

Supporting information for

Physics-based model for scaled cryogenic FETs with band-tail-assisted carrier statistics

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model component	parameters	global value (T independent)				
interface trap distribution	$D_{it,C}$ ($\text{cm}^{-2} \text{eV}^{-1}$)	1.833×10^{12}				
	E_{decay} (eV)	0.2943				
	U_{BT} (V)	0.0291				
band-tail broadening	U_{sp} (V)	0.0998				
	V_{offset} (mV)	4.088				

model component	parameters	T = 12 K	T = 75 K	T = 100 K	T = 150 K	T = 250 K
field-dependent mobility	μ_0 ($\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$)	395.655	349.343	330.663	328.074	314.822
	θ_1	0.55428	0.27194	0.34014	0.34076	0.37306
	θ_2	0.10007	0.33274	0.22963	0.39801	0.57612

Table S1. Extracted fitting parameters for the proposed cryogenic FET model. Temperature-independent global parameters for the interface-trap distribution and band-tail-assisted carrier response are summarized in the upper table, whereas temperature-dependent effective-field mobility parameters are listed in the lower table. The global parameters were shared across all temperatures, while μ_0 , θ_1 , and θ_2 were extracted separately for each temperature.

model	computational form	relative cost / simulator readiness	main limitation relative to present work
Ref. [10]	Analytical 1D Poisson and drift–diffusion model with incomplete ionization, discrete traps, and mobility	Low to moderate; analytical core model, but circuit implementation was not demonstrated	Maxwell–Boltzmann carrier statistics and discrete traps cannot reproduce the gradual deep-cryogenic turn-on of the scaled device.
Ref. [11]	MOS11- and PSP-based compact-model recalibration	Low; SPICE compatible and validated using ring-oscillator and LNA simulations	Cryogenic behavior is captured mainly through parameter recalibration rather than explicit separation of trap, band-tail, and transport physics.
Ref. [13]	BSIM-based model using an effective temperature for band-tail effects	Low; circuit-oriented compact formulation	The effective-temperature and threshold-correction formulations provide less explicit separation among band-tail, interface-trap, and lattice-temperature effects.
Ref. [14]	Modified BSIM-CMG model for short-channel bulk FinFETs	Low circuit-level cost with demonstrated convergence; 3.6–4.4 times the runtime of standard BSIM-CMG in the reported ring oscillator	Uses compact fitting functions rather than a self-consistent physical decomposition of cryogenic mechanisms.
Ref. [15]	Physics-based closed-form model for band-tail-induced SS saturation	Very low; no numerical DOS integration	Limited to subthreshold swing rather than the complete transfer characteristic and above-threshold transport.
Ref. [16]	Lambert-W based band-tail charge and surface-potential model	Low to moderate; analytical band-tail expression, but not demonstrated at circuit level	Validated against TCAD rather than experimental measurements and does not include the same continuous trap and effective-field mobility treatment used here.
Ref. [17]	Self-consistent discretized 1D Poisson solver with Gaussian band-tail and interface-state DOS integration	High; intended primarily for device-level analysis	Computationally intensive and focused mainly on subthreshold-swing inflection and protrusion .
This work	Numerical 1D charge model with continuous interface traps, band-tail-assisted carrier statistics, and field-dependent mobility	Moderate; numerical electrostatic and charge evaluation is required, and circuit implementation has not yet been developed	Higher cost than closed-form compact models, but reproduces the full transfer characteristic and separates the major electrostatic, carrier-statistical, and transport contributions.

Table S2. Qualitative comparison of representative cryogenic FET modeling approaches in terms of computational formulation, simulator readiness, and physical coverage.

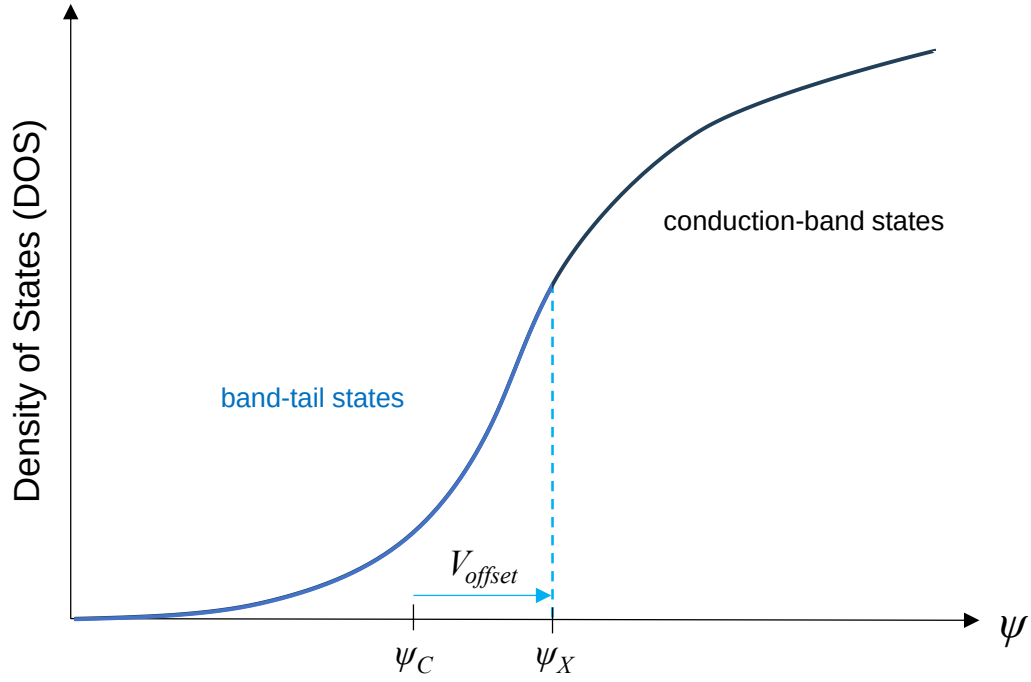


Figure S1. Schematic density-of-states representation underlying the band-tail-assisted statistical carrier response. The vertical axis represents the electronic density of states, including band-tail states below the conduction-band reference and extended conduction-band states. The conduction-band reference potential is given by $\psi_C = V_{ch} + E_g/(2q)$, which follows from $E_C - E_{F,n} = q(\psi_C - \psi)$, where $E_{F,n} = E_F - qV_{ch}$. Thus, $\psi = \psi_C$ corresponds to $E_C = E_{F,n}$. The statistical transition potential is shifted from this reference by V_{offset} , giving $\psi_X = \psi_C + V_{offset}$. For $\psi < \psi_X$, the carrier response is dominated by band-tail-assisted statistical broadening, whereas for $\psi > \psi_X$, the response gradually recovers the conventional conduction-band behavior.

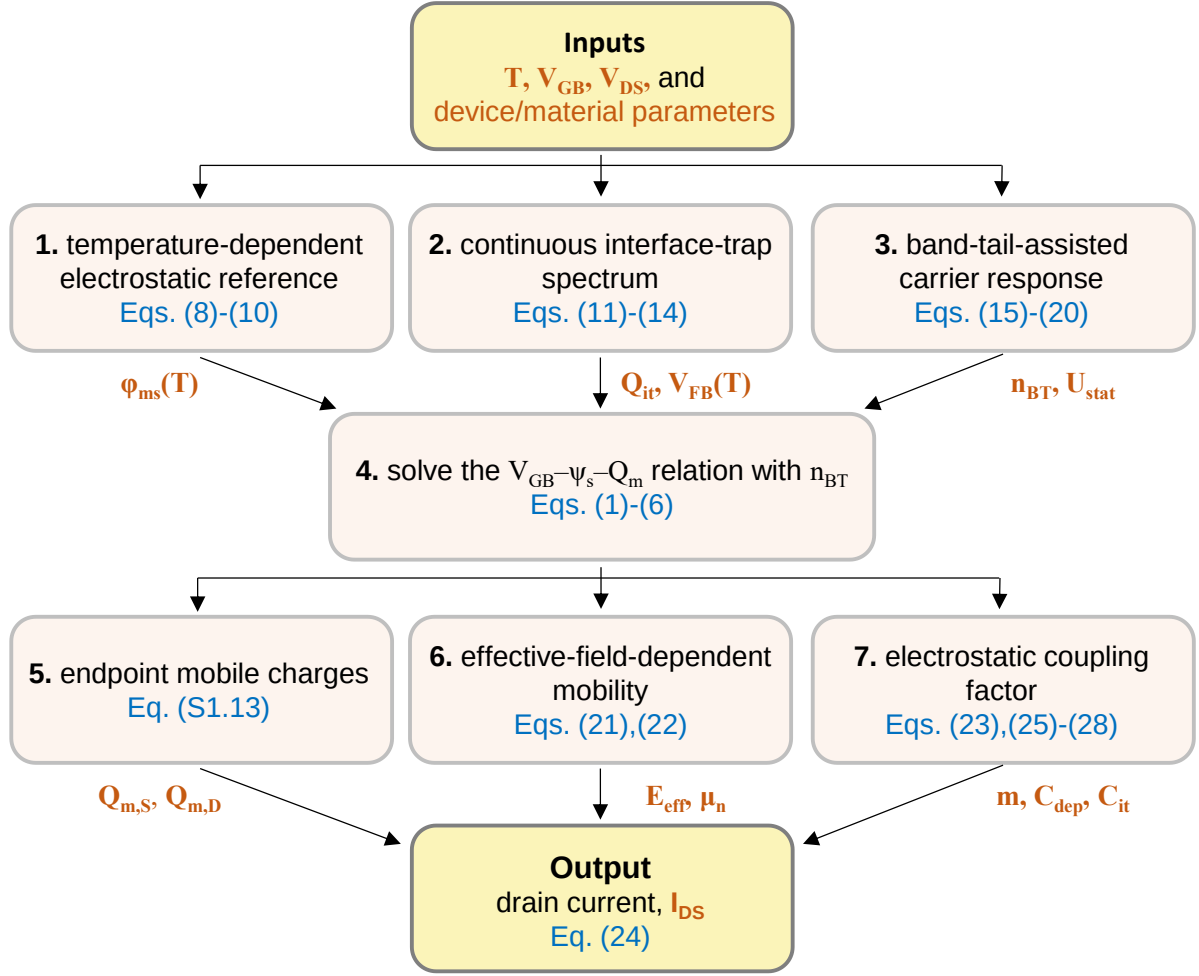


Figure S2. Schematic workflow for calculating the drain current in the proposed cryogenic FET model.

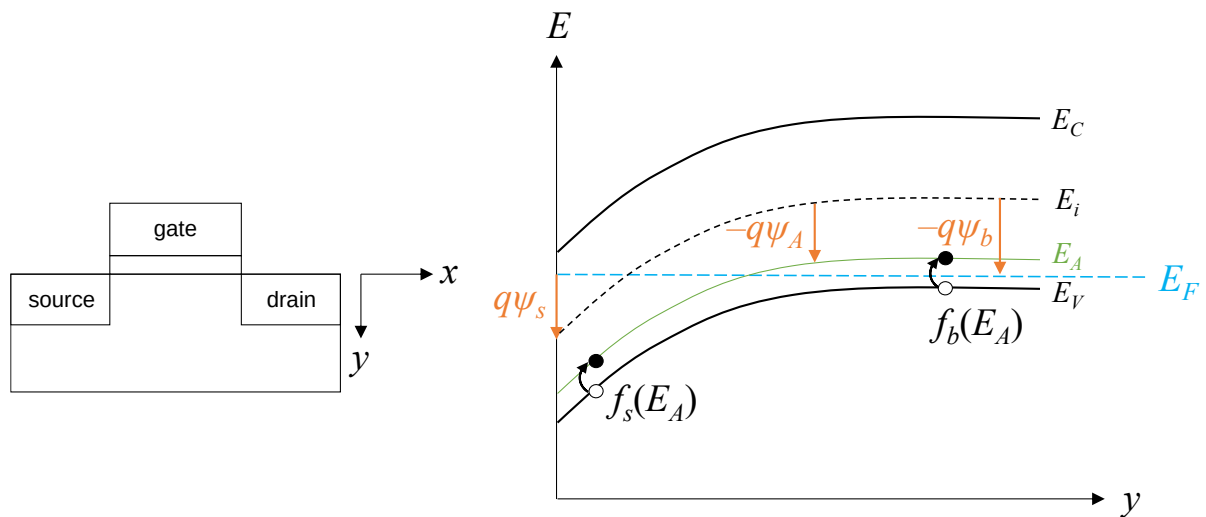
The applied biases, temperature, and device/material parameters are first used to determine the temperature-dependent electrostatic reference, including the metal–semiconductor work-function difference $\phi_{ms}(T)$, from Eqs. (8)–(10). The continuous interface-trap spectrum is then evaluated to obtain Q_{it} and V_{FB} from Eqs. (11)–(14), while the band-tail-assisted carrier response provides U_{stat} and n_{BT} from Eqs. (15)–(20). These quantities are incorporated into the numerical solution of the $V_{GB}-\psi_s-Q_m$ relation, where the electrostatic equation, incomplete dopant ionization, charge neutrality, and surface boundary condition are solved using $n = n_{BT}$. The source- and drain-end mobile charge densities, $Q_{m,S}$ and $Q_{m,D}$, are obtained by evaluating the numerical electrostatic solution at $V_{ch} = 0$ V and $V_{ch} = V_{DS}$, respectively. The effective-field-dependent mobility μ_n is calculated from Eqs. (21) and (22), and the electrostatic coupling factor m is obtained from Eqs. (23) and (25)–(28). Finally, $Q_{m,S}$, $Q_{m,D}$, μ_n and m are substituted into the charge-based current expression in Eq. (24) to calculate I_{DS} .

Supporting Note 1. Baseline cryo-FET model

1.1. Origin and scope of the baseline model

The baseline model used in this work is based on the physics-based cryogenic MOSFET model developed by A. Beckers et al. [Ref. 10]. In their model, the electrostatics of a long-channel bulk MOSFET operating from room temperature down to liquid-helium temperature is formulated by solving the one-dimensional Poisson equation, while the drain current is derived from drift–diffusion transport. The model incorporates key cryogenic effects, including incomplete dopant ionization, temperature-dependent bandgap widening, interface-trap occupation, and mobility degradation. In this Supporting Note, we briefly summarize the essential formulation of the Beckers model up to the charge-based drain-current expression, which is used as the baseline model for comparison with the model developed in this work.

1.2. Electrostatic framework



The electrostatic potential is defined as

$$\psi \equiv \frac{E_F - E_i}{q},$$

where E_F is the Fermi level, E_i is the intrinsic energy level, and q is the elementary charge. Under the gradual-channel approximation, the vertical electrostatics of a FET is described by the one-dimensional Poisson equation,

$$\frac{\partial^2 \psi(y)}{\partial y^2} = -\frac{q}{\epsilon_{si}} (-n + p - N_A^-), \quad (\text{S1.1})$$

where ϵ_{si} is the permittivity of silicon, n and p are the electron and hole concentrations, respectively, and N_A^- is the ionized acceptor concentration.

At cryogenic temperature, the ionized acceptor concentration cannot be approximated as the implanted acceptor concentration N_A . Instead, incomplete dopant ionization is described by the Fermi–Dirac occupation probability of the acceptor level E_A :

$$N_A^- = \frac{N_A}{1 + g_A \exp\left(\frac{E_A - E_{F,n}}{kT}\right)} = \frac{N_A}{1 + g_A \exp\left(\frac{\psi_A - (\psi - V_{ch})}{U_T}\right)}, \quad (\text{S1.2})$$

where g_A is the acceptor degeneracy factor, $E_{F,n} = E_F - qV_{ch}$ is the electron quasi-Fermi level, V_{ch} is the channel potential, and $U_T = kT/q$ is the thermal voltage. The acceptor potential is defined as

$$\psi_A \equiv \frac{E_A - E_i}{q}.$$

Equation (S1.2) accounts for both thermal ionization and field-assisted ionization. In the bulk at cryogenic temperature, acceptor dopants may be partially frozen out because thermal ionization is strongly suppressed. Near the semiconductor surface, however, electrostatic band bending can shift the acceptor level below the Fermi level, thereby inducing nearly complete field-assisted ionization.

1.3. Poisson–Boltzmann equation at cryogenic temperature

The model uses Maxwell–Boltzmann carrier statistics after verifying that incomplete ionization keeps the bulk semiconductor nondegenerate even at deep-cryogenic temperature.

The mobile carrier concentrations are therefore written as

$$n = n_i \exp\left(\frac{\psi - V_{ch}}{U_T}\right), \quad p = n_i \exp\left(-\frac{\psi}{U_T}\right) \quad (\text{S1.3})$$

where the intrinsic carrier concentration is

$$n_i = \sqrt{N_C N_V} \exp\left(-\frac{E_g}{2kT}\right). \quad (\text{S1.4})$$

Here N_C and N_V are the effective densities of states in the conduction and valence bands, respectively, and E_g is the temperature-dependent silicon bandgap derived by Pierret and Neudeck [R1]. Substitution of Eq. (S1.2) and Eq. (S1.3) into Eq. (S1.1) gives the Poisson–Boltzmann equation,

$$\frac{\partial^2 \psi(y)}{\partial y^2} = -\frac{q}{\epsilon_{si}} \left[-n_i \exp\left(\frac{\psi - V_{ch}}{U_T}\right) + n_i \exp\left(-\frac{\psi}{U_T}\right) - N_A^- \right]. \quad (\text{S1.5})$$

1.4. Surface electric field and bulk potential

The surface electric field is obtained by multiplying Eq. (S1.5) by $2\partial\psi/\partial y$ and integrating from the bulk potential ψ_b to the surface potential ψ_s . With $E = -\partial\psi/\partial y$ and $E_b = 0$, this gives

$$E_s^2 = \frac{2qn_i U_T}{\epsilon_{si}} \left[\exp\left(\frac{\psi_s - V_{ch}}{U_T}\right) - \exp\left(\frac{\psi_b - V_{ch}}{U_T}\right) + \exp\left(-\frac{\psi_s}{U_T}\right) - \exp\left(-\frac{\psi_b}{U_T}\right) \right] + \frac{2qN_A}{\epsilon_{si}} \left[\psi_s - \psi_b - U_T \ln\left(\frac{f_s(E_A)}{f_b(E_A)}\right) \right]. \quad (\text{S1.6})$$

The surface and bulk ionization probabilities of the acceptor level are

$$f_s(E_A) = \frac{1}{1 + g_A \exp\left(\frac{\psi_A - (\psi_s - V_{ch})}{U_T}\right)}, \quad (\text{S1.7})$$

$$f_b(E_A) = \frac{1}{1 + g_A \exp\left(\frac{\psi_A - \psi_b}{U_T}\right)}. \quad (\text{S1.8})$$

The logarithmic term in Eq. (S1.6) represents the correction due to incomplete ionization. When complete ionization is assumed, $f_s(E_A) = f_b(E_A) = 1$, and this term vanishes, reducing Eq. (S1.6) to the conventional room-temperature MOS electrostatic expression.

The bulk potential ψ_b is obtained from the bulk charge-neutrality condition. For a p-type body, the resulting expression is

$$\psi_b = U_T \ln\left(\frac{n_i}{N_A}\right) + U_T \ln\left[\frac{1 + \sqrt{1 + 4g_A \frac{N_A}{n_i} \exp\left(\frac{\psi_A}{U_T}\right)}}{2}\right]. \quad (\text{S1.9})$$

The first term corresponds to the conventional complete-ionization result, whereas the second term accounts for the Fermi-level shift caused by incomplete dopant ionization. This correction becomes important at cryogenic temperature because the position of the bulk Fermi level directly affects the threshold voltage.

1.5. Charge-density formulation

The total semiconductor charge density per unit area is obtained from Gauss's law,

$$Q_{sc} = -\epsilon_{si} E_s. \quad (\text{S1.10})$$

Using the charge-sheet and full-depletion approximations, the depletion charge density is written as

$$Q_{dep} = -\epsilon_{si} \sqrt{\frac{2qN_A}{\epsilon_{si}}} \left[\psi_s - \psi_b - U_T \ln\left(\frac{f_s(E_A)}{f_b(E_A)}\right) \right]. \quad (\text{S1.11})$$

The mobile charge density is then obtained from charge neutrality:

$$Q_m = Q_{sc} - Q_{dep}. \quad (S1.12)$$

Neglecting the hole contribution in inversion, the mobile charge density can be expressed as

$$Q_m = -\epsilon_{si} \sqrt{\frac{2qn_i U_T}{\epsilon_{si}} \left[e^{\frac{\psi_s - V_{ch}}{U_T}} - e^{\frac{\psi_b - V_{ch}}{U_T}} \right] + \frac{2qN_A}{\epsilon_{si}} \left[\psi_s - \psi_b - U_T \ln \left(\frac{f_s(E_A)}{f_b(E_A)} \right) \right]} + \epsilon_{si} \sqrt{\frac{2qN_A}{\epsilon_{si}} \left[\psi_s - \psi_b - U_T \ln \left(\frac{f_s(E_A)}{f_b(E_A)} \right) \right]}. \quad (S1.13)$$

Equation (S1.13) is the central charge expression of the baseline cryogenic FET model. It provides the mobile charge density as a function of the surface potential, channel potential, temperature, dopant ionization probability, and intrinsic carrier density.

1.6. Interface-trap contribution and gate-voltage relation

Interface traps are incorporated through their Fermi–Dirac occupation probability. For a discrete acceptor-like interface-trap level $E_{t,j}$, the occupation probability is

$$f_s(E_{t,j}) = \frac{1}{1 + g_t \exp \left(\frac{\psi_{t,j} - (\psi_s - V_{ch})}{U_T} \right)}, \quad (S1.14)$$

where g_t is the trap degeneracy factor and

$$\psi_{t,j} \equiv \frac{E_{t,j} - E_i}{q}.$$

The corresponding interface-trap charge density is

$$Q_{it} = -q \sum_j N_{it,j} f_s(E_{t,j}), \quad (S1.15)$$

where $N_{it,j}$ is the areal density of the j -th trap level. The flat-band voltage including the occupied interface-trap charge is then written as

$$V_{FB} = \phi_{ms} + \frac{q}{C_{ox}} \sum_j \frac{N_{it,j}}{1 + g_t \exp\left(\frac{\psi_{t,j} - (\psi_s - V_{ch})}{U_T}\right)}, \quad (S1.16)$$

where ϕ_{ms} is the metal–semiconductor work-function difference and C_{ox} is the oxide capacitance per unit area.

The gate-to-body voltage is related to the surface potential through the surface boundary condition,

$$V_{GB} = V_{FB} + (\psi_s - \psi_b) - \frac{Q_{sc}}{C_{ox}}. \quad (S1.17)$$

For a given V_{GB} , Eq. (S1.17) is solved to obtain the surface potential ψ_s . The mobile charge density is then calculated from Eq. (S1.13). The source-side mobile charge $Q_{m,S}$ is evaluated at $V_{ch} = 0$, whereas the drain-side mobile charge $Q_{m,D}$ is evaluated at $V_{ch} = V_{DS}$.

1.7. Drift–diffusion current derivation

The drain current is derived using the drift–diffusion transport framework. After transforming the channel-current expression into a charge-based form, the drain–source current can be written as

$$I_{DS} = \frac{W}{L} \int_{\psi_{s,D}}^{\psi_{s,S}} \mu_n Q_m d\psi_s + \frac{W}{L} \int_{Q_{m,D}}^{Q_{m,S}} \mu_n U_T dQ_m. \quad (S1.18)$$

Here, W/L is the device aspect ratio, μ_n is the electron mobility, and $Q_{m,S}$ and $Q_{m,D}$ are the mobile charge densities evaluated at the source and drain ends of the channel, respectively. The corresponding surface potentials are denoted by $\psi_{s,S}$ and $\psi_{s,D}$.

Equation (S1.18) is the baseline charge-based drain-current expression used in this work. The cryogenic effects enter the current equation through $Q_{m,S}$ and $Q_{m,D}$, which are determined

by the electrostatic model including incomplete dopant ionization, temperature-dependent intrinsic carrier density, bandgap widening, and interface-trap occupation.

[R1] R. F. Pierret and G. W. Neudeck, Advanced Semiconductor Fundamentals, vol. 6. Reading, MA, USA: Addison-Wesley, 1987.

Supporting Note 2. Stage-wise parameter extraction procedure

2.1. Current-window partitioning and fitting strategy

The model parameters were extracted using an intentionally stage-wise fitting procedure, rather than a single simultaneous optimization. This strategy was adopted because the proposed model contains several physically distinct parameter groups: the continuous interface-trap distribution, the band-tail-assisted carrier response, and the effective-field-dependent mobility. If all parameters were optimized simultaneously over the entire I_D – V_G curve, different physical mechanisms could compensate for each other, resulting in non-unique parameter extraction. Therefore, the fitting procedure was designed to constrain each parameter group primarily in the current and temperature regimes where its physical effect is most directly observable.

For this purpose, the measured transfer characteristics at $T=12, 75, 100, 150,$ and 250 K were divided into three current-level regions: the subthreshold region, the intermediate-current region, and the high-current region. The subthreshold region was used mainly to capture the gradual cryogenic turn-on and the logarithmic slope of I_D . The intermediate-current region was used to constrain the threshold-transition shape, where electrostatic modulation and carrier formation are both relevant. The high-current region was used to determine the above-threshold current level governed predominantly by mobility degradation. This current-window separation was an essential part of the fitting strategy because it allowed the subthreshold broadening, threshold-region electrostatics, and above-threshold transport to be fitted by physically separated parameter groups.

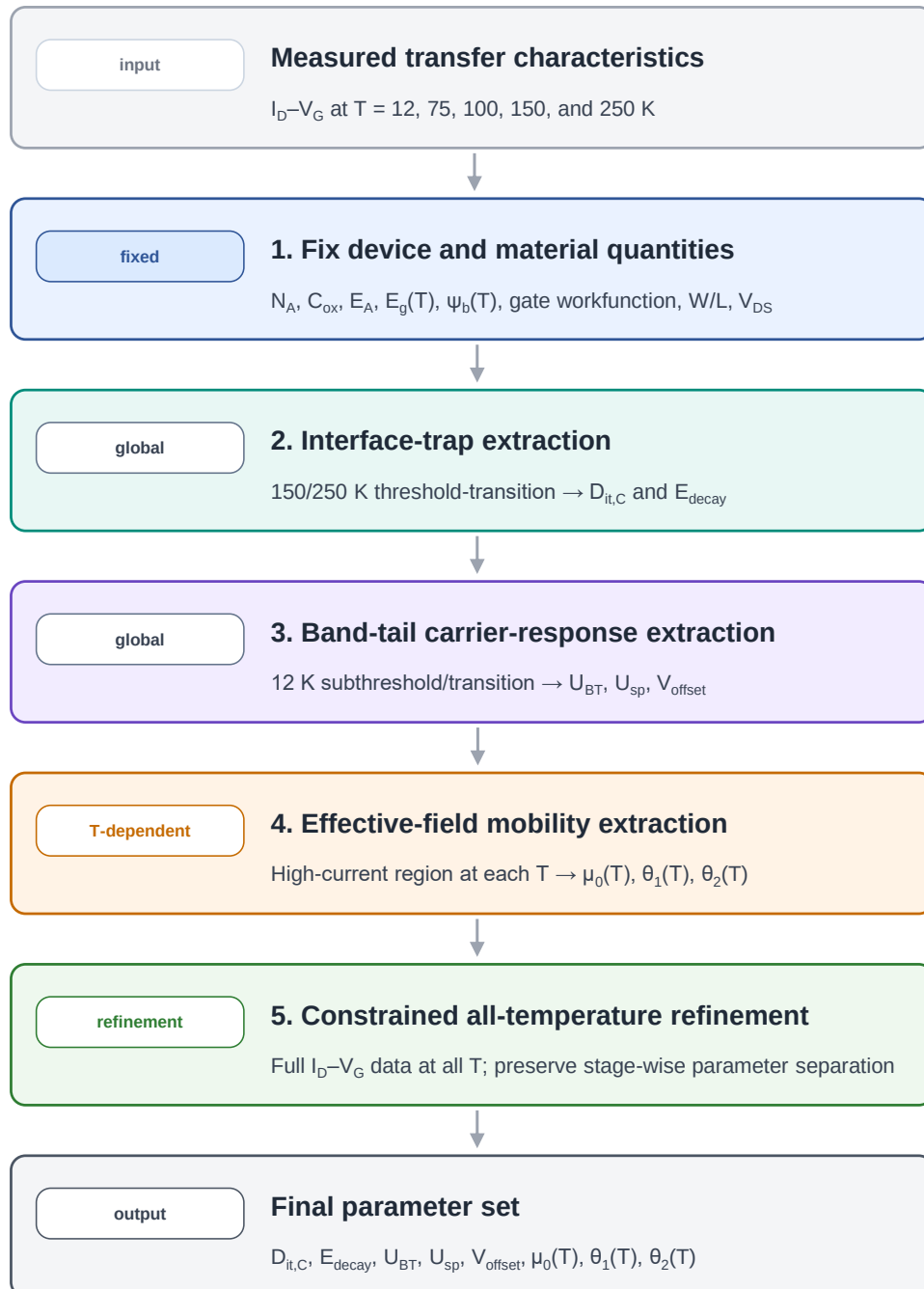
The fitting was implemented in MATLAB using particle swarm optimization (PSO). In each stage, the PSO algorithm was performed within predefined physically reasonable parameter bounds. After the sequential parameter extraction, a constrained global refinement was carried out over all fitted parameters. A local pattern-search refinement was then applied

near the final solution to further improve the overall agreement between the measured and calculated transfer characteristics.

2.2 Stage-wise parameter-extraction procedure

The fitting procedures are as follows.

Stage-wise parameter-extraction procedure



stage	quantities or parameters	primary data used	extraction rationale
1. Fixed device and material quantities	N_A , C_{ox} , E_A , $E_g(T)$, gate workfunction, $\psi_b(T)$, W , L , V_{DS}	device geometry, process information, and established material relations	Establish the electrostatic reference and reduce the number of free parameters.
2. Interface-trap distribution	$D_{it,C}$, E_{decay}	threshold-transition regions of the 150 and 250 K I_D - V_G V_G characteristics	At relatively high temperatures, the interface-trap-induced electrostatic modulation is more clearly observable because deep-cryogenic band-tail broadening is less dominant.
3. Band-tail-assisted carrier response	U_{BT} , U_{sp} , V_{offset}	subthreshold and transition regions of the 12 K I_D - V_G characteristic	The deviation from the conventional U_T -controlled carrier response is strongest at deep-cryogenic temperature.
4. effective-field-dependent mobility	$\mu_0(T)$, $\theta_1(T)$, $\theta_2(T)$	high-current region at each temperature	After the electrostatic and carrier-statistical parameters are constrained, the remaining above-threshold discrepancy is attributed primarily to effective-field-dependent mobility.
5. constrained all-temperature refinement	$D_{it,C}$, E_{decay} , U_{BT} , U_{sp} , V_{offset} , $\mu_0(T)$, $\theta_1(T)$, $\theta_2(T)$	full I_D - V_G characteristics at 12, 75, 100, 150, and 250 K	Improve the overall agreement across the full current and temperature ranges without eliminating the physical parameter separation established in Stages 1–4. The previously extracted parameters are refined only within constrained ranges.

Stage 1: An initial parameter set was assigned for the interface-trap spectrum, band-tail broadening, and field-dependent mobility parameters. The fixed electrostatic and material quantities, including the dopant concentration, oxide capacitance, acceptor ionization energy, work-function reference, temperature-dependent bandgap, and incomplete-ionization-induced bulk potential, were calculated from the device and material parameters and were not treated as free fitting parameters.

Stage 2: The interface-trap distribution was fitted first. In this step, the two parameters of the continuous trap spectrum, $D_{it,C}$ and E_{decay} , were optimized mainly using the 150 and 250 K transfer characteristics. These relatively high-temperature data were selected because the trap-induced threshold-region electrostatic modulation can be identified more clearly without being strongly masked by deep-cryogenic band-tail broadening. This step determined the global interface-trap distribution used for all temperatures.

Stage 3: The band-tail-assisted carrier-response parameters were then extracted from the low-temperature subthreshold characteristics. The 12 K data were used as the primary constraint because the deviation from the conventional U_T -controlled Boltzmann carrier response is most pronounced at deep-cryogenic temperature. In this step, U_{BT} , U_{sp} , and V_{offset} were optimized to reproduce the gradual cryogenic turn-on. Here, U_{BT} determines the magnitude of the statistical broadening, U_{sp} controls the smoothness of the transition from the band-tail-dominated response to the conduction-band-dominated response, and V_{offset} sets the electrostatic position of this transition.

Stage 4: The mobility-related parameters were extracted from the high-current region. After the subthreshold and threshold-region characteristics had been constrained by the interface-trap and band-tail parameters, the remaining discrepancy in the above-threshold current was attributed primarily to effective-field-dependent mobility degradation. Therefore, μ_0 , θ_1 , and θ_2 were optimized separately at each temperature using the high-current part of the measured transfer curves. Unlike the interface-trap and band-tail parameters, these mobility parameters were allowed to vary with temperature.

Stage 5: Finally, the interface-trap, band-tail, and mobility parameters were refined through an all-temperature constrained optimization using the 12, 75, 100, 150, and 250 K transfer characteristics. In this final step, $D_{it,C}$, E_{decay} , U_{BT} , U_{sp} , and V_{offset} were treated as temperature-independent global parameters, whereas μ_0 , θ_1 , and θ_2 were treated as temperature-dependent

parameters. This final refinement was not intended to erase the sequential extraction strategy, but to improve the consistency of the already separated parameter groups over the full temperature range. The final extracted parameters are summarized in Table S1.