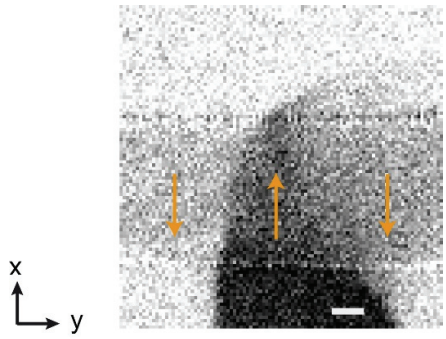
SUPPLEMENTARY INFORMATION

Supplementary Note 1. STXM image of converging 180° domain walls at remanence

The spin-texture comprising two converging 180° domain walls are stable also in absence of external magnetic field, as shown in the static scanning transmission X-ray microscopy (STXM) image (below). As discussed in the main text, small static magnetic fields can be applied in order to control the position of the apex with respect to the antenna and the distance between the domain walls.



Supplementary Figure 1. Static STXM image of two converging 180° Néel walls at remanence. The dark (bright) contrast indicates $M_x > 0$ (< 0), confirming the stabilization of sharp Néel walls. Orange arrows indicate the direction of the magnetization within the domains. Scale bar: 500 nm.

Supplementary Note 2. Time-resolved scanning transmission X-ray microscopy movies

The temporal evolution of the spin-wave propagation along the domain walls in different spin-textures can be better appreciated in the time-resolved STXM movies. Each frame of the movies is a time-resolved image acquired stroboscopically via STXM and shows the normalized M_x contrast calculated as the magnetic deviation $\Delta M_x(t)$ from the time-averaged $< M_x(t) >$ state. The images of each frame are smoothed with a Gaussian filter for enhancing contrast. Movies from the original raw non-smoothed images are also shown. The excitation of the spin-waves was applied through a sinusoidal function with frequency f. The time resolution of each movie was $\Delta t = 1/23f$, as reported in the Methods of the main text. The complete sequence corresponds to a period of sinusoidal excitation of the microstrip.

File name	Structure (raw)	Frequency	Field	Notes	Size (nm^2)
Supplementary Movie 1	Straight	1.28 GHz	0 mT	See Fig. 3a-e	1250x1800
Supplementary Movie 2	Straight, Raw data	1.28 GHz	0 mT	See Fig. 3a-e	1250x1800
Supplementary Movie 3	Curved	1.11 GHz	0 mT	See Fig. 3f-j	1600x2300
Supplementary Movie 4	Curved, Raw data	1.11 GHz	0 mT	See Fig. 3f-j	1600x2300
Supplementary Movie 5	Curved 2	0.96 GHz	0 mT		2650x3350
Supplementary Movie 6	Curved 2 Raw data	0.96 GHz	0 mT		2650x3350

Supplementary Table 1. Propagation along straight and curved domain walls. Propagation of confined spin-waves along straight and curved domain walls is imaged in Fig. 3 of the main text. Here, the corresponding movies are reported, together with the raw data videos for both the configurations. We also show the video (Supplementary Movie 5), together with the corresponding raw data video, of a spin-wave mode propagating along a curved domain wall, with a lateral extension below 100 nm, and clearly detectable up to 3.5 μm outside the microstrip.

File name	Structure (raw)	Frequency	Field	Notes	Size (nm^2)
Supplementary Movie 7	Circ 1	1.28 GHz	2.00 mT	See Fig. 5a-c	2550x3975
Supplementary Movie 8	Circ 1, Raw data	1.28 GHz	2.00 mT	See Fig. 5a-c	2550x3975
Supplementary Movie 9	Circ 2	1.28 GHz	1.75 mT	See Fig. 5a-c	2560x4000
Supplementary Movie 10	Circ 2, Raw data	1.28 GHz	1.75 mT	See Fig. 5a-c	2560x4000
Supplementary Movie 11	Circ 3	1.28 GHz	1.68 mT	See Fig. 5a-c	2560x4000
Supplementary Movie 12	Circ 3, Raw data	1.28 GHz	1.68 mT	See Fig. 5a-c	2560x4000

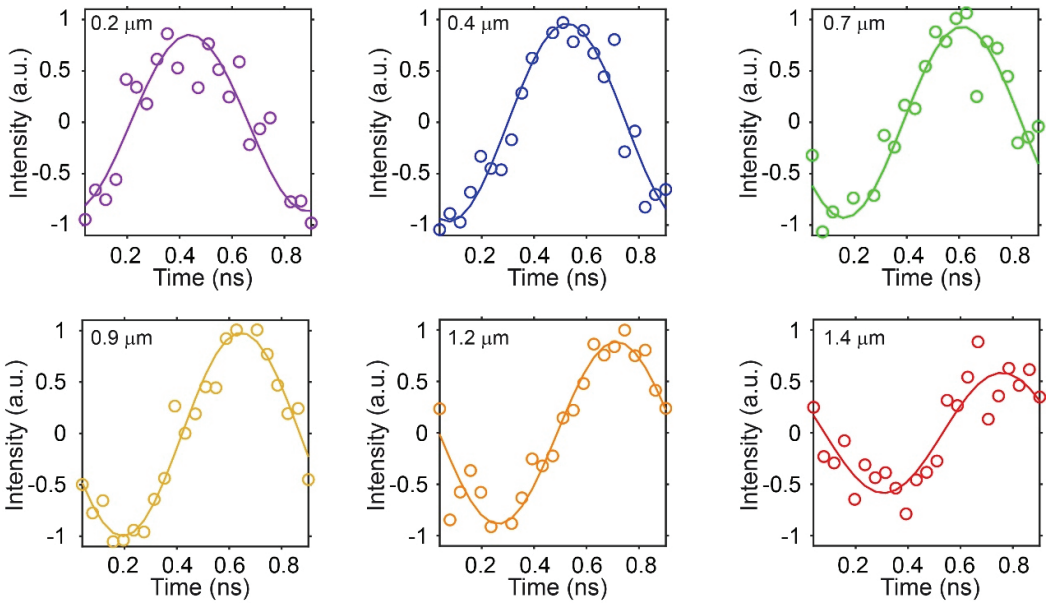
Supplementary Table 2. Tunable spatial superposition of spin-wave modes via external fields.
The movies corresponding to the STXM images of Fig. 5 in the main text are reported for different applied static magnetic fields. The raw data movies are displayed as well.

File name	Structure (raw)	Frequency	Field	Notes	Size (nm^2)
Supplementary Movie 13	CurvedMx	1.11 GHz	0 mT	See Fig. 3f-j	1600x2300
Supplementary Movie 14	CurvedMx, Raw data	1.11 GHz	0 mT	See Fig. 3f-j	1600x2300

Supplementary Table 3. Time-resolved M_x (t) images for the propagation along the curved wall.
The movies showing the time-resolved STXM images of the *x*-component of the magnetization M_x (t) related to the spin-wave propagation along curved domain wall (Figure 3f-j). The raw data movies are displayed as well.

Supplementary Note 3. Data analysis of the scanning transmission X-ray microscopy images

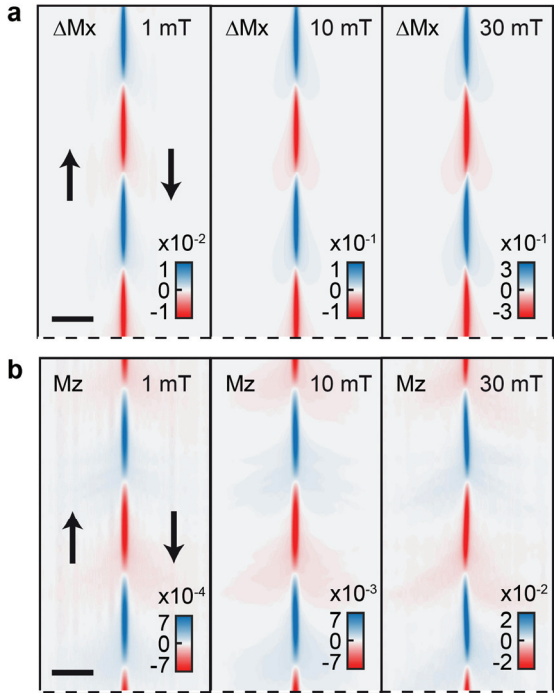
Data analysis was performed in order to extract the spin-wave amplitude, spatial phase shift and wavevector. Each series of time-resolved images of the normalized M_x contrast (see e.g. Supplementary Movie 3) shows the dynamics within one period of excitation. The time-trace of each pixel along the propagating spin-wave was fitted with a sinusoidal function (see e.g. Fig. 3d, i in the main text), whose amplitude and phase were extracted. As an example, the raw data corresponding to the sinusoidal fittings of Fig. 3i in the main text (curved domain wall) are reported together with the fittings in Supplementary Figure 2.



Supplementary Figure 2. STXM time-trace fitting. Raw time-trace (average on 3 adjacent pixels) (empty circles) and sinusoidal fittings (solid lines) at different distances from the microstrip along the curved domain wall. The distance from the microstrip edge is indicated for each panel. The color code is the same as in Fig. 3i, in the main text.

The amplitude of the sinusoidal fits as a function of the position is reported for instance in the spatial maps of Fig. 3b and 3g, in the main text, showing a confined spatial profile of the excitation.

For the calculation of the phase-shift as a function of distance, we plotted the phase of the sinusoidal fit as a function of the distance from the edge of the microstrip, setting a 0 phase shift right outside the microstrip antenna (see Fig. 3e, j in the main text). It is worth to notice that outside the spatial profile of the propagating spin-waves, no coherent spatial phase shift is observed, confirming the localized nature of the excitation. The wavevector was obtained from the linear fitting of the spatial dependence of the phase shift.



Supplementary Figure 3. Micromagnetic simulations for low excitation fields. **a, b** ΔM_x and M_z, respectively, for directly comparing the simulations for lower fields (left and center panel), with the result presented in the main manuscript (right panel). We observe, as expected, that the external field amplitude influences the amplitude of the localized spin-waves. This can be appreciated from the change in the color scale values. Scale bars: 500 nm.