Exact breather waves solutions in a Spatial Symmetric Nonlinear Dispersive Wave Model in (2+1)- Dimensions

Qunyan Zou[1], **Jalil Manafian**[2,3,*], **Somaye Malmir**[4], **K.H. Mahmoud**[5], **A.SA. Alsubaie**[5], **Nilofer Ali Ewadh**[6], **Ihssan Alrekabi**[7]

[1]*School of Information and Artificial Intelligence, Nanchang Institute of science & Technology, 330108, Nanchang, China*

[2]*Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran*

[3]*Natural Sciences Faculty, Lankaran State University, 50, H. Aslanov str., Lankaran, Azerbaijan*

[4]*Department of Physics, Payame Noor University, P.O. Box 19395-4697, Tehran, Iran*

[5]*Department of Physics, College of Khurma University College, Taif University, Taif 21944, Saudi Arabia*

[6]*Optical Techniques Department, College of Health and Medical Techniques, Al-Mustaqbal University, 51001, Babylon, Iraq*

[7]*Mazaya University College, Dhiqar, Iraq*

[*]*Corresponding author: Email: j_manafianheris@tabrizu.ac.ir*

(Dated: October 23, 2024)

Abstract

PACS numbers: 02.60.Lj, 02.70.Wz, 02.90.+p

APPENDIX A: EQUATION (9) OF CASE (2).

The below parameters $A_i, i = 1, ..., 6$ are related to equation (9) of Case (2).

$$A_1 = 4\,k_1(16\,{\alpha_3}^3{\beta_1}^4\beta_3\alpha_1 - 16\,{\alpha_3}^3{\beta_1}^2{\beta_3}^3\alpha_1 - 4\,{\alpha_3}^2{\beta_1}^6\alpha_1 + 72\,{\alpha_3}^2{\beta_1}^4{\beta_3}^2\alpha_1 - 52\,{\alpha_3}^2{\beta_1}^2{\beta_3}^4\alpha_1 - 8\,\alpha_3{\beta_1}^6\beta_3\alpha_1 +$$

$$96\,\alpha_3{\beta_1}^4{\beta_3}^3\alpha_1 - 56\,\alpha_3{\beta_1}^2{\beta_3}^5\alpha_1 - 4\,{\beta_1}^6{\beta_3}^2\alpha_1 + 40\,{\beta_1}^4{\beta_3}^4\alpha_1 - 20\,{\beta_1}^2{\beta_3}^6\alpha_1 + 7\,{\alpha_3}^2{\beta_1}^4\alpha_1 - 2\,{\alpha_3}^2{\beta_1}^2{\beta_3}^2\alpha_1 -$$

$${\alpha_3}^2{\beta_3}^4\alpha_1 + 38\,\alpha_3{\beta_1}^4\beta_3\alpha_1 + 4\,\alpha_3{\beta_1}^2{\beta_3}^3\alpha_1 - 2\,\alpha_3{\beta_3}^5\alpha_1 - 6\,{\beta_1}^6\alpha_1 + 27\,{\beta_1}^4{\beta_3}^2\alpha_1 + 8\,{\beta_1}^2{\beta_3}^4\alpha_1 - {\beta_3}^6\alpha_1 -$$

$$4\,{\alpha_3}^4{\beta_1}^5 + 24\,{\alpha_3}^4{\beta_1}^3{\beta_3}^2 - 4\,{\alpha_3}^4\beta_1{\beta_3}^4 - 16\,{\alpha_3}^3{\beta_1}^5\beta_3 + 96\,{\alpha_3}^3{\beta_1}^3{\beta_3}^3 - 16\,{\alpha_3}^3\beta_1{\beta_3}^5 - 4\,{\alpha_3}^2{\beta_1}^7 + 140\,{\alpha_3}^2{\beta_1}^3{\beta_3}^4 -$$

$$24\,{\alpha_3}^2\beta_1{\beta_3}^6 - 8\,\alpha_3{\beta_1}^7\beta_3 + 32\,\alpha_3{\beta_1}^5{\beta_3}^3 + 88\,\alpha_3{\beta_1}^3{\beta_3}^5 - 16\,\alpha_3\beta_1{\beta_3}^7 - 4\,{\beta_1}^7{\beta_3}^2 + 20\,{\beta_1}^5{\beta_3}^4 + 20\,{\beta_1}^3{\beta_3}^6 - 4\,\beta_1{\beta_3}^8 +$$

$$12\,\alpha_3{}^3\beta_1{}^3\beta_3+4\,\alpha_3{}^3\beta_1\beta_3{}^3-7\,\alpha_3{}^2\beta_1{}^5+38\,\alpha_3{}^2\beta_1{}^3\beta_3{}^2+13\,\alpha_3{}^2\beta_1\beta_3{}^4+2\,\alpha_3\beta_1{}^5\beta_3+40\,\alpha_3\beta_1{}^3\beta_3{}^3+14\,\alpha_3\beta_1\beta_3{}^5-$$

$$6\,\beta_1{}^7+5\,\beta_1{}^5\beta_3{}^2+16\,\beta_1{}^3\beta_3{}^4+5\,\beta_1\beta_3{}^6),$$

$$A_2=-k_3{}^2(16\,\alpha_3{}^3\beta_1{}^4\beta_3\alpha_1-16\,\alpha_3{}^3\beta_1{}^2\beta_3{}^3\alpha_1-2\,\alpha_3{}^2\beta_1{}^6\alpha_1+42\,\alpha_3{}^2\beta_1{}^4\beta_3{}^2\alpha_1-22\,\alpha_3{}^2\beta_1{}^2\beta_3{}^4\alpha_1-2\,\alpha_1\alpha_3{}^2\beta_3{}^6-$$

$$4\,\alpha_3\beta_1{}^6\beta_3\alpha_1+36\,\alpha_3\beta_1{}^4\beta_3{}^3\alpha_1+4\,\alpha_3\beta_1{}^2\beta_3{}^5\alpha_1-4\,\alpha_1\alpha_3\beta_3{}^7-2\,\beta_1{}^6\beta_3{}^2\alpha_1+10\,\beta_1{}^4\beta_3{}^4\alpha_1+10\,\beta_1{}^2\beta_3{}^6\alpha_1-2\,\alpha_1\beta_3{}^8-$$

$$4\,\alpha_3{}^4\beta_1{}^5+24\,\alpha_3{}^4\beta_1{}^3\beta_3{}^2-4\,\alpha_3{}^4\beta_1\beta_3{}^4-4\,\alpha_3{}^3\beta_1{}^5\beta_3+56\,\alpha_3{}^3\beta_1{}^3\beta_3{}^3-4\,\alpha_3{}^3\beta_1\beta_3{}^5-2\,\alpha_3{}^2\beta_1{}^7+6\,\alpha_3{}^2\beta_1{}^5\beta_3{}^2+$$

$$50\,\alpha_3{}^2\beta_1{}^3\beta_3{}^4+10\,\alpha_3{}^2\beta_1\beta_3{}^6-4\,\alpha_3\beta_1{}^7\beta_3+8\,\alpha_3\beta_1{}^5\beta_3{}^3+28\,\alpha_3\beta_1{}^3\beta_3{}^5+16\,\alpha_3\beta_1\beta_3{}^7-2\,\beta_1{}^7\beta_3{}^2+2\,\beta_1{}^5\beta_3{}^4+10\,\beta_1{}^3\beta_3{}^6+$$

$$6\,\beta_1\beta_3{}^8+3\,\alpha_3{}^2\beta_1{}^4\alpha_1+22\,\alpha_3{}^2\beta_1{}^2\beta_3{}^2\alpha_1-5\,\alpha_3{}^2\beta_3{}^4\alpha_1-6\,\alpha_3\beta_1{}^4\beta_3\alpha_1+28\,\alpha_3\beta_1{}^2\beta_3{}^3\alpha_1+2\,\alpha_3\beta_3{}^5\alpha_1-7\,\beta_1{}^4\beta_3{}^2\alpha_1+$$

$$2\,\beta_1{}^2\beta_3{}^4\alpha_1+\beta_3{}^6\alpha_1-4\,\alpha_3{}^3\beta_1{}^3\beta_3+20\,\alpha_3{}^3\beta_1\beta_3{}^3+\alpha_3{}^2\beta_1{}^5-10\,\alpha_3{}^2\beta_1{}^3\beta_3{}^2+21\,\alpha_3{}^2\beta_1\beta_3{}^4-6\,\alpha_3\beta_1{}^5\beta_3-$$

$$8\,\alpha_3\beta_1{}^3\beta_3{}^3+6\,\alpha_3\beta_1\beta_3{}^5-5\,\beta_1{}^5\beta_3{}^2-6\,\beta_1{}^3\beta_3{}^4-\beta_1\beta_3{}^6),$$

$$A_3=4\,k_1\left(\alpha_3+\beta_3\right)\left(4\,\alpha_1\beta_1{}^3\beta_3-4\,\alpha_1\beta_1\beta_3{}^3-\alpha_3\beta_1{}^4+6\,\alpha_3\beta_1{}^2\beta_3{}^2-\alpha_3\beta_3{}^4+3\,\beta_1{}^4\beta_3+2\,\beta_1{}^2\beta_3{}^3-\beta_3{}^5\right),$$

$$A_4=-k_3{}^2\left(\alpha_3+\beta_3\right)\left(4\,\alpha_1\beta_1{}^3\beta_3-4\,\alpha_1\beta_1\beta_3{}^3-\alpha_3\beta_1{}^4+6\,\alpha_3\beta_1{}^2\beta_3{}^2-\alpha_3\beta_3{}^4+3\,\beta_1{}^4\beta_3+2\,\beta_1{}^2\beta_3{}^3-\beta_3{}^5\right),$$

$$A_5=4\,k_1(16\,\alpha_1\alpha_3{}^3\beta_1{}^3\beta_3{}^2-16\,\alpha_1\alpha_3{}^3\beta_1\beta_3{}^4-12\,\alpha_1\alpha_3{}^2\beta_1{}^5\beta_3+72\,\alpha_1\alpha_3{}^2\beta_1{}^3\beta_3{}^3-44\,\alpha_1\alpha_3{}^2\beta_1\beta_3{}^5-24\,\alpha_1\alpha_3\beta_1{}^5\beta_3{}^2+$$

$$96\,\alpha_1\alpha_3\beta_1{}^3\beta_3{}^4-40\,\alpha_1\alpha_3\beta_1\beta_3{}^6-12\,\alpha_1\beta_1{}^5\beta_3{}^3+40\,\alpha_1\beta_1{}^3\beta_3{}^5-12\,\alpha_1\beta_1\beta_3{}^7-4\,\alpha_3{}^4\beta_1{}^4\beta_3+24\,\alpha_3{}^4\beta_1{}^2\beta_3{}^3-4\,\alpha_3{}^4\beta_3{}^5+$$

$$2\,\alpha_3{}^3\beta_1{}^6-26\,\alpha_3{}^3\beta_1{}^4\beta_3{}^2+86\,\alpha_3{}^3\beta_1{}^2\beta_3{}^4-14\,\alpha_3{}^3\beta_3{}^6-6\,\alpha_3{}^2\beta_1{}^6\beta_3-30\,\alpha_3{}^2\beta_1{}^4\beta_3{}^3+118\,\alpha_3{}^2\beta_1{}^2\beta_3{}^5-18\,\alpha_3{}^2\beta_3{}^7-$$

$$18\,\alpha_3\beta_1{}^6\beta_3{}^2+2\,\alpha_3\beta_1{}^4\beta_3{}^4+74\,\alpha_3\beta_1{}^2\beta_3{}^6-10\,\alpha_3\beta_3{}^8-10\,\beta_1{}^6\beta_3{}^3+10\,\beta_1{}^4\beta_3{}^5+18\,\beta_1{}^2\beta_3{}^7-2\,\beta_3{}^9+20\,\alpha_1\alpha_3{}^2\beta_1{}^3\beta_3-$$

$$4\,\alpha_1\alpha_3{}^2\beta_1\beta_3{}^3-12\,\alpha_1\alpha_3\beta_1{}^5+56\,\alpha_1\alpha_3\beta_1{}^3\beta_3{}^2+4\,\alpha_1\alpha_3\beta_1\beta_3{}^4-24\,\alpha_1\beta_1{}^5\beta_3+20\,\alpha_1\beta_1{}^3\beta_3{}^3+4\,\alpha_1\beta_1\beta_3{}^5-$$

$$5\,\alpha_3{}^3\beta_1{}^4+22\,\alpha_3{}^3\beta_1{}^2\beta_3{}^2+3\,\alpha_3{}^3\beta_3{}^4-29\,\alpha_3{}^2\beta_1{}^4\beta_3+42\,\alpha_3{}^2\beta_1{}^2\beta_3{}^3+7\,\alpha_3{}^2\beta_3{}^5-6\,\alpha_3\beta_1{}^6-23\,\alpha_3\beta_1{}^4\beta_3{}^2+28\,\alpha_3\beta_1{}^2\beta_3{}^4+$$

$$5\,\alpha_3\beta_3{}^6-18\,\beta_1{}^6\beta_3-15\,\beta_1{}^4\beta_3{}^3+4\,\beta_1{}^2\beta_3{}^5+\beta_3{}^7),$$

$$A_6=-k_3{}^2(16\,\alpha_1\alpha_3{}^3\beta_1{}^3\beta_3{}^2-16\,\alpha_1\alpha_3{}^3\beta_1\beta_3{}^4+32\,\alpha_1\alpha_3{}^2\beta_1{}^3\beta_3{}^3-32\,\alpha_1\alpha_3{}^2\beta_1\beta_3{}^5+16\,\alpha_1\alpha_3\beta_1{}^3\beta_3{}^4-16\,\alpha_1\alpha_3\beta_1\beta_3{}^6-$$

$$4\,\alpha_3{}^4\beta_1{}^4\beta_3+24\,\alpha_3{}^4\beta_1{}^2\beta_3{}^3-4\,\alpha_3{}^4\beta_3{}^5+4\,\alpha_3{}^3\beta_1{}^4\beta_3{}^2+56\,\alpha_3{}^3\beta_1{}^2\beta_3{}^4-12\,\alpha_3{}^3\beta_3{}^6+20\,\alpha_3{}^2\beta_1{}^4\beta_3{}^3+40\,\alpha_3{}^2\beta_1{}^2\beta_3{}^5-$$

$$12\,\alpha_3{}^2\beta_3{}^7+12\,\alpha_3\beta_1{}^4\beta_3{}^4+8\,\alpha_3\beta_1{}^2\beta_3{}^6-4\,\alpha_3\beta_3{}^8+4\,\alpha_1\alpha_3{}^2\beta_1{}^3\beta_3+12\,\alpha_1\alpha_3{}^2\beta_1\beta_3{}^3+4\,\alpha_1\beta_1{}^3\beta_3{}^3-4\,\alpha_1\beta_1\beta_3{}^5-$$

$$\alpha_3{}^3\beta_1{}^4-2\,\alpha_3{}^3\beta_1{}^2\beta_3{}^2+7\,\alpha_3{}^3\beta_3{}^4+3\,\alpha_3{}^2\beta_1{}^4\beta_3+10\,\alpha_3{}^2\beta_1{}^2\beta_3{}^3+7\,\alpha_3{}^2\beta_3{}^5-\alpha_3\beta_1{}^4\beta_3{}^2+6\,\alpha_3\beta_1{}^2\beta_3{}^4-$$

$$\alpha_3\beta_3{}^6+3\,\beta_1{}^4\beta_3{}^3+2\,\beta_1{}^2\beta_3{}^5-\beta_3{}^7).$$