Supplementary information: Stochastic dynamics and pattern formation of geometrically confined skyrmions

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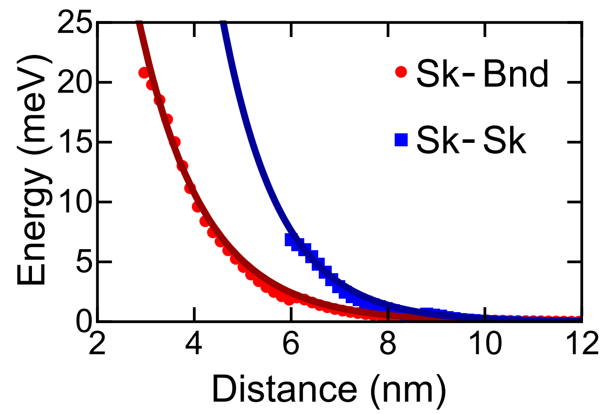
SUPPLEMENTARY METHODS

Calculation of the skyrmion-skyrmion and skyrmion-boundary interaction potentials. For the calculation of the potential, the total energy of a stripe-shaped model system ($75 \times 30 \times 0.4\,\mathrm{nm}^3$) was investigated with micromagnetic simulations for two different cases. In the first case, Neumann boundary conditions were used along the long side, and a homogeneous magnetization was relaxed leading to a canted rim. Afterwards a skyrmion was added to the system at a certain distance from the boundary. Because of the repulsive nature of the interaction, a standard relaxation of the skyrmion position was not suitable for determining the distance dependence. Instead, we fixed the central magnetic moments in the skyrmion (in a circular region of 0.9 nm diameter) at a specific position and relaxed the magnetic texture in the other cells. Analogously, in the second case the skyrmion-skyrmion interaction was calculated by fixing one skyrmion and changing the position of the other skyrmion, using periodic boundary conditions along both directions in the plane. From the obtained total energies we subtracted a reference spectrum for a homogeneously magnetized periodic surrounding around the skyrmion fixed at different positions in the mesh. Small deviations arising due to the finite mesh are mostly canceled out by this procedure.

The results for the interaction strengths are shown in Supplementary Fig. 1. A smooth energy function rapidly decaying with increasing distance between skyrmion and boundary is obtained. For small distances between the boundary and a skyrmion or between two skyrmions, a deformation of the quasiparticles is visible in the micromagnetic simulations, also leading to a bending of the potential in Supplementary Fig. 1. During the construction of the quasiparticle model it is assumed that the thermal energy $k_\mathrm{B}T$ is lower than the energy value where the potential function starts to bend, which is around $5\,\mathrm{meV}$ ($60\,\mathrm{K}$) for the skyrmion-skyrmion interaction potential in Supplementary Fig. 1. In this regime it can be assumed that for sufficiently short simulation times the strong deformation of the skyrmions due to the interactions becomes exceedingly rare, and the basic spin structure is maintained. This temperature regime is in reasonable qualitative agreement with the range where the escape, collapse or creation of skyrmions may also be excluded, discussed in detail in Fig. 3 of the main text. It was shown in Ref.[1] that the interaction potentials in this regime may be approximated by the exponential model function $E(r) = a\exp(-r/b)$. This is confirmed by the simulation results in Supplementary Fig. 1, with the fitted model parameters $a = 0.211\,\mathrm{meV}$,



Supplementary Fig. 1. **Potential energy depending on the skyrmion-skyrmion and skyrmion-boundary distance.** Micromagnetic interaction energies between skyrmion and skyrmion (blue squares), and between skyrmion and boundary (red circles) on a finite nanoisland. Solid lines display exponential fits based on the model function $E(r) = ae^{-r/b}$; the fitting parameters are skyrmion-boundary (Sk-Bnd): $a = 0.211\,\mathrm{meV}$, $b = 1.343\,\mathrm{nm}$; skyrmion-skyrmion (Sk-Sk): $a = 1.246\,\mathrm{meV}$, $b = 1.176\,\mathrm{nm}$.

$b = 1.343\,\mathrm{nm}$ for skyrmion-boundary (Sk-Bnd) repulsion and $a = 1.246\,\mathrm{meV}$, $b = 1.176\,\mathrm{nm}$ for skyrmion-skyrmion (Sk-Sk) interaction. The similar characteristic length scales b between the two cases support that the same physical mechanism is responsible for the interactions, namely the formation of chiral noncollinear spin structures due to the DMI as discussed in the main text. The different values of the parameter a can be reformulated as a shift of the exponential function along the r axis. While this distance is measured between two magnetization cells pointing oppositely to the homogeneous background in the case of two skyrmions, in the case of the skyrmion-boundary repulsion it is measured between the center of the skyrmion and the magnetization at the edge which is only slightly tilted from the homogeneous background, meaning that the same interaction strength is reached at a smaller distance in the latter scenario.

Supplementary references

[1] Lin, S.-Z., Reichhardt, C., Batista, C. D. & Saxena, A. Particle model for skyrmions in metallic chiral magnets: Dynamics, pin-ning, and creep. *Phys. Rev. B* **87**, 214419 (2013).