

Physics-Informed Neural Operators for Rapid Electric Field Prediction in Transcranial Magnetic Stimulation of Healthy and Tumor Subjects

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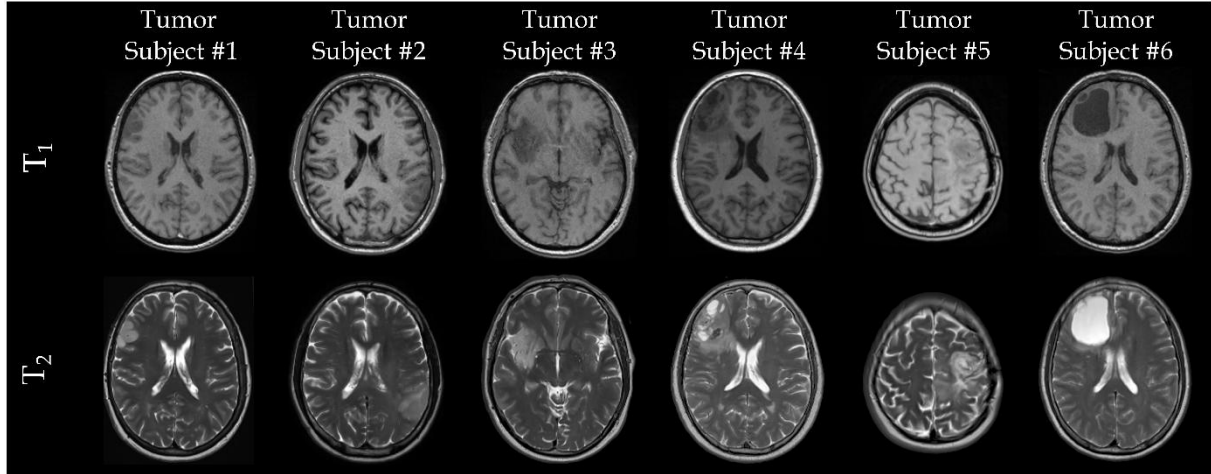
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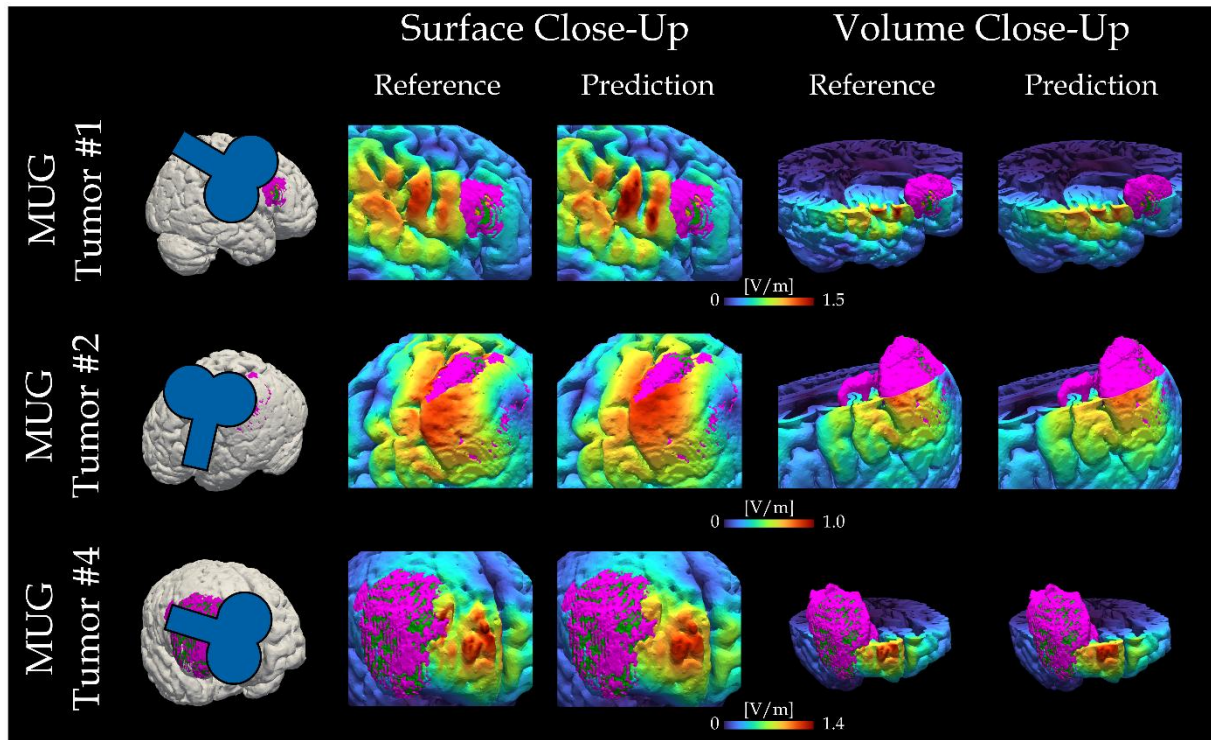
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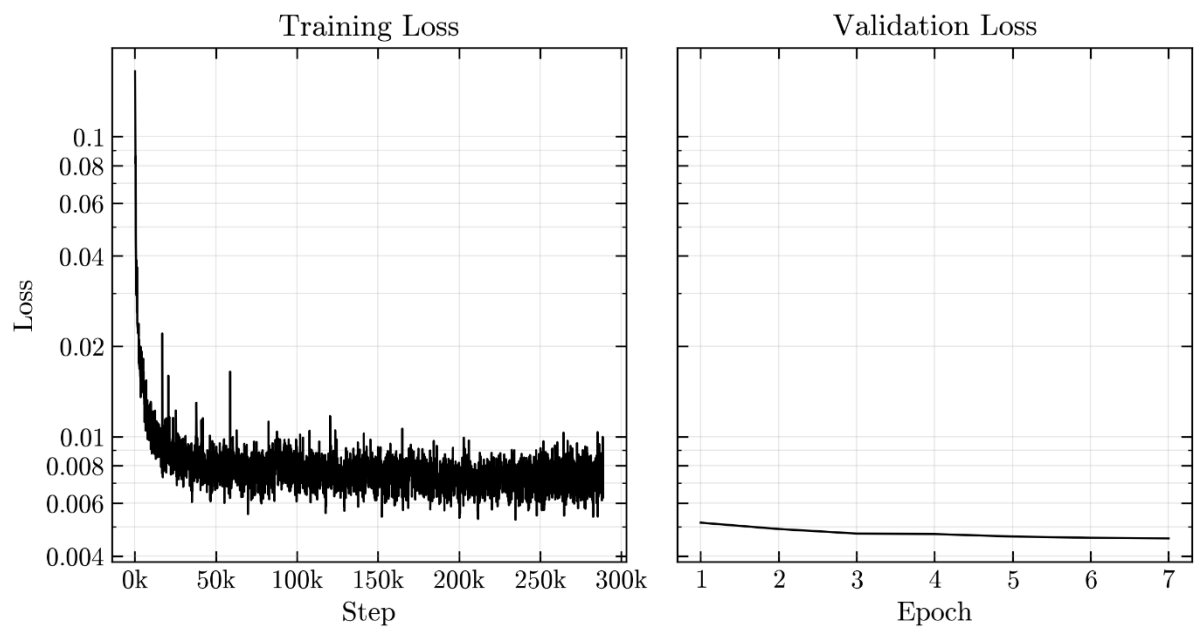
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Supplementary Figure S1: Tumor anatomy for all six tumor patients shown on representative T1- and T2-weighted MR images.



Supplementary Figure S2: Representative qualitative comparison of TMS-induced electric field predictions for three glioma patients. For each example, the predicted magnitude is shown alongside the SimNIBS-based reference. The tumor region is highlighted in pink.



Supplementary Figure S3: Training and validation loss curves for the selected Physics-5% physics-informed neural operator model. The training loss is shown over optimization steps, whereas the validation loss is shown after each epoch. Both curves decrease rapidly and stabilize, indicating stable convergence without evidence of increasing validation loss. The difference in absolute loss magnitude between the two panels reflects the different evaluation settings: during training, the loss was computed on 100,000 sampled gray-matter volume nodes for computational efficiency, whereas during validation the model was queried on the full set of approximately 300,000 gray-matter volume nodes.

Subject	Metric	GM Volume – Foc50		GM Surface – Foc50	
Tumor Subject #1	RMSE↓	0.131 (0.111–0.157)	[0.129–0.132]	0.122 (0.103–0.146)	[0.121–0.123]
	CoG↓	0.37 (0.25–0.53)	[0.363–0.378]	0.37 (0.25–0.53)	[0.366–0.379]
	MRE↓	12.51 (11.16–14.41)	[12.43–12.59]	11.20 (9.88–12.83)	[11.12–11.30]
	MAD↓	6.77 (6.09–7.54)	[6.72–6.80]	6.74 (6.00–7.52)	[6.70–6.78]
	RPE↓	10.02 (6.15–13.82)	[9.58–10.43]	9.44 (5.66–13.30)	[9.00–9.83]
Tumor Subject #2	RMSE↓	0.120 (0.103–0.138)	[0.119–0.121]	0.110 (0.097–0.125)	[0.109–0.110]
	CoG↓	0.44 (0.31–0.63)	[0.429–0.444]	0.41 (0.28–0.61)	[0.401–0.416]
	MRE↓	11.39 (9.93–13.28)	[11.31–11.47]	10.21 (9.10–11.63)	[10.16–10.28]
	MAD↓	6.30 (5.66–7.15)	[6.26–6.34]	6.31 (5.67–7.19)	[6.27–6.35]
	RPE↓	6.54 (3.27–10.10)	[6.14–6.92]	6.27 (3.16–9.73)	[5.91–6.59]
Tumor Subject #3	RMSE↓	0.141 (0.120–0.175)	[0.140–0.143]	0.132 (0.112–0.163)	[0.130–0.133]
	CoG↓	0.44 (0.29–0.65)	[0.430–0.449]	0.45 (0.29–0.68)	[0.442–0.461]
	MRE↓	13.23 (11.26–15.88)	[13.14–13.36]	11.91 (10.08–14.35)	[11.78–12.02]
	MAD↓	6.75 (6.02–7.84)	[6.71–6.80]	6.81 (6.03–7.87)	[6.78–6.86]
	RPE↓	9.50 (5.42–14.29)	[8.96–9.99]	9.01 (4.94–13.73)	[8.53–9.74]
Tumor Subject #4	RMSE↓	0.120 (0.104–0.144)	[0.119–0.121]	0.111 (0.097–0.134)	[0.110–0.112]
	CoG↓	0.40 (0.28–0.57)	[0.388–0.404]	0.40 (0.27–0.57)	[0.390–0.405]
	MRE↓	12.17 (10.47–13.79)	[12.08–12.26]	10.90 (9.50–12.35)	[10.84–10.98]
	MAD↓	6.50 (5.79–7.36)	[6.46–6.55]	6.47 (5.71–7.38)	[6.43–6.53]
	RPE↓	7.19 (3.56–10.77)	[6.72–7.63]	6.79 (3.31–10.43)	[6.33–7.23]
Tumor Subject #5	RMSE↓	0.140 (0.119–0.170)	[0.139–0.141]	0.131 (0.113–0.158)	[0.130–0.132]
	CoG↓	0.50 (0.35–0.73)	[0.492–0.511]	0.51 (0.35–0.75)	[0.503–0.523]
	MRE↓	13.44 (11.27–16.28)	[13.31–13.60]	12.23 (10.25–14.87)	[12.10–12.34]
	MAD↓	7.08 (5.96–8.28)	[7.01–7.15]	7.04 (5.80–8.26)	[6.99–7.10]
	RPE↓	8.27 (4.35–12.74)	[7.94–8.93]	7.90 (3.90–12.04)	[7.54–8.37]
Tumor Subject #6	RMSE↓	0.129 (0.111–0.156)	[0.128–0.130]	0.123 (0.106–0.148)	[0.122–0.124]
	CoG↓	0.48 (0.33–0.72)	[0.469–0.489]	0.48 (0.32–0.71)	[0.470–0.491]
	MRE↓	12.37 (10.62–14.70)	[12.29–12.48]	11.48 (9.80–13.59)	[11.39–11.57]
	MAD↓	6.80 (5.75–7.98)	[6.74–6.86]	6.77 (5.64–7.98)	[6.72–6.84]
	RPE↓	7.08 (3.39–11.80)	[6.66–7.37]	6.60 (3.10–11.23)	[6.24–6.92]

Supplementary Table S1: Per-subject evaluation metrics for the six glioma patients of the *MUG-Tumor* cohort, for gray matter volume and gray matter surface. Metrics were computed within the focality-50% region to highlight the most focal portion of the stimulation. Values are reported as median (IQR) [95% CI], computed across coil placements using bootstrapping. Units are: RMSE in V/m, CoG in mm, MRE in %, MAD in degrees, and RPE in %.

Model	MagVenture MCF-B65		MagStim D70	
	RMSE↓	MAD↓	RMSE↓	MAD↓
Explicit Coil Encoding	0.126	6.69	0.148	6.94
Affine Matrix + MLP	0.139 (+10.3%)	7.43 (+11.1%)	0.250 (+68.9%)	12.58 (+81.3%)
Head Encoder-Only	0.140 (+11.1%)	7.49 (+12.0%)	0.202 (+36.5%)	10.33 (+48.8%)

Supplementary Table S2: Ablation of the coil encoding strategy, evaluated on the IXI validation subjects (IXI-val) using MagVenture MCF-B65 and the unseen MagStim D70 coil. Values are reported as median, with percent changes relative to the explicit coil encoding. Units are: RMSE in V/m and MAD in degrees.