Supplementary Information

LLM-Assisted Value Assessment and Strategy Design for Virtual Power Plant Participation in Coupled Electricity–Carbon Markets

Xuan Zhang, Xinyu Xu, Qin Wang, Haozhe Lu, Zhiyuan Yang, Haijin Wang, Wenbin Su, Binghao He, Junhua Zhao, and Yanli Liu

Supplementary Note 1. Physical operation constraints of resources

This supplementary note gives conventional physical operation constraints for VPP resources in the virtual power plant resource modeling section of the main manuscript, including power and energy limits, dynamic evolution equations, and cyclic consistency conditions. The constraints hold for all $t \in \mathcal{T}$ unless specified.

Distributed renewable resources

Distributed renewable output is split into actual utilization and curtailment:

$$P_t^{RE,avail} = P_t^{RE} + P_t^{RE,curt}, \tag{A.1}$$

where directly accommodated power satisfies:

$$0 \leq P_t^{RE,dir} \leq P_t^{RE,avail}. \tag{A.2}$$

The main manuscript gives the source decomposition of actually utilized power, baseline curtailment boundary, and additionality constraint for energy storage absorbing curtailed renewable energy.

Controllable loads

Controllable-load shifting and curtailment satisfy:

$$-\alpha_t^{shift} P_t^{L,base} \leq P_t^{L,shift} \leq \alpha_t^{shift} P_t^{L,base}, \tag{A.3}$$

$$0 \leq P_t^{L,cut} \leq \alpha_t^{cut} P_t^{L,base}, \tag{A.4}$$

where α_t^{shift} and α_t^{cut} denote maximum load shifting and curtailment ratios, respectively. The actual electricity consumption power after dispatch is non-negative:

$$P_t^L \geq 0. \tag{A.5}$$

The main manuscript gives the power decomposition of controllable loads and the daily energy conservation constraint for load shifting.

Energy storage system

The state of charge of the energy storage system evolves as:

$$E_t^{ES} = E_{t-1}^{ES} + \eta^{ch} P_t^{ES,ch} \Delta t - \frac{P_t^{ES,dis} \Delta t}{\eta^{dis}}, \tag{A.6}$$

Stored energy and charging/discharging power obey state-of-charge and rated power limits:

$$\underline{E}^{ES} \leq E_t^{ES} \leq \overline{E}^{ES}, \tag{A.7}$$

$$0 \leq P_t^{ES,ch} \leq u_t^{ES} P^{ES,max}, \tag{A.8}$$

$$0 \leq P_t^{ES,dis} \leq \left(1 - u_t^{ES}\right) P^{ES,max}, \tag{A.9}$$

where $\underline{E}^{ES}$ and $\overline{E}^{ES}$ denote lower and upper state-of-charge limits, respectively; $u_t^{ES} \in \{0, 1\}$ is the charging/discharging state variable preventing simultaneous charging and discharging. Cyclic consistency is:

$$E_0^{ES} = E_T^{ES}. \tag{A.10}$$

The main manuscript gives the source decomposition of the energy storage charging power, the low-carbon attribute ledger, and the corresponding low-carbon electricity accounting constraints.

Electric vehicles

For any electric vehicle $v \in \mathcal{V}$, onboard battery energy evolves as:

$$E_{v,t}^{EV} = E_{v,t-1}^{EV} + \eta_v^{EV} P_{v,t}^{EV} \Delta t, \tag{A.11}$$

Plug-in battery energy equals the initial energy:

$$E_{v,t_v^{arr}-1}^{EV} = E_v^{EV,0}, \tag{A.12}$$

Charging power and battery energy constraints are:

$$0 \le P_{v,t}^{EV} \le P_v^{EV,max}, \tag{A.13}$$

$$P_{v,t}^{EV} = 0, \quad t \in \mathcal{T} \setminus \mathcal{T}_v^{plug}, \tag{A.14}$$

$$0 \le E_{v,t}^{EV} \le E_v^{EV,max}. \tag{A.15}$$

Equation (A.11) and constraint (A.15) hold for plug-in periods $t \in \mathcal{T}_v^{plug}$. The main manuscript gives the target-energy equality constraint at departure and the equal-service constraint for the natural charging baseline.

Supplementary Note 2. Conventional constraints and settlement formulas for optimization

This appendix provides the conventional bidding limits, market settlement formulas, grid-interaction constraints, and market-assessment constraints used in the deterministic co-optimization model in the optimization modeling subsection of the main manuscript. Unless otherwise specified, all constraints hold for each $t \in \mathcal{T}$.

Bidding Limits and Market Settlement

The day-ahead energy, frequency regulation, and reserve bidding variables satisfy

$$-\overline{P}^{\text{grid}} \le Q_t^{\text{DA}} \le \overline{P}^{\text{grid}}, \tag{B.1}$$

$$0 \le Q_t^{\text{REG}} \le \overline{Q}^{\text{REG}}, \quad 0 \le Q_t^{\text{RES}} \le \overline{Q}^{\text{RES}}, \tag{B.2}$$

where $\overline{P}^{\text{grid}}$ is the maximum exchange power at the grid-connection point, and $\overline{Q}^{\text{REG}}$ and $\overline{Q}^{\text{RES}}$ are the upper bidding limits of frequency regulation and reserve capacity, respectively.

The day-ahead energy market revenue is

$$R^{\text{DA}} = \sum_{t \in \mathcal{T}} \pi_t^{\text{DA}} Q_t^{\text{DA}} \Delta t. \tag{B.3}$$

The deviation between the planned grid exchange power and the day-ahead submitted power is not repeatedly settled in R^{DA}, but is assessed through the deviation cost defined below.

The frequency regulation revenue and reserve revenue are

$$R^{\text{REG}} = \sum_{t \in \mathcal{T}} \left(\pi_t^{\text{REG,C}} Q_t^{\text{REG}} + \pi_t^{\text{REG,M}} M_t \right) \Delta t, \tag{B.4}$$

$$R^{\text{RES}} = \sum_{t \in \mathcal{T}} \pi_t^{\text{RES}} Q_t^{\text{RES}} \Delta t, \tag{B.5}$$

where M_t denotes the actual frequency regulation mileage. The carbon market revenue R^{C} is calculated from the carbon-credit quantity Γ_t^{C} in the optimization modeling subsection of the main manuscript.

Operation Cost and Grid Interaction

The operation cost and market assessment cost are written as

$$C^{\text{op}} = C^{\text{pref}} + C^{\text{L,comp}} + C^{\text{ES}} + C^{\text{EV}} + C^{\text{AS,resp}}, \tag{B.6}$$

$$C^{\text{pen}} = C^{\text{DA,dev}} + C^{\text{AS,vio}}, \tag{B.7}$$

where C^{pref} is the LLM-corrected preference penalty, $C^{\mathrm{L,comp}}$ and C^{EV} are the compensation costs of controllable loads and electric vehicles, C^{ES} is the energy-storage operation cost, and $C^{\mathrm{AS,resp}}$ is the ancillary-service response cost.

The net exchange power between the VPP and the grid is

$$P_t^{\mathrm{grid}} = P_t^{\mathrm{grid,sell}} - P_t^{\mathrm{grid,buy}}. \tag{B.8}$$

With positive P_t^{grid} denoting electricity export, the internal power balance is

$$P_t^{\mathrm{grid}} = P_t^{\mathrm{RE}} + P_t^{\mathrm{ES,dis}} - P_t^{\mathrm{ES,ch}} - P_t^{\mathrm{L}} - \sum_{v\in\mathcal{V}} P_{v,t}^{\mathrm{EV}}. \tag{B.9}$$

A binary variable w_t is used to avoid simultaneous electricity purchasing and selling:

$$0 \le P_t^{\mathrm{grid,sell}} \le w_t \overline{P}^{\mathrm{grid}}, \tag{B.10}$$

$$0 \le P_t^{\mathrm{grid,buy}} \le (1-w_t) \overline{P}^{\mathrm{grid}}, \tag{B.11}$$

$$w_t \in \{0,1\}. \tag{B.12}$$

Deviation and Ancillary-Service Assessment

The day-ahead energy deviation is represented by nonnegative positive- and negative-deviation variables:

$$\Delta_t^{\mathrm{DA},+} \ge P_t^{\mathrm{grid}} - Q_t^{\mathrm{DA}}, \quad \Delta_t^{\mathrm{DA},+} \ge 0, \tag{B.13}$$

$$\Delta_t^{\mathrm{DA},-} \ge Q_t^{\mathrm{DA}} - P_t^{\mathrm{grid}}, \quad \Delta_t^{\mathrm{DA},-} \ge 0. \tag{B.14}$$

The corresponding day-ahead deviation assessment cost is

$$C^{\mathrm{DA,dev}} = \sum_{t\in\mathcal{T}} \left(\pi_t^{\mathrm{dev},+} \Delta_t^{\mathrm{DA},+} + \pi_t^{\mathrm{dev},-} \Delta_t^{\mathrm{DA},-} \right) \Delta t. \tag{B.15}$$

Let $\mathcal{I}^{\mathrm{AS}} = \{\mathrm{ES}, \mathrm{L}, \mathrm{EV}\}$ denote the set of resources that can provide ancillary-service response. Their available response capacities satisfy

$$0 \le R_t^{i,\mathrm{AS}} \le \overline{R}_t^{i,\mathrm{AS}}, \quad i \in \mathcal{I}^{\mathrm{AS}}, \tag{B.16}$$

where $\overline{R}_t^{i,\mathrm{AS}}$ is determined by the corresponding physical operation limits in the virtual power plant resource modeling section of the main manuscript and Supplementary Note 1, together with the LLM-corrected preference constraints.

The actual frequency regulation and reserve responses are bounded by submitted capacity and available internal flexibility:

$$0 \le R_t^{\mathrm{REG,act}} \le Q_t^{\mathrm{REG}}, \tag{B.17}$$

$$0 \le R_t^{\mathrm{RES,act}} \le Q_t^{\mathrm{RES}}, \tag{B.18}$$

$$R_t^{\mathrm{REG,act}} + R_t^{\mathrm{RES,act}} \le \sum_{i\in\mathcal{I}^{\mathrm{AS}}} R_t^{i,\mathrm{AS}}. \tag{B.19}$$

The frequency regulation mileage is constrained by the actual available regulation capacity:

$$0 \le M_t \le \rho_t^{\mathrm{REG}} R_t^{\mathrm{REG,act}}, \tag{B.20}$$

where ρ_t^{REG} is the mileage coefficient.

Ancillary-service non-performance is represented by nonnegative slack variables:

$$\Delta_t^{\mathrm{REG}} \ge Q_t^{\mathrm{REG}} - R_t^{\mathrm{REG,act}}, \quad \Delta_t^{\mathrm{REG}} \ge 0, \tag{B.21}$$

$$\Delta_t^{\mathrm{RES}} \ge Q_t^{\mathrm{RES}} - R_t^{\mathrm{RES,act}}, \quad \Delta_t^{\mathrm{RES}} \ge 0. \tag{B.22}$$

The ancillary-service non-performance assessment cost is

$$C^{\mathrm{AS,vio}} = \sum_{t\in\mathcal{T}} \left(k^{\mathrm{REG}} \Delta_t^{\mathrm{REG}} + k^{\mathrm{RES}} \Delta_t^{\mathrm{RES}} \right) \Delta t, \tag{B.23}$$

where k^{REG} and k^{RES} are the non-performance assessment coefficients for frequency regulation and reserve capacity, respectively.