Supplementary materials - Magnetism and hidden quantum geometry in charge neutral twisted trilayer graphene

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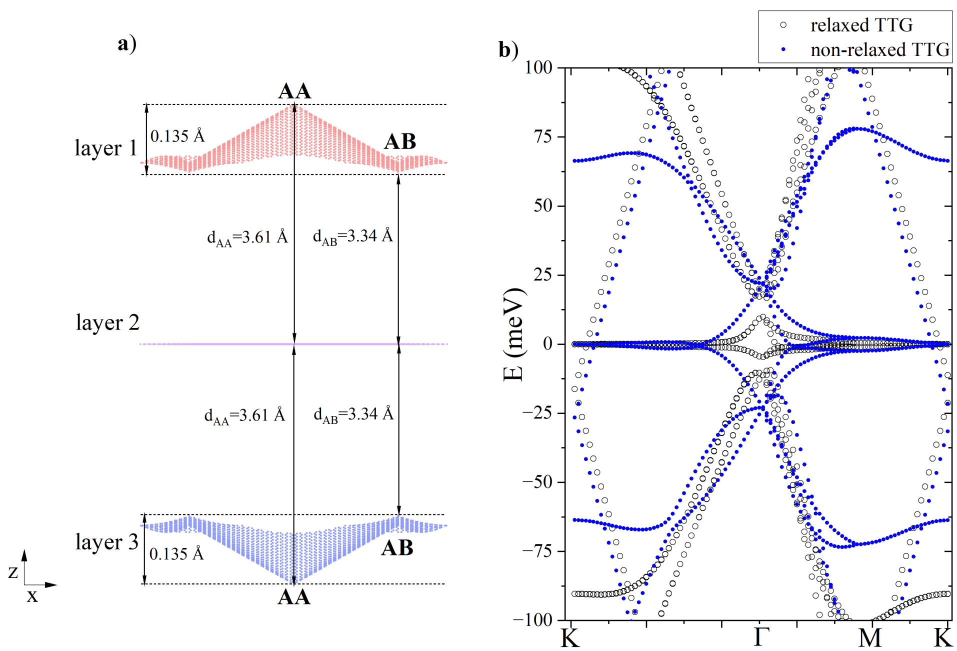

Supplementary Figure 1. **Relaxation details.** (a) Schematic side view of relaxed TTG layers. The interlayer distance varies across the moiré unit cell depending on the local stacking. The middle layer remains rigid and top/bottom layers are adjusted. The layers are closest in the AB-stacked regions, where the interlayer spacing is $d_{\mathrm{AB}} = d_{\min} = 3.34\,\text{Å}$. The largest separation occurs in the AA-stacked regions, where $d_{\mathrm{AA}} = d_{\max} = 3.61\,\text{Å}$. (b) Band structure of non-relaxed and relaxed TTG denoted by blue dots and black circles, respectively. No gap between the flat and remote bands is observed for non-relaxed structure. Relaxation leads to flattening of the flat band, renormalization of the remote bands, and gaps opening between the flat and the remote bands an the Γ point.

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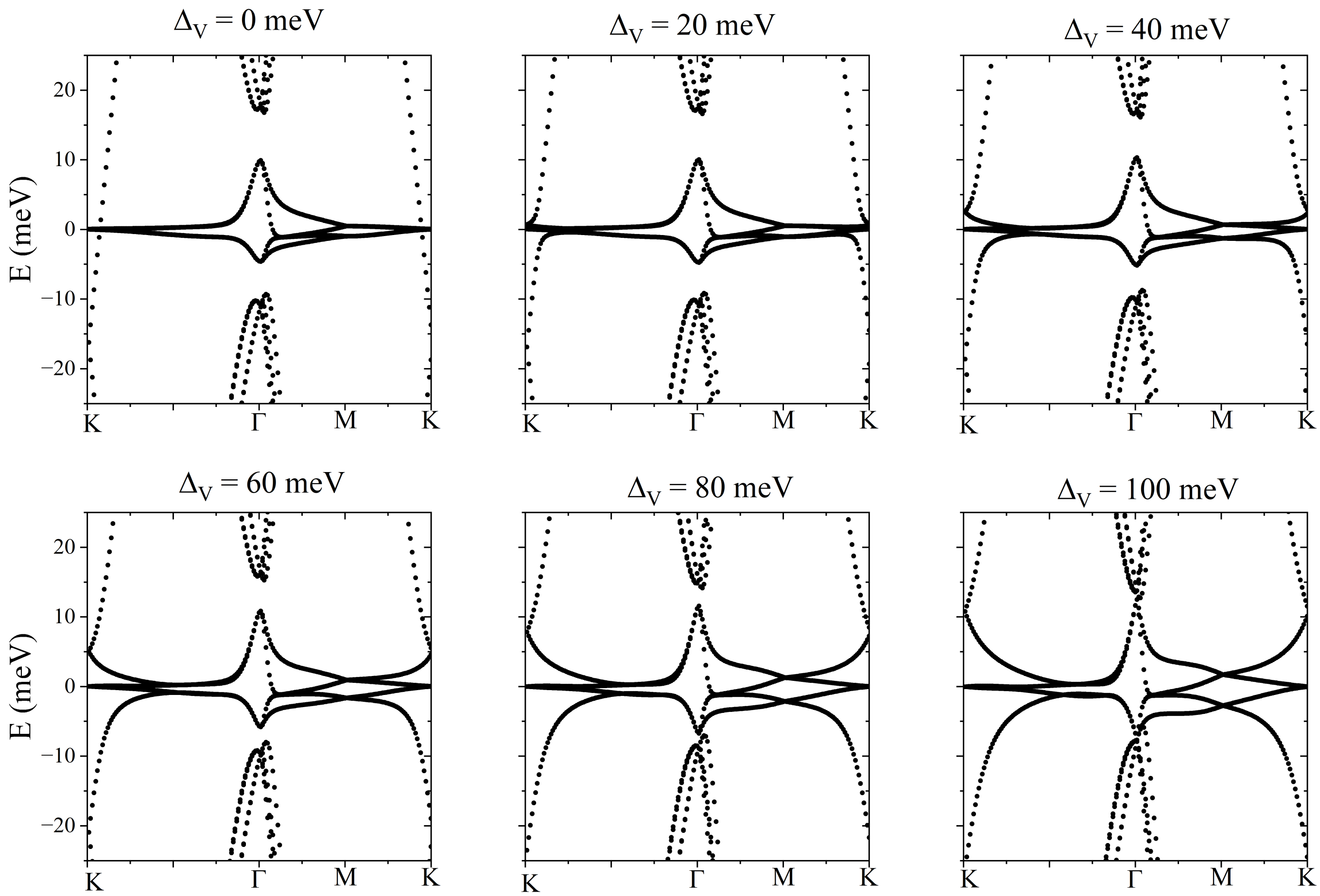
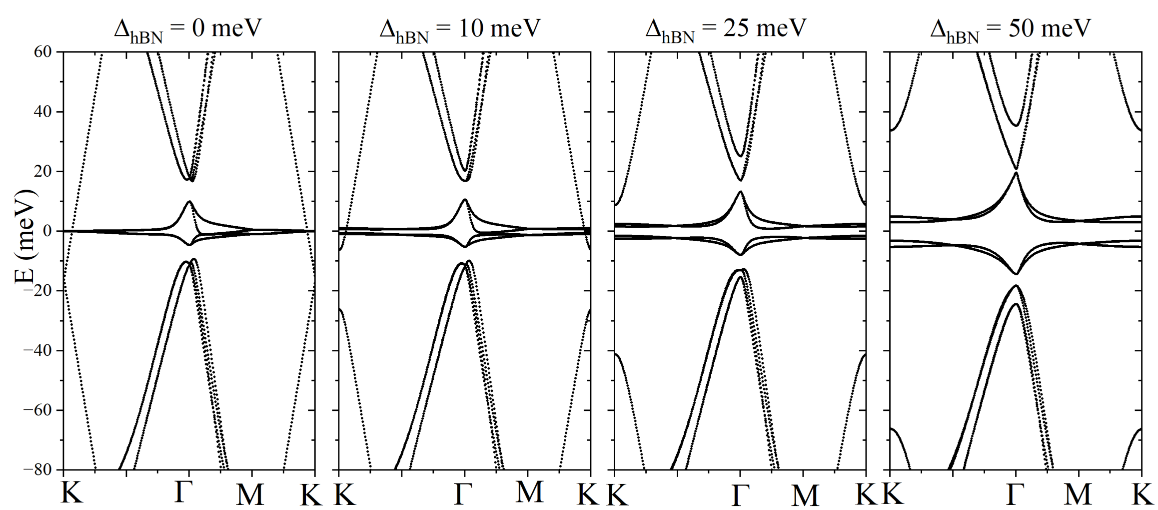
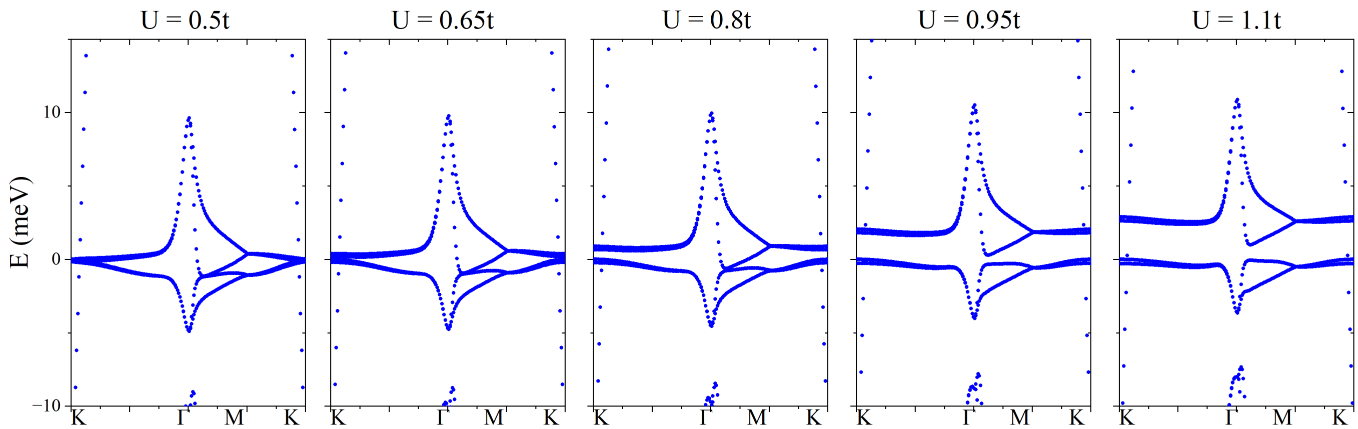


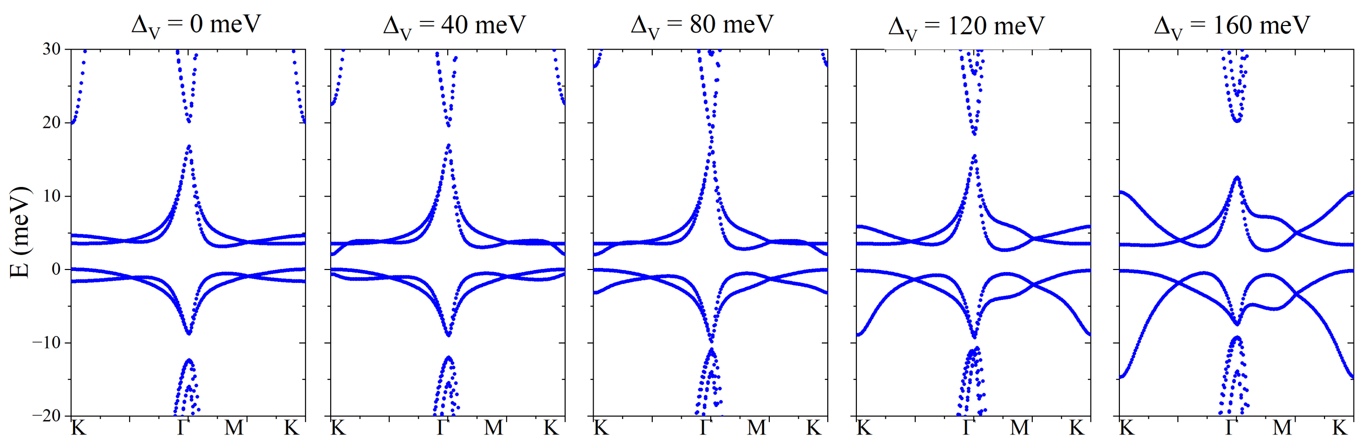
Supplementary Figure 2. **Effect of electric field.** Renormalization of the non-interacting bandstructure as a function of the applied electric field Δ_V. We show here an evolution of the band gaps opening within the flat band and splitting off of the Dirac cones. For Δ_V that's large enough ($\geq$ 100 meV) the gap between the flat band and the remote bands closes at the Γ point.



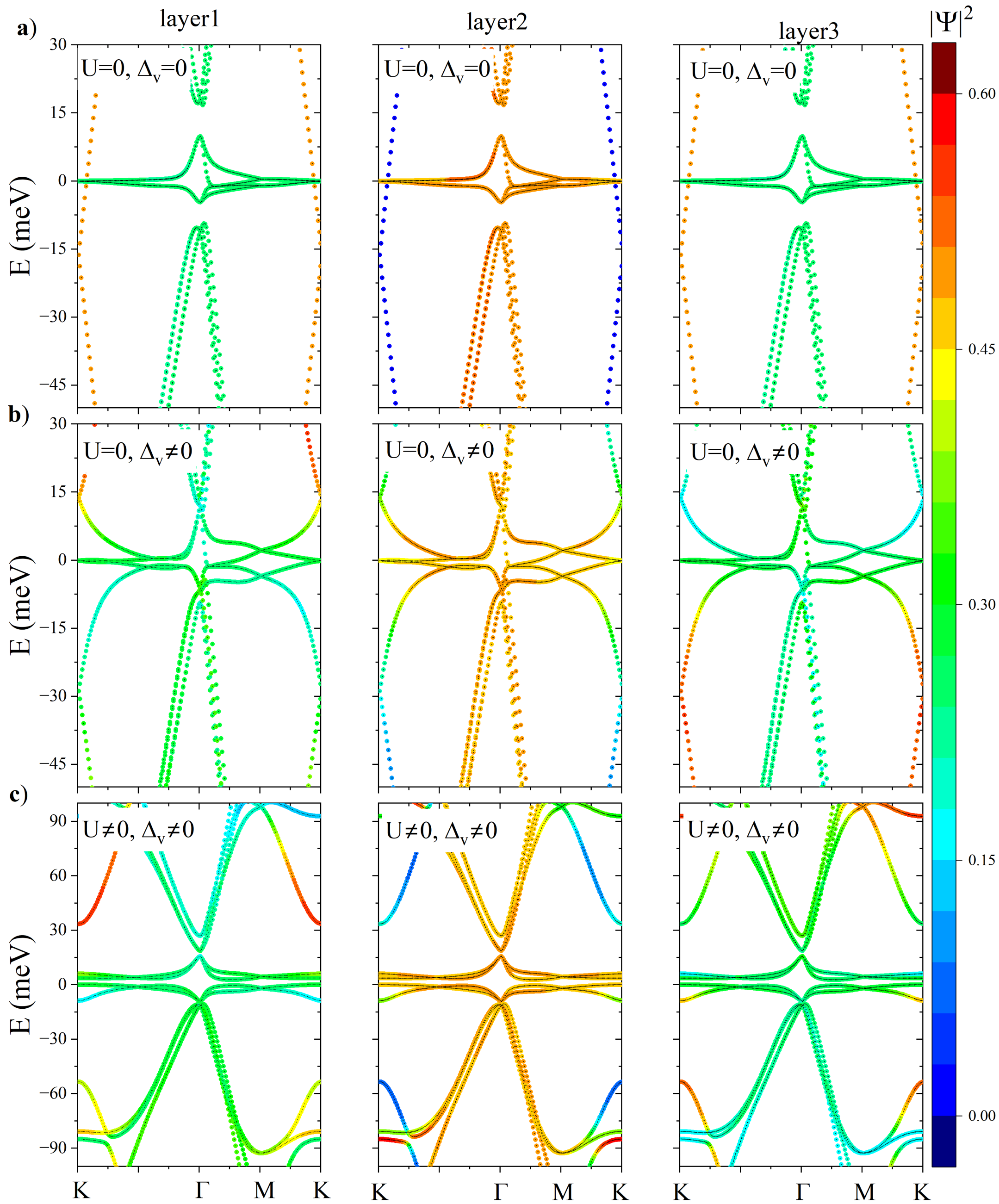
Supplementary Figure 3. **Effect of hBN.** Analogous to Sup. Fig. 2 evolution of bandstructure as a function of the strength of the staggered potential Δ_{hBN} induced by an hBN substrate. Increase of Δ_{hBN} leads to a gap opening within the flat band and between the Dirac cones at K-points.



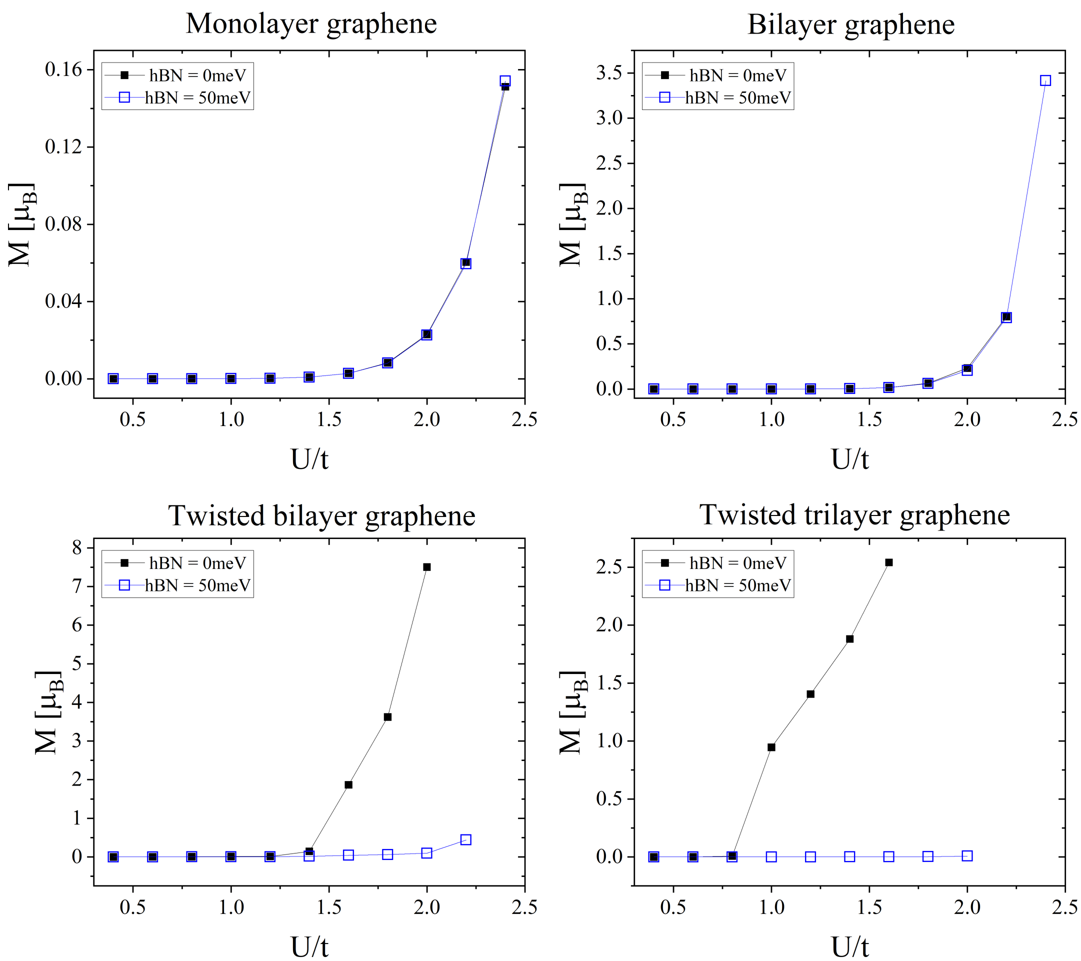
Supplementary Figure 4. **Effect of interaction.** Renormalization of the mean-field band structure as a function of the interaction strength U, with $\Delta_V = 0$ and $\Delta_{\mathrm{hBN}} = 0$. As U increases, Mott-like gap within the flat band opens and the bands along $K - \Gamma$ and $M - K$ flatten. We observe no gap opening within high-velocity Dirac cones, therefore no global gap is generated.



Supplementary Figure 5. **Effect of electric field on a mean-field band structure** Spectrum of interacting TTG with hBN encapsulation as a function of applied electric field Δ_V. Hubbard parameter $U = 0.95t$, $\Delta_{\mathrm{hBN}} = 50$ meV. As the Δ_V increases, the gap between the flat and remote bands decreases, and finally closes for $\Delta_V = 80$meV. However, further increase of the electric field leads to the gap opening again.



Supplementary Figure 6. **Layer-resolved wave function localization of Bloch states.** a) Non-interacting TTG, (b) non-interacting TTG with applied electric field, (c) interacting TTG with applied electric field. The color scale shows the localization of the wave function $|\Psi|^2$ in layer 1 (left panels), layer 2 (middle panels) and layer 3 (right panels). In the case of (a) mirror symmetry is preserved and the wave function is equally distributed between the top and bottom layer. The wave functions of the Dirac cones are localized solely on the top and bottom layers, while the flat band's wave functions are localized 50% on the middle layer and 25% on the top and bottom. (b) Applying electric field breaks the mirror symmetry and the wave function distribution is no longer equal between the top and bottom layer. (c) Including interactions further renormalizes this distribution, however one can still notice the clear trend of majority of the Dirac cone's wave function being localized in the top and bottom layers and the flat band wave function being localized predominantly in the middle layer.



Supplementary Figure 7. **AF transition in various graphene-based systems** with (blue empty squares) and without (black filled squares) the hBN substrate. Total absolute magnetization of different graphene systems in the units of Bohr magneton as a function of the interaction U scaled by the nearest-neighbor hopping t. In twisted bilayer and trilayer graphene systems, the inclusion of a staggered potential has a similar effect - it delays the transition from the paramagnetic to the antiferromagnetic phase. In contrast, for monolayer and untwisted bilayer graphene, the impact of the staggered potential is negligible. This is because the para-/antiferromagnetic transition in these systems occurs at much larger interaction strengths U, resulting in a high U/Δ_{hBN} ratio (~ 40). Consequently, interaction effects dominate over those induced by the staggered potential.