# Foot Trajectory as a Key Factor for Diverse Gait Patterns in Quadruped Robot Locomotion

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**Supplementary Video S1.**

One-minute Summary

## Gait Estimation using Linear Regression

To verify how much gait data—specifically, RGB values based on phase relationships, locomotion velocity, and CoT (an indicator of energy efficiency)—can be estimated from foot trajectory parameters ($z_{\mathrm{btm}}, z_{\mathrm{top}}, x_{\mathrm{fore}}, x_{\mathrm{hind}}$), we constructed a gait estimation model using linear regression and assessed its estimation accuracy. Utilizing the gait data obtained from the simulations in Figs. 2 and 4, we developed a linear regression model with foot trajectory parameters ($z_{\mathrm{btm}}, z_{\mathrm{top}}, x_{\mathrm{fore}}, x_{\mathrm{hind}}$) as the input (explanatory variable) and gait visual RGB colorization, locomotion velocity, and CoT as the output (target variables). The input data ranges were scaled to [-1.0, 1.0], while the locomotion velocity and CoT data ranges were scaled to [0.0, 1.0]. In addition to the foot trajectory parameters ($z_{\mathrm{btm}}, z_{\mathrm{top}}, x_{\mathrm{fore}}, x_{\mathrm{hind}}$), we included their six cross-terms as explanatory variables to examine the effects of these interactions. The regression model can be expressed as follows, with the weight coefficients estimated using the simulation data:

$$\begin{bmatrix} R \\ G \\ B \\ V \\ \mathrm{CoT} \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_{12} & r_{13} & r_{14} & r_{23} & r_{24} & r_{34} \\ g_1 & g_2 & g_3 & g_4 & g_{12} & g_{13} & g_{14} & g_{23} & g_{24} & g_{34} \\ b_1 & b_2 & b_3 & b_4 & b_{12} & b_{13} & b_{14} & b_{23} & b_{24} & b_{34} \\ v_1 & v_2 & v_3 & v_4 & v_{12} & v_{13} & v_{14} & v_{23} & v_{24} & v_{34} \\ c_1 & c_2 & c_3 & c_4 & c_{12} & c_{13} & c_{14} & c_{23} & c_{24} & c_{34} \end{bmatrix} \begin{bmatrix} z_{\mathrm{btm}} \\ z_{\mathrm{top}} \\ x_{\mathrm{fore}} \\ x_{\mathrm{hind}} \\ z_{\mathrm{btm}} z_{\mathrm{top}} \\ z_{\mathrm{btm}} x_{\mathrm{fore}} \\ z_{\mathrm{btm}} x_{\mathrm{hind}} \\ z_{\mathrm{top}} x_{\mathrm{fore}} \\ z_{\mathrm{top}} x_{\mathrm{hind}} \\ x_{\mathrm{fore}} x_{\mathrm{hind}} \end{bmatrix} + \begin{bmatrix} r_0 \\ g_0 \\ b_0 \\ v_0 \\ c_0 \end{bmatrix} \quad \text{(S1)}$$

We here used 75% of the total data as training data to estimate the model parameters and then evaluated the model's estimation accuracy with the remaining 25% as test data.

Fig. S1 A shows the validation results for the estimation accuracy of both the training data (top) and test data (bottom) using the parameters of the linear regression model in Eq. (S1) estimated from the training data. The plots are organized from left to right as follows: R, G, and B values; locomotion velocity; and CoT. The horizontal and vertical axes represent the true and predicted values, respectively, while the color of the points corresponds to the RGB values of the gait. The dotted line in each graph indicates the point where the true value matches the predicted value exactly. As illustrated in the graphs, we assessed estimation accuracy for both the training and test data using $R^2$ scores. Fig. S1 B displays the color map of the weight coefficients from the obtained linear regression model. From these results, we found that (i) the R, G, and B values demonstrated prediction accuracy with $R^2 > 0.6$ for test data, while the velocity and CoT showed prediction accuracy with $R^2 < 0.5$ for test data; (ii) the G and B values were primarily explained by $z_{\mathrm{top}}$, whereas the cross term $z_{\mathrm{top}} z_{\mathrm{btm}}$ was a major explanatory variable for the R value; and (iii) we confirmed that the cross term $z_{\mathrm{top}} x_{\mathrm{hind}}$ was significant explanatory variables for locomotion velocity and CoT (with $R^2$ scores not particularly high). The locomotion velocity was also involved $(x_{\mathrm{fore}}, x_{\mathrm{hind}})$. These findings suggest that the primary parameter for generating trot and pace gaits is $z_{\mathrm{top}}$, which indicates how much the leg is elevated. The weights for $z_{\mathrm{top}}$ (trajectory height) switch between the trot and the pace gait, while $(x_{\mathrm{fore}}, x_{\mathrm{hind}})$—representing the front-back foot trajectory—and the cross-term $z_{\mathrm{top}}x_{\mathrm{hind}}$ would contribute to locomotion velocity and CoT. The present analysis, based on simple assumptions such as the use of a linear regression model and the gait representation in terms of colors according to phase relationships, was insufficient to explain all the phenomena that contribute to gait generation. Further consideration of the effects of complex nonlinear interactions will be necessary to adress what has not been explained here.

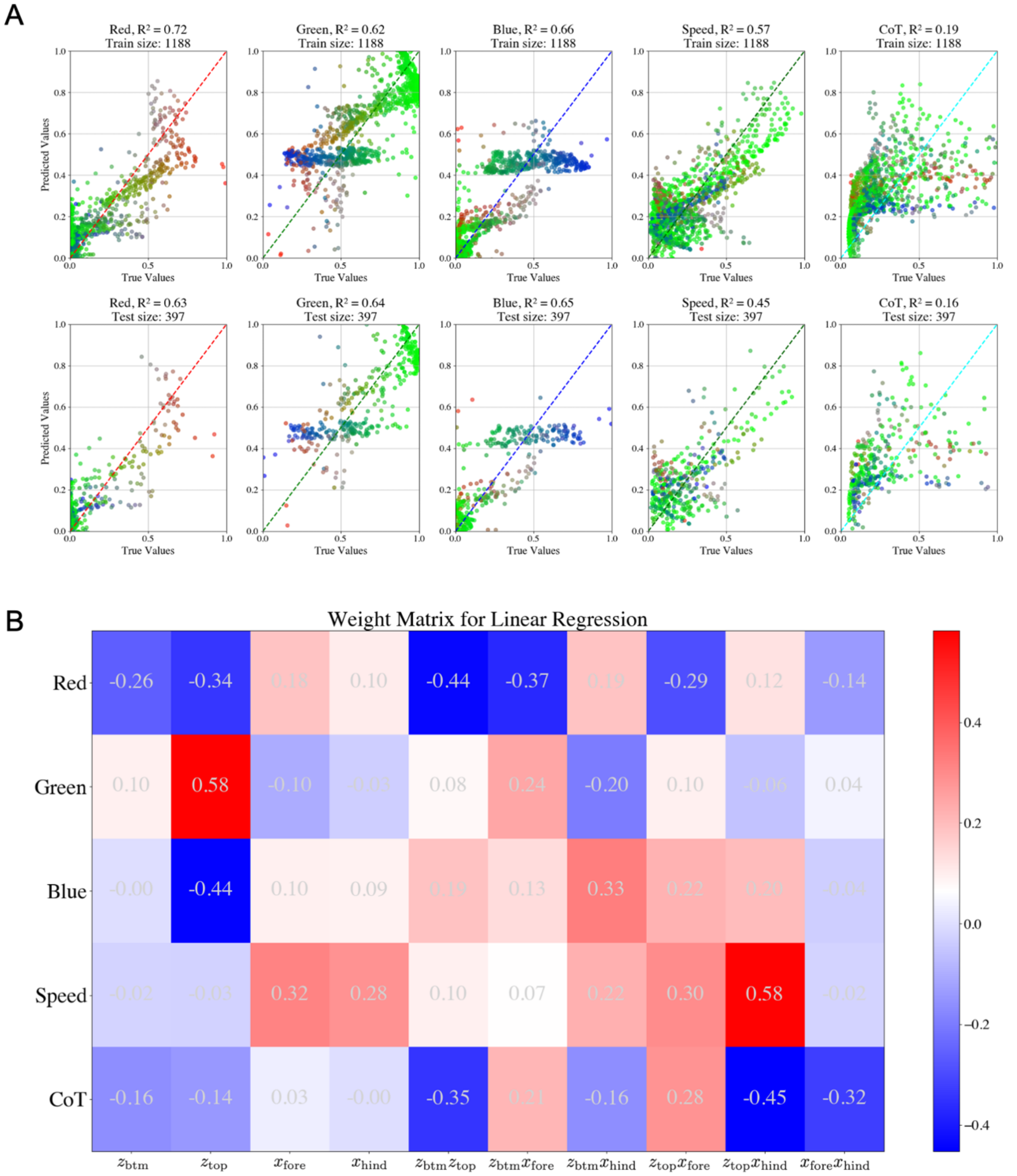


**Fig.S1: Gait estimation using linear regression with the height ($z_{\mathrm{btm}}, z_{\mathrm{top}}$) and width ($x_{\mathrm{fore}}, x_{\mathrm{hind}}$) of the foot trajectory as input variables.** (A) Gait visual RGB colorization (R, G, B values from left to right), locomotion velocity, and Cost of Transport (CoT) estimation results utilizing linear regression models for the simulation results in Figs 2 and 4 of the main text. The top graphs display the estimation accuracy for the training data (75%), while the bottom graphs display the estimation accuracy for the test data (25%). The horizontal and vertical axes represent the true and predicted values, respectively. The color of the points corresponds the RGB values of the gait. The dotted line in each graph indicates the point where the true value matches the predicted value exactly. As demonstrated in the graphs, we evaluated estimation accuracy for both the training and test data using $R^2$. (B) Color map of the weight coefficients from the obtained linear regression model. In this model, ($z_{\mathrm{btm}}, z_{\mathrm{top}}, x_{\mathrm{fore}}, x_{\mathrm{hind}}$) and their six cross terms were used as input variables for estimation.