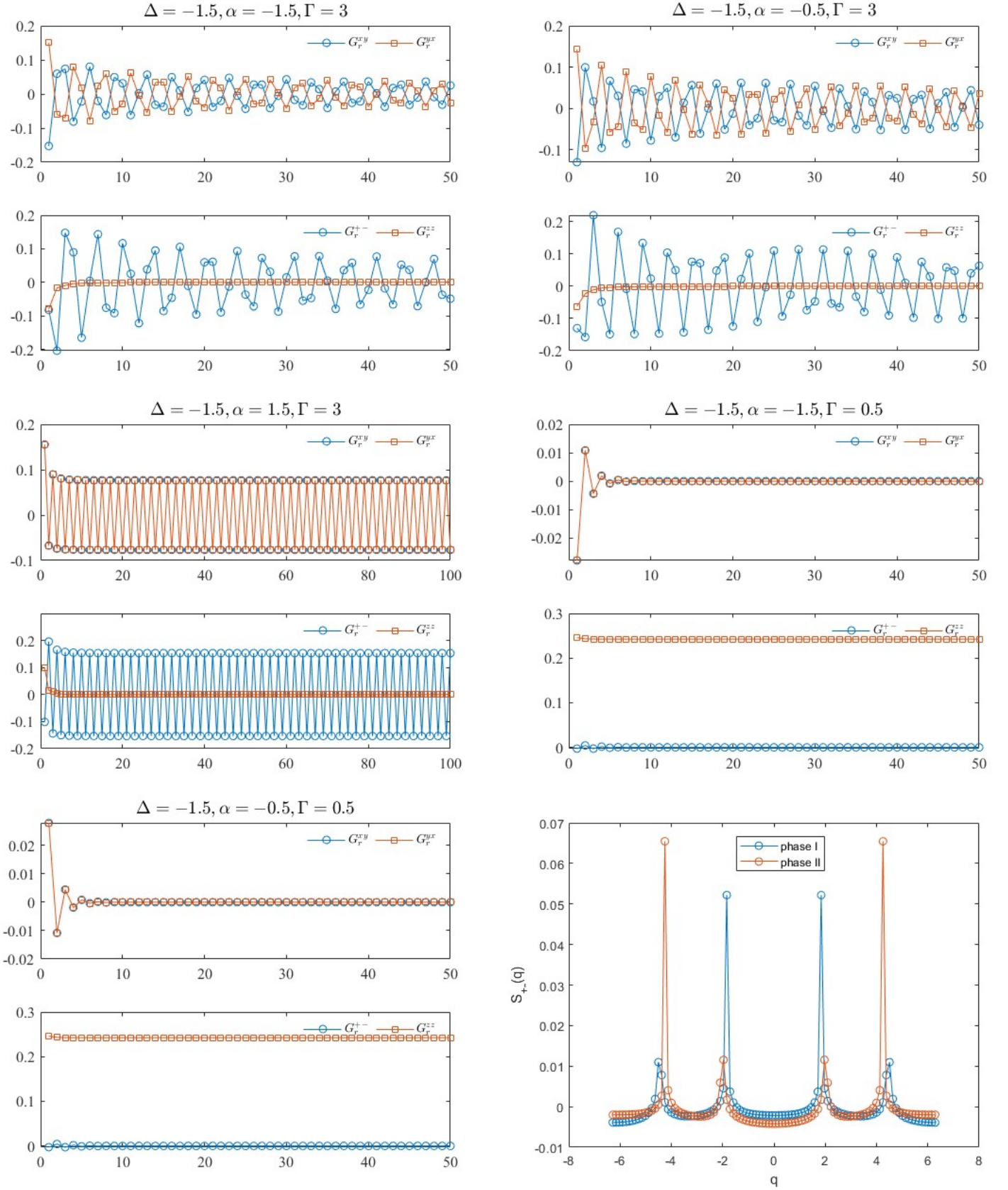


FIG. s3: (color online) Two-point correlation G_r^{xy} , G_r^{yx} , G_r^{+-} and G_r^{zz} as a function of r in different phase I (upper left), II (upper right), III (middle left), IV (middle right), V (lower left), shown on Fig. 6 (c), and the spin structure factor calculated based on G_r^{+-} in phase I and II (lower right).