

Supplementary Information

Supplementary Note 1. Magnetic field dependence of the gate-based thermometer

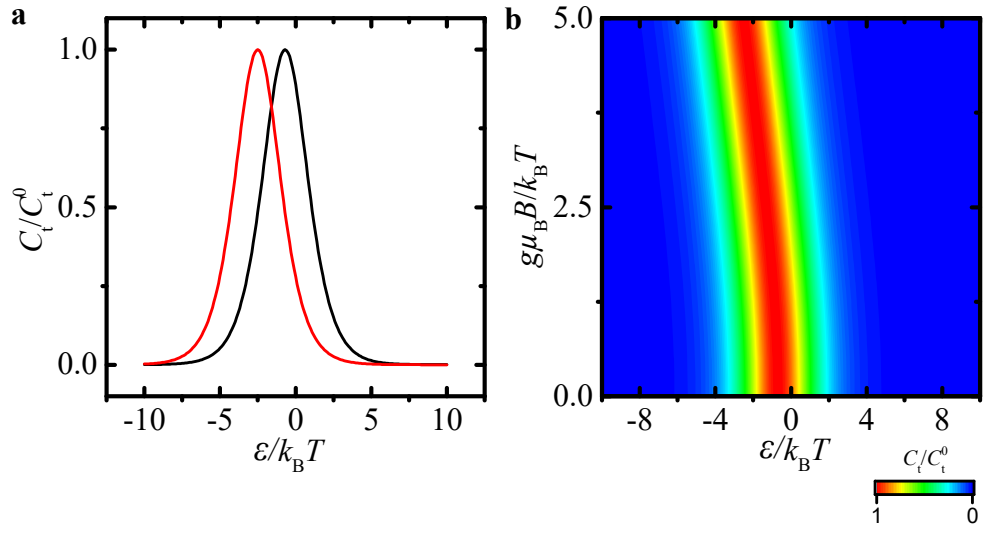
In this section, we discuss the magnetic field dependence of the gate-based thermometer. We express the adiabatic probability distribution P_1^0 in the quantum dot as a function of the reduced detuning $\hat{\varepsilon} = \varepsilon/k_B T$ and reduced magnetic field $\hat{B} = g\mu_B B/k_B T$ taking in to account the spin degree of freedom (S=1/2):

$$P_1^0 = \frac{\exp[(\hat{\varepsilon} - \hat{B})/2] + \exp[(\hat{\varepsilon} + \hat{B})/2]}{\exp[(\hat{\varepsilon} - \hat{B})/2] + \exp[(\hat{\varepsilon} + \hat{B})/2] + \exp(-\hat{\varepsilon}/2)} \quad (1)$$

and calculate the tunneling capacitance $C_t = (e\alpha)^2 \partial P_1^0 / \partial \varepsilon$,

$$C_t = C_t^0 \frac{4 \exp[(\hat{B} + 2\hat{\varepsilon})/2] (1 + \exp(\hat{B}))}{\exp(\hat{B}/2) + \exp(\hat{\varepsilon}) + \exp(\hat{B} + \hat{\varepsilon})} \quad (2)$$

In Supplementary Figure 1a, we plot the reduced tunneling capacitance C_t/C_t^0 as a function of detuning for two different values of the reduced magnetic field $\hat{B} = 0$ (black) and $\hat{B} = 5$ (red). We see that the full width at half maximum (FWHM) and the height of the capacitance peak remain constant whereas the center of the peak has been shifted towards lower detuning at high $\hat{B}$. In Supplementary Figure 1b, we plot a colormap of C_t/C_t^0 as a function of reduced detuning and magnetic field. We see that the capacitance peak shifts to lower detuning and this shifts tends to $\hat{\varepsilon}(\hat{B}) = \hat{\varepsilon}(0) - \hat{B}/2$ at high $\hat{B}$.



Supplementary Figure 1. **a** Reduced capacitance curve as a function of reduced detuning for $\hat{B} = 0, 5$ respectively. **b** Reduced capacitance as a function of reduced detuning and reduced magnetic field.