Supplementary Information for Exceptional-Point Stability Boundaries from Quantum Dissipation to Cosmological Acceleration

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Contents

Note on supplementary content. This document contains explicit derivations of key results referenced in the main text, including: (i) the Lindblad-to-classical mapping, (ii) information efficiency maximum derivation, (iii) conventions for (ω, Γ) parameters, (iv) QCD damping mechanisms, (v) the electron mass derivation, (vi) renormalization group analysis, (vii) information efficiency beyond Gaussian channels, (viii) neutrino sector details, (ix) experimental protocols, and (x) cross-scale statistical validation. Numerical values for Figure 3 are compiled in Supplementary Table S1.

1 Extended Mathematical Derivations

1.1 Lindblad-to-Second-Order Mapping

Here $\omega > 0$ denotes the characteristic frequency magnitude. The dimensionless ratio

$$\chi \equiv \frac{\gamma}{2\omega} \tag{1}$$

plays an analogous role in dynamical stability. Throughout, the critical boundary corresponds to the vanishing of the quadratic discriminant in the associated second-order response.

The GKSL master equation for a harmonic oscillator with amplitude damping

$$\dot{\rho} = -i[H, \rho] + \gamma \left(a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\} \right), \qquad H = \omega\, a^\dagger a, \tag{2}$$

yields first-moment evolution

$$\dot{x} = -\frac{\gamma}{2}x + \omega p, \qquad \dot{p} = -\omega x - \frac{\gamma}{2}p, \tag{3}$$

where $x = \langle a + a^\dagger \rangle$ and $p = -i\langle a - a^\dagger \rangle$.

Differentiating the first equation and substituting the second gives

$$\ddot{x} = -\gamma\dot{x} - \left(\omega^2 + \frac{\gamma^2}{4}\right) x, \tag{4}$$

i.e.

$$\ddot{x} + \gamma\dot{x} + \Omega^2 x = 0, \qquad \Omega^2 \equiv \omega^2 + \frac{\gamma^2}{4}. \tag{5}$$

Equation (5) is an exact rewriting of the first-moment dynamics and involves no approximation. For the scalar second-order mode, the critical (double-root) condition is the vanishing of the discriminant,

$$\gamma^2 - 4\Omega^2 = 0 \qquad \Longleftrightarrow \qquad \gamma = 2\Omega, \tag{6}$$

equivalently $\chi_\Omega \equiv \gamma/(2\Omega) = 1$ in terms of the effective frequency Ω. At this point the impulse response takes the standard critical form

$$h(t) \propto t\, e^{-\Omega t}. \tag{7}$$

Clarification. Under the definition $\Omega^2 = \omega^2 + \gamma^2/4$, imposing the scalar second-order double-root condition $\gamma = 2\Omega$ forces $\omega^2 = \Omega^2 - \gamma^2/4 = 0$. Thus, in this specific first-moment amplitude-damping mapping, the scalar "critical" form corresponds to the $\omega \to 0$ limit of the underlying (x, p) generator rather than to a defective EP in the 2×2 Liouvillian block.

Remark (scope). For the specific amplitude-damping first-moment generator above, the 2×2 system for (x, p) has eigenvalues $-\gamma/2 \pm i\omega$ and is not defective for $\omega \neq 0$. Exceptional-point behavior in open quantum systems can arise in extended Lindblad settings (e.g. coupled modes, gain/loss imbalance, or reduced effective channels), and is asserted here at the level of the shared quadratic response structure used throughout the manuscript.

1.2 Quadratic Eigenproblem in Open QFT

Consider the standard damped mode

$$\ddot{q} + \gamma\dot{q} + \Omega_0^2\, q = 0, \tag{8}$$

where $\Omega_0 > 0$ is the (undamped) mode frequency. With the ansatz $q(t) = e^{-i\Omega t}$ one obtains the quadratic eigenproblem

$$-\Omega^2 - i\gamma\Omega + \Omega_0^2 = 0 \quad \Rightarrow \quad \Omega^2 + i\gamma\Omega - \Omega_0^2 = 0. \tag{9}$$

The roots are

$$\Omega_\pm = -\frac{i\gamma}{2} \pm \sqrt{\Omega_0^2 - \frac{\gamma^2}{4}}. \tag{10}$$

They coalesce when the discriminant vanishes,

$$\Omega_0^2 - \frac{\gamma^2}{4} = 0 \qquad \Longleftrightarrow \qquad \gamma = 2\Omega_0, \tag{11}$$

at which point the retarded propagator has a second-order pole,

$$G_R(\Omega) \propto \frac{1}{(\Omega + i\Omega_0)^2}, \tag{12}$$

and the inverse transform yields the critical time-domain form

$$h(t) \propto t\, e^{-\Omega_0 t}. \tag{13}$$

1.3 Cosmological Growth Equation: Full Derivation

The growth equation for matter perturbations in an expanding universe:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0. \tag{14}$$

Identifying $\gamma_\delta = 2H$ and defining the characteristic rate $|\omega_\delta| \equiv \sqrt{4\pi G\rho_m}$ (noting the inverted-sign restoring term in the growth equation):

$$\chi_\delta = \frac{\gamma_\delta}{2|\omega_\delta|} = \frac{H}{\sqrt{4\pi G\rho_m}}. \tag{15}$$

In flat ΛCDM, the Friedmann equation is

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda). \tag{16}$$

Define $\Omega_m = 8\pi G\rho_m/(3H^2)$ and $\Omega_\Lambda = 8\pi G\rho_\Lambda/(3H^2)$. The deceleration parameter:

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2}\Omega_m - \Omega_\Lambda. \tag{17}$$

Setting $q = 0$:

$$\Omega_m = 2\Omega_\Lambda. \tag{18}$$

In flat cosmology, $\Omega_m + \Omega_\Lambda = 1$, so $\Omega_m = 2/3$ and $\Omega_\Lambda = 1/3$. From the definition:

$$\Omega_m = \frac{8\pi G\rho_m}{3H^2} = \frac{2}{3} \quad \Rightarrow \quad H^2 = 4\pi G\rho_m. \tag{19}$$

Therefore:

$$\chi_\delta = \frac{H}{\sqrt{4\pi G\rho_m}} = \frac{H}{H} = 1. \tag{20}$$

This establishes the identity $\chi_\delta = 1 \Longleftrightarrow q = 0$ within flat ΛCDM.

2 Information Efficiency Maximum: Formal Statement and Derivation

The information-efficiency functional $\eta(\chi) \equiv I(\chi)/\Sigma(\chi)$, first introduced in the context of critical-damping optimization in information-processing systems [13], exhibits a strict local maximum near the exceptional-point boundary in representative linear Gaussian channel models, under generic smoothness and positivity assumptions on $I(\chi)$ and $\Sigma(\chi)$.

Let $I(\chi)$ denote an information-throughput measure for a linear response channel with additive Gaussian noise and finite bandwidth, and let $\Sigma(\chi)$ denote the corresponding entropy production rate proxy. Assume $I(\chi)$ and $\Sigma(\chi)$ are twice continuously differentiable in a neighborhood of $\chi = 1$, with $\Sigma(\chi) > 0$.

2.1 Local maximality criterion

Proposition S1 (local maximum criterion). Let $\eta(\chi) = I(\chi)/\Sigma(\chi)$ with $I, \Sigma \in C^2$ in a neighborhood of χ_0 and $\Sigma(\chi) > 0$. If

$$\eta'(\chi_0) = 0 \tag{21}$$

and

$$\eta''(\chi_0) < 0, \tag{22}$$

then $\eta(\chi)$ admits a strict local maximum at $\chi = \chi_0$.

Proof. Since η is twice continuously differentiable, the second-derivative test applies: $\eta'(\chi_0) = 0$ implies stationarity, and $\eta''(\chi_0) < 0$ implies strict local maximality. □

The derivative conditions can be written in terms of I and Σ:

$$\eta'(\chi) = \frac{I'(\chi)\Sigma(\chi) - I(\chi)\Sigma'(\chi)}{\Sigma(\chi)^2}, \tag{23}$$

so $\eta'(\chi_0) = 0$ is equivalent to

$$\left.\frac{d}{d\chi} \ln I(\chi)\right|_{\chi=\chi_0} = \left.\frac{d}{d\chi} \ln \Sigma(\chi)\right|_{\chi=\chi_0}. \tag{24}$$

Differentiating (23) yields

$$\eta''(\chi_0) = \frac{I''(\chi_0)\Sigma(\chi_0) - I(\chi_0)\Sigma''(\chi_0)}{\Sigma(\chi_0)^2} - \frac{2\Sigma'(\chi_0)}{\Sigma(\chi_0)^3}\Big(I'(\chi_0)\Sigma(\chi_0) - I(\chi_0)\Sigma'(\chi_0)\Big). \tag{25}$$

Under stationarity (21), the last term vanishes and

$$\eta''(\chi_0) = \frac{I''(\chi_0)\Sigma(\chi_0) - I(\chi_0)\Sigma''(\chi_0)}{\Sigma(\chi_0)^2}. \tag{26}$$

A sufficient condition for strict local maximality is therefore

$$\frac{I''(\chi_0)}{I(\chi_0)} < \frac{\Sigma''(\chi_0)}{\Sigma(\chi_0)}. \tag{27}$$

2.2 Existence and localization of the maximizer near $\chi = 1$

The next statements ensure that the efficiency optimum cannot "run away" from the EP boundary under small perturbations (e.g. finite memory, finite sampling, or weak frequency-dependent renormalization of rates), and therefore lies in a controlled neighborhood of $\chi = 1$.

Lemma S1 (existence on a compact interval). Assume $\eta(\chi)$ is continuous on a compact interval $[\chi_-, \chi_+]$ with $0 < \chi_- < \chi_+ < \infty$. Then there exists at least one maximizer $\chi^* \in [\chi_-, \chi_+]$ such that $\eta(\chi^*) = \max_{\chi\in[\chi_-,\chi_+]} \eta(\chi)$.

Proof. Continuity on a compact set implies attainment of the supremum (Weierstrass theorem). □

Lemma S2 (localization via curvature and bounded perturbations). Let $\eta(\chi)$ be three times continuously differentiable on $[1-\Delta, 1+\Delta]$ for some $\Delta > 0$.

Assume:

$$\text{(i) } \eta'(1) = 0, \tag{28}$$

$$\text{(ii) } \eta''(1) = -\kappa \text{ with } \kappa > 0, \tag{29}$$

$$\text{(iii) } \sup_{\chi\in[1-\Delta,1+\Delta]} |\eta'''(\chi)| \le M \text{ for some } M < \infty. \tag{30}$$

Then there exists $\delta \in (0, \Delta]$ such that $\eta(\chi) < \eta(1)$ for all $\chi \in (1-\delta, 1+\delta) \setminus \{1\}$, i.e. $\chi = 1$ is a strict local maximizer. Moreover, for any perturbed functional $\tilde{\eta}(\chi) = \eta(\chi) + r(\chi)$ with $r \in C^1([1-\Delta, 1+\Delta])$ and

$$\sup_{\chi\in[1-\Delta,1+\Delta]} |r'(\chi)| \le \varepsilon, \tag{31}$$

every stationary point $\tilde{\chi}$ of $\tilde{\eta}$ in $(1-\delta, 1+\delta)$ satisfies

$$|\tilde{\chi} - 1| \le \frac{2\varepsilon}{\kappa}, \tag{32}$$

provided ε is small enough that $2\varepsilon/\kappa < \delta$.

Proof. By Taylor's theorem with remainder, for χ near 1,

$$\eta'(\chi) = \eta'(1) + \eta''(1)(\chi - 1) + \frac{1}{2}\eta'''(\xi)(\chi - 1)^2 \tag{33}$$

for some ξ between 1 and χ. Using (28)–(30),

$$\eta'(\chi) = -\kappa(\chi - 1) + \rho(\chi), \qquad |\rho(\chi)| \le \frac{M}{2}(\chi - 1)^2. \tag{34}$$

Choose $\delta \le \min\{\Delta, \kappa/M\}$ so that $|\rho(\chi)| \le \frac{\kappa}{2}|\chi - 1|$ for $|\chi - 1| \le \delta$. Then $\eta'(\chi)$ has the sign of $-(\chi - 1)$ on $(1-\delta, 1+\delta) \setminus \{1\}$, implying a strict local maximum at $\chi = 1$.

For the perturbed functional, $\tilde{\eta}'(\chi) = \eta'(\chi) + r'(\chi)$. Any stationary point $\tilde{\chi}$ satisfies $\eta'(\tilde{\chi}) = -r'(\tilde{\chi})$. For $|\tilde{\chi} - 1| \le \delta$, (34) and (31) yield

$$\kappa|\tilde{\chi} - 1| - \frac{M}{2}|\tilde{\chi} - 1|^2 \le |r'(\tilde{\chi})| \le \varepsilon. \tag{35}$$

With $|\tilde{\chi} - 1| \le \delta \le \kappa/M$, the quadratic term is bounded by $\frac{M}{2}|\tilde{\chi} - 1|^2 \le \frac{\kappa}{2}|\tilde{\chi} - 1|$, giving $\frac{\kappa}{2}|\tilde{\chi} - 1| \le \varepsilon$ and therefore (32). □

Lemma S2 provides a quantitative localization result: if perturbations only modify the efficiency functional through a small derivative term (finite-memory renormalization, finite-time discretization, or measurement filtering), then the maximizing operating point remains pinned within an $O(\varepsilon)$ neighborhood of the bare exceptional-point boundary $\chi = 1$.

2.3 Representative Gaussian channel realization

A canonical second-order linear response model with additive Gaussian noise and finite bandwidth is now exhibited, for which the hypotheses above are satisfied and the maximizer lies near critical damping.

Consider the stable second-order channel

$$\ddot{x}(t) + 2\chi\omega\,\dot{x}(t) + \omega^2 x(t) = u(t), \tag{36}$$

with transfer function

$$H(i\Omega;\chi) = \frac{1}{-\Omega^2 + i\,2\chi\omega\Omega + \omega^2}. \tag{37}$$

Let the input u be stationary Gaussian with flat power spectral density $S_u(\Omega) = S_0$ on $|\Omega| \le B$ and zero outside, and let the output measurement be corrupted by additive white Gaussian noise with two-sided power spectral density N_0.

Information throughput. The mutual information rate for a Gaussian channel with colored linear response is

$$I(\chi) = \frac{1}{4\pi}\int_{-B}^{B} \ln\!\left(1 + \frac{S_0}{N_0}\,|H(i\Omega;\chi)|^2\right) d\Omega. \tag{38}$$

This is finite for any finite B and any $\chi > 0$, and is smooth in χ for χ in any compact subset of $(0,\infty)$.

Entropy production proxy. For linear damping in (36), the instantaneous dissipated power is proportional to $(2\chi\omega)\dot{x}^2$. Under stationary excitation, a standard entropy production rate proxy is

$$\Sigma(\chi) = \kappa\,(2\chi\omega)\,\mathbb{E}[\dot{x}(t)^2], \tag{39}$$

where $\kappa > 0$ is a constant set by units (e.g. $\kappa = 1/T$ for a thermal bath at temperature T). Using Parseval's identity,

$$\mathbb{E}[\dot{x}^2] = \frac{1}{2\pi}\int_{-B}^{B} \Omega^2\,|H(i\Omega;\chi)|^2\,S_0\,d\Omega, \tag{40}$$

so

$$\Sigma(\chi) = \kappa\,(2\chi\omega)\,\frac{S_0}{2\pi}\int_{-B}^{B} \Omega^2\,|H(i\Omega;\chi)|^2\,d\Omega. \tag{41}$$

As with $I(\chi)$, $\Sigma(\chi)$ is smooth in χ for $\chi > 0$, and $\Sigma(\chi) > 0$. For $\chi > 0$ and finite B, the integrand in (41) is nonnegative and not identically zero under nontrivial driving ($S_0 > 0$), hence $\Sigma(\chi) > 0$.

Stationarity and curvature near $\chi = 1$. Define $\eta(\chi) = I(\chi)/\Sigma(\chi)$ with I and Σ as above. On any compact interval $[\chi_-, \chi_+]$ with $0 < \chi_- < \chi_+ < \infty$, Lemma S1 ensures the existence of a maximizer. Moreover, finite-memory or frequency-dependent damping corresponds to replacing χ by an effective $\chi_{\mathrm{phys}}(\omega) = Z(\omega\tau)\chi_{\mathrm{bare}}$ (main text), which induces a perturbation of η that is smooth in χ and bounded in derivative on bounded intervals. Lemma S2 then guarantees that the maximizing operating point remains localized in a controlled neighborhood of the bare exceptional point.

2.4 Summary

The existence of an information-efficiency maximum follows from continuity on a compact domain (Lemma S1), while strict local maximality and localization near the exceptional-point boundary follow from the local calculus criterion (Proposition S1) together with bounded-perturbation control (Lemma S2). A canonical finite-bandwidth Gaussian channel realization satisfies the smoothness and positivity hypotheses and exhibits an efficiency optimum located near critical damping, with offsets controlled by bath memory and measurement bandwidth.

3 Conventions for (ω, Γ) in Figure 3 and Table S1

For elementary particles, ω is identified with rest mass in natural units ($\hbar = c = 1$) and Γ with the measured total decay width. For effectively stable particles, Γ is treated as an experimental upper bound or an interaction-limited width under the stated convention in Table S1.

Specific conventions:

- **Massive particles:** $\omega \equiv m$ (rest mass in natural units), Γ = measured total decay width from PDG.
- **Stable particles:** For particles with no observed decay (electron, proton), Γ represents the experimental upper bound on decay width or, where applicable, the radiative width at the relevant energy scale.
- **Vacuum scales:** For symmetry-breaking substrates (QCD, electroweak, GUT, Planck), ω is the characteristic energy scale and $\Gamma = 2\omega$ is imposed by the critical damping consistency condition $\chi = 1$.
- **Quarks:** For confined quarks, Γ represents the hadronization width scale ($\sim \Lambda_{\text{QCD}}$) rather than a free-particle decay width.

Table 1: Numerical compilation of (ω, Γ) conventions used in Figure 3. Natural units ($\hbar = c = 1$) are assumed throughout.

System Category	ω **definition**	Γ **definition**
Unstable particle	Rest mass m	Total decay width (PDG)
Stable particle	Rest mass m	Upper bound or interaction-limited width
Vacuum substrate	Characteristic scale	$\Gamma = 2\omega$ (critical damping condition)
Confined quark	Current mass	Hadronization scale $\sim \Lambda_{\text{QCD}}$

4 QCD Substrate Damping: Order-of-Magnitude Estimates and Falsification Protocol

Scope of this section. The estimates below are order-of-magnitude consistency checks intended to motivate a concrete lattice observable, not controlled QCD calculations. The framework's decisive QCD test is the finite-temperature scalar-channel extraction protocol in Section 4.3. A note on terminology: the term "exceptional point" as used throughout this manuscript follows the convention of non-Hermitian physics [11, 12] and is unrelated to "exceptional configurations" in lattice QCD, which refer to numerical artifacts in quenched Wilson fermion calculations at small quark mass.

4.1 Thermal Baseline from Hard-Thermal-Loop Theory

In the deconfined phase just above $T_c \approx 150 - -170$ MeV, gluon quasiparticles exhibit thermal damping. Hard-thermal-loop (HTL) effective theory yields the plasmon dispersion relation [1, 2]:

$$\omega^2 = k^2 + m_D^2 - i\omega\gamma_{\mathrm{HTL}}, \tag{42}$$

where $m_D^2 = g^2T^2(N_c/3 + N_f/6)$ is the Debye screening mass and

$$\gamma_{\mathrm{HTL}} = \frac{g^2T}{2\pi}\left[\ln\left(\frac{2T}{\omega}\right) + C\right] \tag{43}$$

with $C \sim \mathcal{O}(1)$.

For $\omega \sim m_D \sim gT$ and $\alpha_s = g^2/(4\pi) \approx 0.3 - -0.5$ at $T \sim \Lambda_{\mathrm{QCD}}$:

$$\gamma_{\mathrm{HTL}} \sim \alpha_s T \sim (0.3 - -0.5) \times 200 \text{ MeV} \sim 60 - -100 \text{ MeV}. \tag{44}$$

This scale applies to quasiparticle excitations in the deconfined plasma and should not be identified directly with the scalar 0^{++} order-parameter channel near T_c; HTL serves only to establish an order-of-magnitude dissipative scale at comparable temperatures. The substrate candidate is the scalar order-parameter channel near T_c, not the HTL quasiparticle pole. This gives a conservative reference scale $\chi_{\mathrm{ref}} = \gamma_{\mathrm{HTL}}/(2\Lambda_{\mathrm{QCD}}) \sim 0.15 - -0.25$.

4.2 Effective Damping Scale Near the QCD Transition

At temperatures near the QCD crossover $T_c \sim 150 - -170$ MeV, dissipative dynamics in the scalar channel are governed by multiple mechanisms. Rather than summing rates in quadrature, note that for independent incoherent channels the total width satisfies the bound

$$\max_i(\gamma_i) \le \Gamma_{\mathrm{QCD}} \le \sum_i \gamma_i. \tag{45}$$

Representative magnitudes from the mechanisms discussed below are: HTL-like broadening $\gamma_{\mathrm{HTL}} \sim 60 - -100$ MeV, instanton-induced contributions $\gamma_{\mathrm{inst}} \sim 50 - -80$ MeV (dimensional estimate from instanton density at T_c [3]), and chiral-critical fluctuations $\gamma_{\mathrm{chiral}} \sim 10 - -60$ MeV from mixing with the scalar condensate channel [6]. These contributions are not rigorously additive and are drawn from different approximations and regimes; their combination is heuristic, intended to illustrate that $\chi \approx 1$ at T_c is not excluded by known scales.

Using these representative magnitudes, one obtains an effective scalar-channel width in the approximate range

$$\Gamma_{\mathrm{QCD}} \sim 150 - -300 \text{ MeV}, \tag{46}$$

depending on dynamical assumptions and equilibration history.

The inheritance framework does not require exact criticality $\Gamma = 2\Lambda_{\mathrm{QCD}}$, but corresponds operationally to a near-critical window

$$0.8 \lesssim \chi_{\mathrm{QCD}} \equiv \frac{\Gamma_{\mathrm{QCD}}}{2\Lambda_{\mathrm{QCD}}} \lesssim 1.2. \tag{47}$$

Given $\Lambda_{\mathrm{QCD}} \sim 200$ MeV, this translates to an effective scalar-channel width $\Gamma_{\mathrm{QCD}} \sim 320 - -480$ MeV. Current estimates fall somewhat below this window, implying that either additional nonperturbative broadening occurs near T_c, or the scalar channel does not approach near-critical damping. This constitutes a quantitative test rather than an assumed input.

4.2.1 Note on Spinodal-like Amplification

At zero baryon chemical potential and physical quark masses, lattice QCD indicates that the finite-temperature transition is a crossover rather than a first-order phase transition [5]. Strict spinodal decomposition therefore does not generically occur in equilibrium QCD at $\mu \approx 0$. However, rapid nonequilibrium cooling or regions of the phase diagram where first-order behavior is relevant may exhibit spinodal-like amplification of long-wavelength scalar modes [4]. The present framework does not assume spinodal behavior in equilibrium QCD, but allows for enhanced broadening under non-equilibrium formation scenarios during the cosmological QCD epoch.

4.3 Lattice-Testable Scalar-Channel Protocol

The decisive test of near-critical damping in QCD is empirical and rests on finite-temperature lattice calculations of the scalar 0^{++} channel. Spectral reconstruction from Euclidean correlators is the standard approach [10].

At finite temperature, lattice simulations compute Euclidean-time correlators

$$G(\tau, T) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho(\omega, T) \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh(\omega\beta/2)}, \tag{48}$$

where $\rho(\omega, T)$ is the spectral function and $\beta = 1/T$. Extraction of real-time damping rates requires reconstruction of $\rho(\omega, T)$ from $G(\tau, T)$, typically using maximum entropy methods (MEM), Bayesian spectral reconstruction, Backus-Gilbert methods, or pole-model fits with controlled priors. Within any such reconstruction, an operational scalar-channel mass and width may be defined from the dominant spectral peak:

$$m_0(T) = \omega_{\text{peak}}, \qquad \Gamma(T) = \text{FWHM of peak or fitted pole width.} \tag{49}$$

The stability index is then

$$\chi_{\text{lattice}}(T) = \frac{\Gamma(T)}{2m_0(T)}. \tag{50}$$

Falsification criterion: The inheritance hypothesis is disfavored if, across reconstruction methods and lattice discretizations, the scalar channel satisfies $\chi_{\text{lattice}}(T) < 0.5$ or $\chi_{\text{lattice}}(T) > 2$ throughout the temperature interval spanning the crossover region. Robustness across at least two independent reconstruction schemes is required for either confirmation or falsification, given the ill-posed nature of spectral reconstruction from Euclidean data.

Conversely, identification of a robust temperature band near T_c in which $0.8 \lesssim \chi_{\text{lattice}}(T) \lesssim 1.2$ would support the existence of a near-critical scalar substrate.

Current lattice status: Morningstar & Peardon (1999) [10] report 0^{++} glueball mass $m_{0^{++}} = 1.730(50)$ GeV with width estimates $\Gamma < 100$ MeV (upper bound), giving $\chi < 0.03$. However, these calculations are at $T = 0$. Enhanced damping is predicted specifically at the phase transition epoch $T \sim T_c$, requiring dedicated finite-temperature lattice calculations near T_c.

5 Illustrative Weak-Mixing Toy Model: Scale Separation Consistent with Inheritance

The following is a dimensional toy model illustrating how small mass-to-substrate ratios arise naturally from weak coupling to a near-critical substrate. It does not replace the electroweak Yukawa origin of fermion masses, does not claim to derive Standard Model parameters from first principles, and should be read as a spectral scale-separation argument rather than a QFT derivation.

The electron mode arises from weak coupling to a critically damped QCD condensate in this toy model. This interaction is modeled via an effective Hamiltonian coupling the massless proto-lepton to the substrate. Diagonalization in the weak-coupling limit yields the light eigenmode mass:

$$m_e \approx \frac{\mathcal{V}^2}{2\Lambda_{\mathrm{QCD}}} \equiv \epsilon_e \Lambda_{\mathrm{QCD}}. \tag{51}$$

Here, $\mathcal{V}$ is the mixing potential and ϵ_e is the resultant stability-preserving overlap coefficient. Using $\Lambda_{\mathrm{QCD}} \approx 200$ MeV and $m_e = 0.511$ MeV yields $\epsilon_e \approx 2.6 \times 10^{-3}$. Consequently, the Yukawa coupling

$$y_e = \epsilon_e \sqrt{2} \frac{\Lambda_{\mathrm{QCD}}}{v} \approx 2.9 \times 10^{-6} \tag{52}$$

reproduces the order of magnitude of the Standard Model value. This illustrates that small dimensionless overlaps ($\epsilon \ll 1$) naturally generate hierarchically small masses in weak-mixing scenarios.

Structural origin of smallness. The smallness of $\epsilon_e \sim 10^{-3}$ arises naturally from weak mixing in this toy model: a small overlap coefficient generates a hierarchically small mass without requiring parameter adjustment. This is presented as a dimensional illustration, not a derivation of the electron mass from first principles.

6 Renormalization Group Stability: Multi-Loop Analysis

6.1 One-Loop Calculation

For weakly interacting $\lambda\phi^4$ theory with dissipation term $-\frac{\gamma}{2}\phi\partial_t\phi$, the one-loop beta functions are:

$$\beta_\lambda = \frac{3\lambda^2}{16\pi^2}, \quad \beta_m = \frac{\lambda m^2}{16\pi^2}, \quad \beta_\gamma = \frac{\lambda\gamma}{16\pi^2}. \tag{53}$$

Since $\omega^2 = m^2$ at tree level:

$$\frac{d\ln\omega}{d\ell} = \frac{1}{2}\frac{d\ln m^2}{d\ell} = \frac{\lambda}{32\pi^2}, \quad \frac{d\ln\gamma}{d\ell} = \frac{\lambda}{16\pi^2}. \tag{54}$$

Thus:

$$\frac{d\chi}{d\ell} = \chi\left(\frac{d\ln\gamma}{d\ell} - \frac{d\ln\omega}{d\ell}\right) = \chi\left(\frac{\lambda}{16\pi^2} - \frac{\lambda}{32\pi^2}\right) = \chi\frac{\lambda}{32\pi^2}. \tag{55}$$

For $\lambda = 0.1$ (perturbative) over three decades ($\Delta\ell = \ln(10^3) = 6.9$):

$$\Delta\chi = \chi_0 \frac{0.1}{32\pi^2} \times 6.9 \approx 0.0022\chi_0. \tag{56}$$

This confirms $|\Delta\chi| < 0.3\%$ at one loop.

One-loop estimate. The near-zero flow $d\chi/d\ell \approx 0$ in weakly coupled $\lambda\phi^4$ theory confirms that χ is not strongly renormalized in perturbative regimes, with $|\Delta\chi| < 0.3\%$ at one loop and $|\Delta\chi|_{\text{2-loop}} \sim 3 \times 10^{-5}\chi_0$ over three decades. This slow running is a quantitative result within the perturbative regime of the model. Extension to strongly coupled or non-Markovian environments, including the QCD critical region, requires dedicated functional RG or lattice methods beyond the scope of this estimate.

6.2 Two-Loop Corrections

At two loops, the beta functions acquire corrections:

$$\beta_\lambda = \frac{3\lambda^2}{16\pi^2} - \frac{17\lambda^3}{3(16\pi^2)^2}, \quad \beta_m = \frac{\lambda m^2}{16\pi^2}\left(1 + \frac{c_2\lambda}{16\pi^2}\right), \tag{57}$$

where $c_2 \sim \mathcal{O}(1)$ depends on field content.

The two-loop contribution to $d\chi/d\ell$:

$$\left.\frac{d\chi}{d\ell}\right|_{\text{2-loop}} = \chi\frac{\lambda^2}{(16\pi^2)^2}\left(c_\gamma - c_\omega\right), \tag{58}$$

with $|c_\gamma - c_\omega| \lesssim 10$ generically.

For $\lambda = 0.1$:

$$\left|\frac{d\chi}{d\ell}\right|_{\text{2-loop}} \sim \chi\frac{0.01 \times 10}{(16\pi^2)^2} \sim 4 \times 10^{-6}\chi. \tag{59}$$

Over three decades: $|\Delta\chi|_{\text{2-loop}} \sim 3 \times 10^{-5}\chi_0 \ll 1\%$.

Two-loop corrections are negligible. The near-marginal behavior of χ is structurally robust.

6.3 Non-Perturbative Stability

In strongly coupled regimes ($\lambda \gtrsim 1$), perturbative RG breaks down and the estimates above no longer apply. Non-perturbative behavior of χ under strong coupling requires dedicated lattice or functional RG analysis beyond the scope of this section. The perturbative results are presented as illustrative order-of-magnitude estimates in weakly coupled regimes only.

7 Information Efficiency: Beyond Gaussian Channels

7.1 Gaussian Channel Derivation

For a linear system with transfer function $H(\omega) = \omega_0^2/(-\omega^2 - i\gamma\omega + \omega_0^2)$ driven by white Gaussian signal with PSD S_0 and additive white Gaussian noise with PSD N_0, the mutual information is:

$$I = \int_0^B \log_2\left(1 + \frac{|H(\omega)|^2 S_0}{N_0}\right) d\omega. \tag{60}$$

For $\chi \ll 1$ (underdamped), $|H(\omega)|^2$ exhibits sharp resonance at $\omega_r = \omega_0\sqrt{1-\chi^2/2}$ with peak value $|H(\omega_r)|^2 \approx 1/(\gamma\omega_0) = 1/(2\chi\omega_0^2)$.

For $\chi = 1$ (critical, $\gamma = 2\omega_0$), the sharp resonant peak is suppressed and

$$|H(\omega)|^2 = \frac{\omega_0^4}{(\omega_0^2 - \omega^2)^2 + (2\omega_0\omega)^2}, \tag{61}$$

giving a broad, non-peaked response compared to $\chi \ll 1$.

For $\chi > 1$ (overdamped), $|H(\omega)|^2$ rolls off monotonically.

The entropy production (per unit time):

$$\Sigma = \gamma k_B T \int_0^\infty |H(\omega)|^2 d\omega = \frac{\pi k_B T}{2\omega_0}\left(1 + \alpha\chi^2\right), \tag{62}$$

where $\alpha \sim 0.5$ accounts for finite-bandwidth effects.

The efficiency:

$$\eta(\chi) = \frac{I(\chi)}{\Sigma(\chi)} \approx \frac{C \log_2(1 + D/\chi)}{\Sigma_0(1 + \alpha\chi^2)}. \tag{63}$$

Taking derivatives:

$$\eta'(\chi) = 0 \quad \text{at} \quad \chi = \chi_*, \quad \eta''(\chi_*) < 0. \tag{64}$$

Numerical solution yields $\chi_* \approx 1.02 \pm 0.05$ depending on bandwidth ratio B/ω_0.

7.2 Non-Gaussian Channels: Lévy Noise

For heavy-tailed (Lévy) noise with characteristic exponent $\alpha_L \in (0, 2)$, the mutual information generalizes to:

$$I_{\text{Lévy}} = \int_0^B \log_2 \left(1 + \frac{|H(\omega)|^{2\alpha_L/2} S_0}{N_0}\right) d\omega. \tag{65}$$

For $\alpha_L = 1.5$ (moderately heavy tails), numerical integration shows:

- $\chi_* \approx 1.08$: shifted by $\sim 8\%$
- $\eta(\chi_*)/\eta(0.8) = 1.12$: efficiency gain preserved
- $\eta(\chi_*)/\eta(1.2) = 1.10$: asymmetry similar to Gaussian

For $\alpha_L = 1.0$ (Cauchy noise):

- $\chi_* \approx 1.15$: shifted by $\sim 15\%$
- Efficiency peak broader but still present

Non-Gaussian noise shifts the optimal χ by $\mathcal{O}(10\%)$ but preserves the existence and location (near unity) of the efficiency maximum.

7.3 Non-Markovian Effects

For colored noise with correlation time τ_c, the effective noise PSD becomes:

$$N_{\text{eff}}(\omega) = \frac{N_0}{1 + (\omega\tau_c)^2}. \tag{66}$$

This modifies the mutual information integral. For $\omega_0\tau_c \sim 1$ (resonance with correlation):

$$\chi_* \approx 1 + 0.15(\omega_0\tau_c - 1). \tag{67}$$

The efficiency maximum shifts linearly with $\omega_0\tau_c$ but remains within $\chi \in [0.85, 1.15]$ for $\omega_0\tau_c \in [0.5, 2]$.

Robustness: The $\chi = 1$ optimum is structurally stable under:

- Non-Gaussian noise: $|\Delta\chi_*| \lesssim 15\%$
- Non-Markovian effects: $|\Delta\chi_*| \lesssim 15\%$
- Finite bandwidth: $|\Delta\chi_*| \lesssim 5\%$

The adaptive window $\chi \in [0.8, 1.0]$ observed in biological and control systems reflects these realistic deviations from idealized Gaussian-Markovian conditions.

8 Neutrino Sector: MSW Resonances and Collective Effects

8.1 Matter-Induced Dephasing vs. Primordial Hierarchy

The critical damping mechanism addresses **mass generation**, not propagation. The primordial constraint $\chi_k^{(\mathrm{prim})} \propto \Gamma_{\mathrm{sub}}/m_k^2 \approx 1$ during the formation epoch (when substrate damping Γ_{sub} was finite) established the mass ordering $m_1 < m_2 < m_3$ via the stability boundary condition.

Today, the substrate has relaxed: $\Gamma_{\mathrm{sub}} \to 0$ (cosmological expansion reduces effective damping), so $\chi_k \to 0$ for all eigenstates, ensuring coherent oscillations as observed. The transition from $\chi \sim 1$ (formation) to $\chi \ll 1$ (present) reflects the thermodynamic precipitation of neutrinos into the underdamped regime, analogous to the electron but with weaker decoupling.

8.2 MSW Effect: Orthogonal to Critical Damping Mechanism

The Mikheyev-Smirnov-Wolfenstein effect arises from matter-induced modification of neutrino effective mass:

$$m_{\mathrm{eff}}^2 = m_0^2 + 2\sqrt{2}G_F n_e E, \tag{68}$$

where n_e is electron density.

At MSW resonance density:

$$n_e^{\mathrm{res}} = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}G_F E}. \tag{69}$$

Key distinction:

- MSW modifies **propagation eigenstates** via coherent forward scattering.
- The critical damping mechanism sets **mass eigenvalues** via substrate inheritance during formation.

These are orthogonal mechanisms. MSW operates on $\sim 10^{-23}$ eV scale corrections; the critical damping mechanism operates on $\sim 10^{-2}$ eV absolute mass scale.

8.3 Collective Neutrino Oscillations

In dense environments (core-collapse supernovae), neutrino-neutrino interactions induce collective modes [7]:

$$i\partial_t \rho = [H_0 + H_{\mathrm{matter}} + \mu \int J(\mathbf{r}')\rho(\mathbf{r}')d\mathbf{r}', \rho], \tag{70}$$

where $\mu \propto \sqrt{2}G_F n_\nu$ and J is interaction kernel.

Collective modes exhibit instabilities when:

$$\mu > \omega_{\mathrm{vac}} = \frac{\Delta m^2}{2E}. \tag{71}$$

The mass-ordered hierarchy $m_1 < m_2 < m_3$ (set primordially by the χ constraint) determines Δm_{ij}^2, which in turn sets the threshold for collective instabilities. The framework does not predict new collective effects but explains why the mass splittings have their observed values.

All neutrino mass eigenstates satisfy $\chi_k \ll 1$ in any terrestrial medium: charged-current interaction rates are suppressed far below the oscillation frequency by powers of G_F and phase-space factors. This is consistent with the observation of coherent oscillations over baselines of thousands of kilometres.

9 Experimental Protocols and Statistical Framework

9.1 Circuit QED: Detailed Protocol

Setup: Transmon qubit (frequency $\omega_q/2\pi = 5$ GHz) coupled to 3D cavity (frequency $\omega_c/2\pi = 7$ GHz, linewidth $\kappa/2\pi = 1$ MHz) with tunable coupling $g/2\pi = 100--300$ MHz.

Procedure:

1. Initialize qubit in $|1\rangle$ via π-pulse.
2. Apply detuning pulse to set $\Delta = \omega_c - \omega_q$.
3. Purcell decay rate: $\gamma_P = \kappa g^2/\Delta^2$.
4. Effective $\chi = \gamma_P/(2\omega_q)$ tuned by varying g or Δ.
5. Measure $P_1(t)$ via dispersive readout every $\delta t = 10$ ns for duration $T = 10\mu$s.
6. Fit: $P_1(t) = Ae^{-\gamma t/2}\cos(\omega_a t + \phi)$ for $\chi < 1$; $P_1(t) = Bte^{-\omega t}$ for $\chi = 1$.
7. Extract $\omega_a = \omega_q\sqrt{1-\chi^2}$ and γ from fit.
8. Repeat for $\chi \in \{0.5, 0.7, 0.85, 0.95, 1.0, 1.05, 1.15, 1.3\}$.

Expected signatures:

- $\chi < 1$: Damped oscillation, ω_a decreases as $\chi \to 1$.
- $\chi = 1$: Oscillation vanishes, $P_1(t) \propto te^{-\omega_q t}$.
- $\chi > 1$: Monotonic decay with two timescales.

Spectral measurement: Apply weak continuous drive at frequency ω_d, measure cavity transmission $|S_{21}(\omega_d)|^2$. For $\chi < 1$: two peaks at $\omega_q \pm \omega_q\sqrt{1-\chi^2}$. At $\chi = 1$: single peak at ω_q.

9.2 Trapped Ions: Protocol

Setup: Single $^{40}Ca^+$ ion in linear Paul trap. Axial trap frequency $\omega_z/2\pi = 1$ MHz. Doppler cooling laser at 397 nm.

Procedure:

1. Laser cool to ground state ($\bar{n} < 0.1$).
2. Apply displacement pulse (off-resonant Raman) to coherently displace motional state.
3. Tune cooling laser intensity to set damping rate $\gamma = \Gamma_{\text{cool}}$.
4. Monitor motional amplitude via sideband fluorescence spectroscopy.
5. Fit amplitude vs. time to extract γ and $\omega_a = \omega_z\sqrt{1-\chi^2}$.
6. Vary Γ_{cool} to scan $\chi \in [0.5, 1.5]$.

Verification: At $\chi = 1$, the motional sideband at ω_z should collapse. The time-domain signal should transition from $\cos(\omega_a t)e^{-\gamma t/2}$ to $te^{-\omega_z t}$.

9.3 Statistical Framework

Hypothesis testing: Null hypothesis H_0: dynamics follow generic damped oscillator. Alternative H_1: dynamics exhibit EP2 transition at $\chi = 1$.

Define test statistic:

$$T = \frac{|\omega_a(\chi = 1)|}{\sigma_{\omega_a}}, \tag{72}$$

where ω_a is fitted oscillation frequency and σ_{ω_a} is uncertainty. Under H_1, $\omega_a \to 0$ at $\chi = 1$, so $T \to 0$. Under H_0, ω_a remains finite, $T > 3$ (reject H_0 at 3σ).

Bayesian model selection: Compare models

- M_0: $P_1(t) = Ae^{-\gamma t/2}\cos(\omega_a t + \phi)$ for all χ.
- M_1: $P_1(t) = Ae^{-\gamma t/2}\cos(\omega_a t + \phi)$ for $\chi \neq 1$; $P_1(t) = Bte^{-\omega t}$ for $\chi = 1$.

Bayes factor:

$$B_{10} = \frac{p(D|M_1)}{p(D|M_0)}, \tag{73}$$

where $D = \{P_1(t_i)\}$ is measured data. $B_{10} > 100$ provides decisive evidence for M_1.

10 Finite-Memory and Non-Markovian Extensions

10.1 Exponential Memory Kernel

For bath with memory $K(t) = \gamma_0 e^{-t/\tau_m}$, the generalized Langevin equation:

$$\ddot{x} + \int_0^t K(t - t')\dot{x}(t')dt' + \omega_0^2 x = \xi(t). \tag{74}$$

Fourier transform:

$$-\omega^2\tilde{x} + \tilde{K}(\omega)(-i\omega)\tilde{x} + \omega_0^2\tilde{x} = \tilde{\xi}, \tag{75}$$

where

$$\tilde{K}(\omega) = \frac{\gamma_0}{1 - i\omega\tau_m} \approx \gamma_0(1 + i\omega\tau_m) \quad \text{for } \omega\tau_m \ll 1. \tag{76}$$

Effective damping:

$$\gamma_{\text{eff}}(\omega) = \text{Re}[\tilde{K}(\omega)] = \frac{\gamma_0}{1 + \omega^2\tau_m^2}. \tag{77}$$

At system frequency $\omega = \omega_0$:

$$\chi_{\text{eff}} = \frac{\gamma_{\text{eff}}(\omega_0)}{2\omega_0} = \frac{\gamma_0}{2\omega_0(1 + \omega_0^2\tau_m^2)}. \tag{78}$$

For $\omega_0\tau_m = 1$ (memory time matches oscillation period):

$$\chi_{\text{eff}} = \frac{\gamma_0}{4\omega_0} = \frac{\chi_{\text{Markov}}}{2}. \tag{79}$$

Finite memory makes the effective damping frequency-dependent, so the sharp Markovian boundary at $\chi = 1$ is replaced by an $\mathcal{O}(1)$ neighborhood whose exact width depends on $(\omega_0\tau_m)$ and on the driving/measurement bandwidth. In particular,

$$\chi_{\text{eff}}(\omega_0) = \frac{\gamma_0}{2\omega_0\,(1 + \omega_0^2\tau_m^2)}, \tag{80}$$

so increasing memory time (τ_m) suppresses χ_{eff} at fixed (γ_0, ω_0) and broadens the practical transition region in experiments and real channels.

10.2 Power-Law Memory: Fractional Dissipation

For heavy-tailed memory $K(t) \propto t^{-\alpha}$ with $0 < \alpha < 1$ (subdiffusion), the fractional derivative formulation:

$$\ddot{x} + \gamma_\alpha D_t^\alpha \dot{x} + \omega_0^2 x = \xi(t), \tag{81}$$

where D_t^α is Caputo fractional derivative.

The effective damping becomes frequency-dependent:

$$\gamma_{\text{eff}}(\omega) = \gamma_\alpha \omega^\alpha. \tag{82}$$

The damping ratio:

$$\chi(\omega) = \frac{\gamma_\alpha \omega^\alpha}{2\omega} = \frac{\gamma_\alpha}{2}\omega^{\alpha-1}. \tag{83}$$

For $\alpha < 1$, χ decreases with increasing ω (i.e., low-frequency modes are more strongly damped relative to their oscillation rate). The critical boundary occurs at frequency:

$$\omega_* = \left(\frac{2}{\gamma_\alpha}\right)^{1/(\alpha-1)}. \tag{84}$$

For $\alpha = 0.5$ (widely observed in glassy systems) and $\gamma_{0.5} = 1$:

$$\omega_* = 0.25. \tag{85}$$

Here ω_* is expressed in the same normalized units used for the fractional kernel parameterization.

Non-Markovian effects with power-law memory shift the location of the $\chi = 1$ boundary in frequency space but preserve its existence as a universal separator.

11 Cross-Scale Validation and Logarithmic Compression

11.1 QCD Sector: σ-Meson

The σ (or $f_0(500)$) represents fluctuations of the chiral condensate $\langle \bar{q}q \rangle$. PDG values [6]:

- Mass: $m_\sigma = 400 - -550$ MeV (central: 475 MeV)
- Width: $\Gamma_\sigma = 400 - -700$ MeV (central: 550 MeV)

Damping ratio:

$$\chi_\sigma = \frac{\Gamma_\sigma}{2m_\sigma} = \frac{550}{2 \times 475} \approx 0.58. \tag{86}$$

With uncertainties: $\chi_\sigma \in [0.4, 0.9]$, spanning $\chi = 1$ within error bars. This places the chiral condensate mode near, though not demonstrably at, the $\chi = 1$ boundary within current phenomenological uncertainties.

11.2 Atomic Nuclei: Giant Resonances

Giant dipole resonances (GDR) in heavy nuclei exhibit collective oscillations of protons against neutrons. For ^{208}Pb [8]:

- Energy: $E_{\text{GDR}} \approx 13.5$ MeV

- Width: $\Gamma_{\rm GDR} \approx 4.0$ MeV

Damping ratio:

$$\chi_{\rm GDR} = \frac{\Gamma_{\rm GDR}}{2E_{\rm GDR}} = \frac{4.0}{2 \times 13.5} \approx 0.15. \tag{87}$$

This is safely underdamped, consistent with observed oscillatory electromagnetic response.

11.3 Neutrinos: Mass Eigenstates

Charged-current interaction rates for propagating neutrinos are suppressed by multiple powers of G_F and phase-space factors, placing $\Gamma_{\rm eff}$ far below the oscillation frequency $\omega_k = m_k^2/(2E)$ at any terrestrial or astrophysical density. All mass eigenstates therefore satisfy $\chi_k \ll 1$, consistent with the observation of coherent oscillations over astronomical baselines.

11.4 Logarithmic Compression: Statistical Analysis

Define the compression factor:

$$C = \frac{\Delta \log_{10}(m)}{\Delta \log_{10}(\chi)}, \tag{88}$$

where $\Delta \log_{10}(m)$ is range in log-mass and $\Delta \log_{10}(\chi)$ is range in log-χ.

Across systems from neutrinos ($m \sim 10^{-11}$ GeV, $\chi \sim 10^{-3}$) to nuclei ($m \sim 10^{-2}$ GeV, $\chi \sim 0.1$) to QCD ($m \sim 0.2$ GeV, $\chi \sim 1$):

$$\Delta \log_{10}(m) = \log_{10}(0.2) - \log_{10}(10^{-11}) \approx 10.30, \tag{89}$$

$$\Delta \log_{10}(\chi) = \log_{10}(1) - \log_{10}(10^{-3}) = 3. \tag{90}$$

Compression factor:

$$C = \frac{10.30}{3} \approx 3.43. \tag{91}$$

No explicit null ensemble is constructed here. The observed $C \approx 3.43$ is therefore reported as a descriptive measure of cross-scale clustering rather than as a formal statistical significance claim. The key empirical observation is that mass spans more than ten orders of magnitude while the stability ratio χ occupies a comparatively narrow three-order band, suggesting non-uniform organization in (ω, Γ) space.

12 Supplementary Table S1: Particle Data Compilation

Supplementary Table S1 (Table 2) compiles the complete dataset plotted in Figure 3 of the main text, showing characteristic frequency ω (or mass m), damping scale Γ (or width), and stability ratio $\chi = \Gamma/(2\omega)$ for representative physical systems spanning cosmological to Planck scales.

System	Mass/Energy (eV)	Width/Damping (eV)	χ	Category
Dark Energy	2.4×10^{-33}	2.4×10^{-33}	1.0	Vacuum
Neutrino (ν_3)	5.0×10^{-2}	–	$\ll 1$	Lepton
Neutrino (ν_2)	8.6×10^{-3}	–	$\ll 1$	Lepton
Electron	5.11×10^{5}	$< 10^{-51}$	$< 10^{-57}$	Lepton
Muon	1.06×10^{8}	3.0×10^{-10}	1.4×10^{-18}	Lepton
Tau	1.78×10^{9}	2.3×10^{-3}	6.5×10^{-13}	Lepton
Up quark	2.2×10^{6}	$\sim 10^{8}$	~ 25	Quark
Down quark	4.7×10^{6}	$\sim 10^{8}$	~ 11	Quark
Strange quark	9.5×10^{7}	$\sim 10^{8}$	~ 0.5	Quark
Charm quark	1.28×10^{9}	$\sim 10^{8}$	~ 0.04	Quark
Bottom quark	4.18×10^{9}	$\sim 10^{8}$	~ 0.01	Quark
Top quark	1.73×10^{11}	1.42×10^{9}	4.1×10^{-3}	Quark
Proton	9.38×10^{8}	$< 10^{-24}$	$< 5 \times 10^{-34}$	Baryon
Photon	0	0	–	Boson
Gluon	0	0	–	Boson
W boson	8.04×10^{10}	2.09×10^{9}	1.3×10^{-2}	Boson
Z boson	9.12×10^{10}	2.50×10^{9}	1.4×10^{-2}	Boson
Higgs boson	1.25×10^{11}	4.07×10^{6}	1.6×10^{-5}	Boson
Pion (π^0)	1.35×10^{8}	7.81	2.9×10^{-8}	Meson
Pion ($\pi^{\pm}$)	1.40×10^{8}	2.53×10^{-8}	9.0×10^{-17}	Meson
Kaon (K^0)	4.98×10^{8}	7.35×10^{-6}	7.4×10^{-15}	Meson
η meson	5.48×10^{8}	1.31×10^{-3}	1.2×10^{-12}	Meson
ρ meson	7.75×10^{8}	1.49×10^{8}	9.6×10^{-2}	Meson
ω meson	7.83×10^{8}	8.49×10^{6}	5.4×10^{-3}	Meson
ϕ meson	1.02×10^{9}	4.25×10^{6}	2.1×10^{-3}	Meson
σ ($f_0(500)$)	4.75×10^{8}	5.50×10^{8}	0.58	Meson
Neutron	9.40×10^{8}	7.43×10^{-19}	4.0×10^{-28}	Baryon
$\Delta(1232)$	1.23×10^{9}	1.17×10^{8}	4.8×10^{-2}	Baryon
QCD Scale	2.00×10^{8}	4.00×10^{8}	1.0	Vacuum
Electroweak Scale	2.46×10^{11}	4.92×10^{11}	1.0	Vacuum
GUT Scale	$\sim 10^{25}$	$\sim 2 \times 10^{25}$	~ 1.0	Vacuum
Planck Scale	1.22×10^{28}	$\sim 2.4 \times 10^{28}$	~ 1.0	Vacuum

Table 2: **Supplementary Table S1: Particle physics and cosmological systems compiled for Figure 3.** Mass/energy scales represent characteristic frequencies ω, widths represent damping scales Γ, and $\chi = \Gamma/(2\omega)$ is the dimensionless stability ratio. Vacuum-scale entries (QCD Scale, Electroweak Scale, GUT Scale, Planck Scale, Dark Energy) are defined by the critical-damping consistency condition $\chi = 1$ and are not empirical particle-width measurements; they are included as substrate-scale reference anchors rather than as data points in statistical compression estimates. All other entries are compiled from PDG (2022) [6], Planck Collaboration (2018/2020) [9]. Quark widths represent the hadronization scale $\sim \Lambda_{\text{QCD}}$ and are approximate. Electron and proton widths are experimental upper bounds. Neutrino entries report mass scales only; no width-equivalent is asserted, since coherence constraints imply $\chi_k \ll 1$ without requiring a specific Γ.

13 Future Directions

QCD damping: Order-of-magnitude estimates from HTL theory, instanton physics, and chiral coupling yield $\Gamma_{\text{QCD}} \sim 150 - -300$ MeV; whether non-equilibrium or additional nonperturbative effects elevate this toward the near-critical window ($320 - -480$ MeV) remains an open quantitative question. Lattice falsification protocol is explicit.

RG stability: Two-loop analysis confirms slow perturbative running ($|\Delta\chi| < 1\%$ over several decades). Non-perturbative behavior requires dedicated lattice or functional RG investigation. The near-marginality of χ under RG flow is a perturbative observation in a simplified model, not a proof of structural protection.

Information efficiency: Extension to non-Gaussian (Lévy) noise and non-Markovian (colored noise) effects shows the $\chi = 1$ optimum shifts by $\lesssim 15\%$ but remains structurally present. The adaptive window $\chi \in [0.8, 1.0]$ reflects realistic deviations.

Neutrinos: MSW and collective effects are orthogonal to the primordial mass-setting mechanism. All mass eigenstates satisfy $\chi_k \ll 1$, consistent with observed coherent oscillations. The transition from $\chi \sim 1$ during formation to $\chi \ll 1$ today reflects cosmological precipitation.

Experimental protocols: Circuit QED, trapped ion, and optomechanical procedures are specified in detail with statistical frameworks for hypothesis testing and Bayesian model selection.

Cross-scale validation: Logarithmic compression analysis documents the observed confinement of stability ratios within a comparatively narrow band in χ across 10+ orders of mass scale ($C \approx 3.43$); formal statistical quantification relative to an explicit null model remains an open item for future work.

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