

Supplementary Note 1: The system Hamiltonian

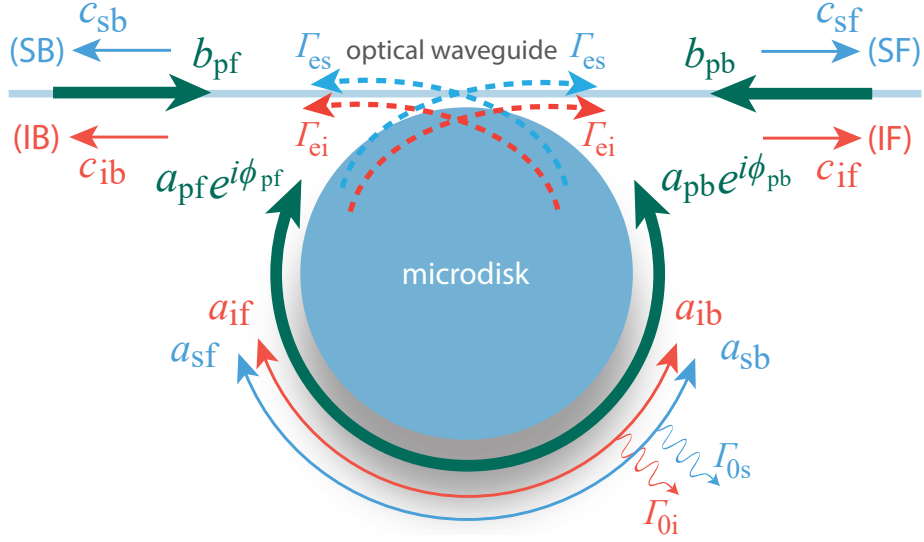
In this section, we describe the Hamiltonian which governs the photon pair creation process between the coupled counter-propagating modes of an optical microresonator, as well as the coupling between the device and an optical waveguide. This Hamiltonian will serve as the springboard for our theoretical treatment of the intracavity quantum state (see Supplementary Note 2) and second-order correlations that manifest (see Supplementary Note 4).

We begin by considering the establishment of Kerr-type nonlinear optical interactions between the pump (p), signal (s), and idler (i) whispering-gallery modes (WGMs), as illustrated in Supp. Fig. 1. The passive cavity modes are specified by their resonance frequencies, ω_{0m} ($m = p, s, i$), along with their intrinsic decay rates, Γ_{0m} , external coupling rates, Γ_{em} , and resonance splitting, $2\beta_m$. Furthermore, input pump waves (at a carrier frequency of ω_p) propagating in the forward, b_{pf} , and backward, b_{pb} , directions are coupled from the optical waveguide into the device and coherently build up in their respective intracavity modes. In particular, we assume that there exists a mutual coupling between intracavity optical fields propagating in the forward (clockwise), a_{mf} , and backward (counterclockwise), a_{mb} , directions, which renormalizes the pairs of degenerate traveling-wave modes into pairs of standing-wave modes. The coupling occurs at a rate of β_m , so that the standing-wave modes have distinct frequencies, $\omega_{0m}^{\pm} = \omega_{0m} \pm |\beta_m|$, which are shifted from the uncoupled system. When the intracavity pump waves undergo spontaneous four-wave mixing (SFWM), signal and idler photon pairs are created in their respective coupled counter-propagating modes. The Hamiltonian, $H = H_0 + H_I$, describing the relevant Kerr nonlinear interactions within the cavity is given by [1]

$$H_0 = \sum_{m=p,s,i} \left\{ \hbar\omega_{0m} \left(a_{mf}^{\dagger} a_{mf} + a_{mb}^{\dagger} a_{mb} \right) - \left(\hbar\beta_m a_{mf}^{\dagger} a_{mb} + \hbar\beta_m^* a_{mb}^{\dagger} a_{mf} \right) - \hbar\sqrt{\Gamma_{em}} \left((a_{mf}^{\dagger} b_{mf} + a_{mb}^{\dagger} b_{mb}) e^{-i\omega_{0m}t} + (b_{mf}^{\dagger} a_{mf} + b_{mb}^{\dagger} a_{mb}) e^{i\omega_{0m}t} \right) \right\}, \quad (1)$$

$$H_I = \frac{\hbar g_p}{2} \left((a_{pf}^{\dagger})^2 a_{pf}^2 + (a_{pb}^{\dagger})^2 a_{pb}^2 + 4a_{pf}^{\dagger} a_{pf} a_{pb}^{\dagger} a_{pb} \right) + 2\hbar(a_{pf}^{\dagger} a_{pf} + a_{pb}^{\dagger} a_{pb}) \left(g_{ps}(a_{sf}^{\dagger} a_{sf} + a_{sb}^{\dagger} a_{sb}) + g_{pi}(a_{if}^{\dagger} a_{if} + a_{ib}^{\dagger} a_{ib}) \right) + \hbar g_{psi} \left(a_{sf}^{\dagger} a_{if}^{\dagger} a_{pf}^2 + a_{sb}^{\dagger} a_{ib}^{\dagger} a_{pb}^2 \right) + \hbar g_{psi}^* \left((a_{pf}^{\dagger})^2 a_{sf} a_{if} + (a_{pb}^{\dagger})^2 a_{sb} a_{ib} \right), \quad (2)$$

where the intracavity field operators are normalized such that $a_{mj}^{\dagger} a_{mj}$ ($m = p, s, i$ and $j = f, b$) represent the photon number operators of the system. The input pump fields are normalized such that $b_{mj}^{\dagger} b_{mj}$ denote the input photon fluxes and satisfy the commutation relation $[b_{mj}(t), b_{mj'}^{\dagger}(t')] = \delta_{mm'} \delta_{jj'} \delta(t - t')$. We note that the number of photons contained in the pump modes greatly exceed the signal and idler modes, so that self-phase modulation (SPM) and cross-phase modulation (XPM) initiated from these modes may be neglected. The vacuum coupling rates for the pump-initiated self-phase modulation (SPM), cross-phase modulation and SFWM are respectively denoted as g_p , g_{pm} ($m = s, i$), and g_{psi} . However, given the similarity in field profiles and frequencies between the interacting cavity modes, we approximate $g_p \approx g_{pm} \approx g_{psi} \equiv g = \frac{c\eta n_2 \hbar \omega_p \sqrt{\omega_s \omega_i}}{n_s n_i V}$, where c is the speed of light in vacuum, η is the spatial overlap fraction, $n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c n_p^2}$ is the Kerr nonlinear coefficient [2], n_s (n_i) is the index of refraction of the microresonator at



Supplementary Figure 1. Photon pair creation between coherently coupled counter-propagating whispering-gallery modes (WGM). Forward, b_{pf} , and backward, b_{pb} , propagating pump waves (green) evanescently couple into the microdisk, on resonance with the pump cavity mode, respectively building up strong intracavity fields in the forward, a_{pf} , and backward, a_{pb} , directions. The counter-propagating intracavity pump fields are mutually coupled at a rate of β_p . Through spontaneous four-wave mixing (SFWM), signal (blue) and idler (red) photons are created within their respective cavity modes. Furthermore, the counter-propagating intracavity signal (idler) fields are mutually coupled at a rate of β_s (β_i). Thus, the photon pairs coherently cycle between propagation directions within the cavity and ultimately exit into the four transmitted fields of the optical waveguide: c_{sf} for signal forward (SF), c_{sb} for signal backward (SB), c_{if} for idler forward (IF), and c_{ib} for idler backward (IB). Hence, correlations are established between photon pairs in the four path configurations: SF-IF, SF-IB, SB-IF, SB-IB.

the signal (idler) wavelength, and $\bar{V}$ is the effective mode volume.

Supplementary Note 2: The intracavity quantum state

To determine the biphoton quantum state inside our optical microresonator, first-order time-dependent perturbation theory was applied to the closed system Hamiltonian (see Supplementary Note 1) in an undepleted pump regime. At the instant of pair creation (here defined as $t = 0$), the state takes the form $|\psi(t = 0)\rangle = c_f |f\rangle_s |f\rangle_i + c_b |b\rangle_s |b\rangle_i$, where f denotes the forward-propagating traveling-wave mode and b denotes the backward-propagating traveling-wave mode. The subscripts s and i differentiate the signal and idler photons. Complex coefficients c_f and c_b are set by the relative amplitude and phase between the pump's forward and backward traveling-wave modes.

The initial state can be intuited by viewing the nonlinear interaction Hamiltonian H_I (see Eq. 1 & 2 for the complete Hamiltonian) in the basis of forward and backward propagating modes. The terms that contribute to the creation of signal and idler photons via spontaneous four-wave mixing (SFWM) are given by $H_{\text{SFWM}} = \hbar g_{\text{psi}} \left(a_{\text{sf}}^\dagger a_{\text{if}}^\dagger a_{\text{pf}}^2 + a_{\text{sb}}^\dagger a_{\text{ib}}^\dagger a_{\text{pb}}^2 \right)$ where g_{psi} is the vacuum coupling rate for SFWM. These terms allow for photon pairs to be created in the co-propagating directions only, as is required to conserve angular momentum.

We implement time-dependent perturbation theory to verify the initial state. The linear portion of the closed-system Hamiltonian is

$$H_0 = \sum_{m=p,s,i} \hbar \omega_{0m} \left(a_{\text{mf}}^\dagger a_{\text{mf}} + a_{\text{mb}}^\dagger a_{\text{mb}} \right) - \hbar \left(\beta_m a_{\text{mf}}^\dagger a_{\text{mb}} + \beta_m^* a_{\text{mb}}^\dagger a_{\text{mf}} \right). \quad (3)$$

where ω_{0m} is the center frequency of the m^{th} resonance and β_m is the coupling between forward and backward modes of the m^{th} resonance.

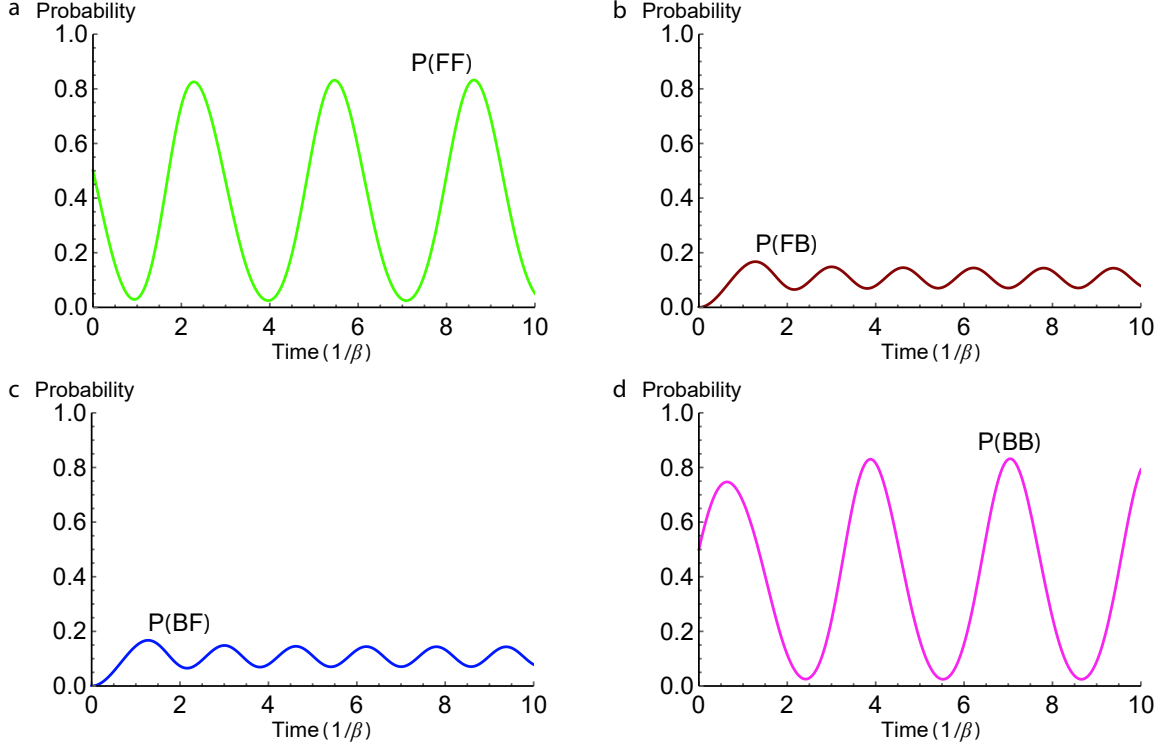
Meanwhile, the perturbative Hamiltonian is H_{SFWM} , given that the other terms in H_I have a negligible effect in the weakly pumped system considered here. To first order, time-dependent perturbation theory then gives an unnormalized state

$$|\psi(t)\rangle = U_0(t, 0) |\text{Vacuum}\rangle + \frac{1}{i\hbar} \int_0^t dt' U_0(t, t') H_{\text{SFWM}}(t') U_0(t', 0) |\text{Vacuum}\rangle, \quad (4)$$

where $U_0(t_2, t_1)$ is defined to be the unitary time evolution operator $\exp[-iH_0(t_2 - t_1)/\hbar]$. To evaluate the integral, it is helpful to diagonalize H_0 and write H_{SFWM} in the basis that performs this diagonalization. In the microresonator system, this amounts to transforming into the standing-wave basis. The basis states, denoted by $+$ and $-$ are respectively the higher and lower energy eigenstates of H_0 . Furthermore, we apply the approximation that $\beta e^{i\phi_\beta} = \beta_p = \beta_s = \beta_i \equiv \beta$. Then the result of the transformation on H_0 and H_{SFWM} is

$$H_0 = \sum_{m=p,s,i} \hbar \left((\omega_{0m} - \beta) a_{\text{m}-}^\dagger a_{\text{m}-} + (\omega_{0m} + \beta) a_{\text{m}+}^\dagger a_{\text{m}+} \right), \quad (5)$$

$$H_{\text{SFWM}} = \frac{1}{2} \hbar g_{\text{psi}} \left(a_{\text{s}-}^\dagger a_{\text{i}-}^\dagger + a_{\text{s}+}^\dagger a_{\text{i}+}^\dagger e^{-2i\phi_\beta} \right) \left(a_{\text{p}-}^2 + a_{\text{p}+}^2 e^{2i\phi_\beta} \right) + \hbar g_{\text{psi}} \left(a_{\text{s}-}^\dagger a_{\text{i}+}^\dagger + a_{\text{s}+}^\dagger a_{\text{i}-}^\dagger \right) a_{\text{p}-} a_{\text{p}+}. \quad (6)$$



Supplementary Figure 2. A depiction of the time evolution of created signal and idlers assuming a closed system. To arrive at this result, $t = 0$ was taken to be the instant of photon creation, forward and backward-propagating pumps were treated as equal amplitude, $\phi_\beta = \pi/4$, and $g_{\text{psi}} = 2\pi \times 560$ radians per second. The probability of finding the photons in the path configurations (a) signal-forward and idler-forward, (b) signal-forward and idler-backward, (c) signal-backward and idler-forward, and (d) signal-backward and idler-backward is shown by the vertical axes. The horizontal axes display time in units of $1/\beta$ the reciprocal of the modal coupling. The state begins in a superposition of forward-forward and backward-backward path configurations, providing evidence for the initial state supplied in the main text.

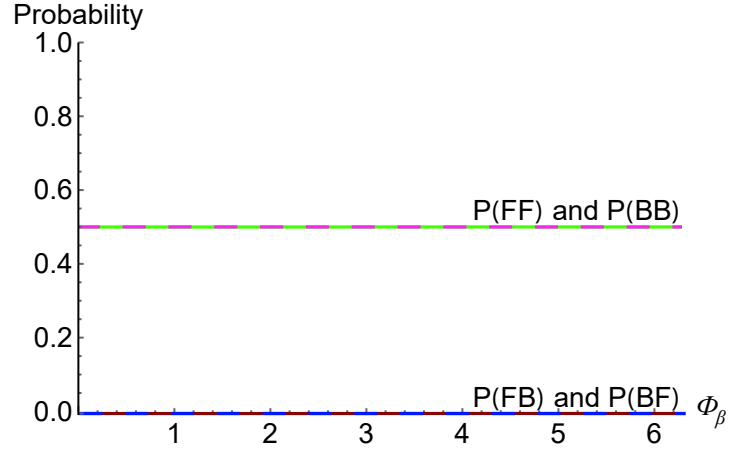
Now Eq. 4 becomes

$$|\psi(t)\rangle = |\text{Vacuum}\rangle + \frac{1}{i\hbar} \int_0^t dt' e^{-iE_t(t-t')/\hbar} H_{\text{SFWM}}(t') e^{-iE_i t'} |\text{Vacuum}\rangle, \quad (7)$$

where E_i and E_f are the energies of the initial and final states, respectively. Treating the pump as classical and undepleted, E_i can be taken to zero by introducing a plane wave oscillation $\exp[\mp 2i\omega_p t']$ to $H_{\text{SFWM}}(t')$. E_f is the total energy of the created signal and idler photons. The integral can then be evaluated, and will reflect a linear growth in time as the probability of photon creation increases. If we again define the instant of creation as $t = 0$ and then normalize the first order terms, we can observe how the state of the created photons evolves in time. Supplementary Figure 2 shows the probability of detecting signal and idler photons in each of the four pairs of path configurations as a function of time.

Because the probability of detecting counter-propagating signal and idler photon pairs is zero at $t = 0$, we can affirm that the photons are created in a superposition of the forward-forward and backward-backward path configurations. That is, the initial biphoton state is indeed of the form $|\psi(t=0)\rangle =$

$c_f |f\rangle_s |f\rangle_i + c_b |b\rangle_s |b\rangle_i$, as stated in the main text. Notably, this is independent of ϕ_β , as seen in Supp. Fig. 3.



Supplementary Figure 3. Probability as a function of ϕ_β at $t = 1/10000\beta$. The probability of created photons being in a co-propagating path configuration is $1/2$ for all values of ϕ_β , while for counter-propagating path configurations it is 0 for all values of ϕ_β .

Supplementary Note 3: Quantum state evolution

Beginning with the quantum state $|\psi(t=0)\rangle = c_f |f\rangle_s |f\rangle_i + c_b |b\rangle_s |b\rangle_i$, time evolution can be easily performed by switching to the standing-wave basis using the following transformations,

$$|f\rangle_m = \frac{1}{\sqrt{2}} \left(|-\rangle_m - e^{-i\phi_\beta} |+\rangle_m \right), \quad (8)$$

$$|b\rangle_m = \frac{1}{\sqrt{2}} \left(e^{i\phi_\beta} |-\rangle_m + |+\rangle_m \right). \quad (9)$$

The lower-energy standing wave mode is represented by $|-\rangle$. $|+\rangle$ likewise refers to the higher-energy standing-wave mode. For the following calculation, it will be assumed for simplicity that $\phi_\beta = 0$. Implementing the basis transformation yields an initial state

$$|\psi(t=0)\rangle = \frac{1}{2} \left((c_b + c_f) (|-\rangle_s |-\rangle_i + |+\rangle_s |+\rangle_i) + (c_b - c_f) (|-\rangle_s |+\rangle_i + |+\rangle_s |-\rangle_i) \right). \quad (10)$$

Because the state is now in terms of energy eigenstates of the cavity, the unitary time evolution operator $U_0(t, 0) = \exp[-iH_0t/\hbar]$ can be easily applied. The time evolved state in the standing-mode basis is thus,

$$|\psi(t)\rangle = \frac{1}{2} e^{-i(\omega_s + \omega_i)t} \left((c_b + c_f) (e^{2i\beta t} |-\rangle_s |-\rangle_i + e^{-2i\beta t} |+\rangle_s |+\rangle_i) + (c_b - c_f) (|-\rangle_s |+\rangle_i + |+\rangle_s |-\rangle_i) \right). \quad (11)$$

By transforming back to the basis of traveling waves and simplifying, this becomes

$$\begin{aligned} |\psi(t)\rangle = e^{-i(\omega_s + \omega_i)t} & \left((c_f \cos^2(\beta t) - c_b \sin^2(\beta t)) |f\rangle_s |f\rangle_i + \right. \\ & (c_b \cos^2(\beta t) - c_f \sin^2(\beta t)) |b\rangle_s |b\rangle_i + \\ & \left. i(c_f + c_b) \sin(2\beta t) (|f\rangle_s |b\rangle_i + |b\rangle_s |f\rangle_i) \right). \end{aligned} \quad (12)$$

This result appears as **equation (1)** in the main text with the overall phase ignored.

Supplementary Note 4: Theory of coherent oscillations in photon pair correlations

In Supplementary Note 3, we showed that the quantum states generated within the microresonator exhibit cyclically evolving path-entanglement (see Eq. 7). When the photon pairs exit the system (see Supp. Fig. 1), they are transmitted into both directions of the optical waveguide which establishes the following single photon pathways: signal forward (SF), signal backward (SB), idler forward (IF), and idler backward (IB). The cyclically evolving path-entanglement within the microresonator will consequently manifest as coherent oscillations between pairs of propagation pathways. To theoretically demonstrate this phenomenon, we employ the Hamiltonian in Eq. 1-2, in order to write the Heisenberg-Langevin equations of motion. These equations describe the wave dynamics within the microdisk, and are given as

$$\frac{da_{\text{pf}}}{dt} = (-i\omega_{0\text{p}} - \Gamma_{\text{tp}}/2)a_{\text{pf}} + i\beta_{\text{p}}a_{\text{pb}} - ig(a_{\text{pf}}^\dagger a_{\text{pf}} + 2a_{\text{pb}}^\dagger a_{\text{pb}})a_{\text{pf}} + \zeta_{\text{pf}}(t), \quad (13)$$

$$\frac{da_{\text{pb}}}{dt} = (-i\omega_{0\text{p}} - \Gamma_{\text{tp}}/2)a_{\text{pb}} + i\beta_{\text{p}}^*a_{\text{pf}} - ig(a_{\text{pb}}^\dagger a_{\text{pb}} + 2a_{\text{pf}}^\dagger a_{\text{pf}})a_{\text{pb}} + \zeta_{\text{pb}}(t), \quad (14)$$

$$\frac{da_{\text{sf}}}{dt} = (-i\omega_{0\text{s}} - \Gamma_{\text{ts}}/2)a_{\text{sf}} + i\beta_{\text{s}}a_{\text{sb}} - 2ig(a_{\text{pf}}^\dagger a_{\text{pf}} + a_{\text{pb}}^\dagger a_{\text{pb}})a_{\text{sf}} - iga_{\text{if}}^\dagger a_{\text{pf}}^2 + \zeta_{\text{sf}}(t), \quad (15)$$

$$\frac{da_{\text{sb}}}{dt} = (-i\omega_{0\text{s}} - \Gamma_{\text{ts}}/2)a_{\text{sb}} + i\beta_{\text{s}}^*a_{\text{sf}} - 2ig(a_{\text{pf}}^\dagger a_{\text{pf}} + a_{\text{pb}}^\dagger a_{\text{pb}})a_{\text{sb}} - iga_{\text{ib}}^\dagger a_{\text{pb}}^2 + \zeta_{\text{sb}}(t), \quad (16)$$

$$\frac{da_{\text{if}}}{dt} = (-i\omega_{0\text{i}} - \Gamma_{\text{ti}}/2)a_{\text{if}} + i\beta_{\text{i}}a_{\text{ib}} - 2ig(a_{\text{pf}}^\dagger a_{\text{pf}} + a_{\text{pb}}^\dagger a_{\text{pb}})a_{\text{if}} - iga_{\text{sf}}^\dagger a_{\text{pf}}^2 + \zeta_{\text{if}}(t), \quad (17)$$

$$\frac{da_{\text{ib}}}{dt} = (-i\omega_{0\text{i}} - \Gamma_{\text{ti}}/2)a_{\text{ib}} + i\beta_{\text{i}}^*a_{\text{if}} - 2ig(a_{\text{pf}}^\dagger a_{\text{pf}} + a_{\text{pb}}^\dagger a_{\text{pb}})a_{\text{ib}} - iga_{\text{sb}}^\dagger a_{\text{pb}}^2 + \zeta_{\text{ib}}(t), \quad (18)$$

where $\zeta_{\text{mj}}(t) \equiv i\sqrt{\Gamma_{\text{em}}}b_{\text{mj}}(t) + \sqrt{\Gamma_{0\text{m}}}u_{\text{mj}}(t)$ ($\text{m} = \text{p}, \text{s}, \text{i}$ and $\text{j} = \text{f}, \text{b}$) and u_{mj} are noise operators associated with intrinsic cavity losses and obey the commutation relation $[u_{\text{mj}}(t), u_{\text{m'j'}}^\dagger(t')] = \delta_{\text{mm'}}\delta_{\text{jj'}}\delta(t - t')$.

In order to solve Eq. 13-18, we treat the pumps as classical and undepleted (with respect to the Kerr nonlinear interactions), transform to a frame rotating at the carrier frequency of the input pump waves, and apply exponential transformations to eliminate the SPM and XPM terms, which gives

$$\frac{da_{\text{pf}}}{dt} = (i\Delta - \Gamma_{\text{tp}}/2)a_{\text{pf}} + i\beta_{\text{p}}a_{\text{pb}} + i\sqrt{\Gamma_{\text{ep}}}b_{\text{pf}}, \quad (19)$$

$$\frac{da_{\text{pb}}}{dt} = (i\Delta - \Gamma_{\text{tp}}/2)a_{\text{pb}} + i\beta_{\text{p}}^*a_{\text{pf}} + i\sqrt{\Gamma_{\text{ep}}}b_{\text{pb}}, \quad (20)$$

$$\frac{da_{\text{sf}}}{dt} = (i\Delta - \Gamma_{\text{ts}}/2)a_{\text{sf}} + i\beta_{\text{s}}a_{\text{sb}} - iga_{\text{if}}^\dagger a_{\text{pf}}^2 + \zeta_{\text{sf}}(t), \quad (21)$$

$$\frac{da_{\text{sb}}}{dt} = (i\Delta - \Gamma_{\text{ts}}/2)a_{\text{sb}} + i\beta_{\text{s}}^*a_{\text{sf}} - iga_{\text{ib}}^\dagger a_{\text{pb}}^2 + \zeta_{\text{sb}}(t), \quad (22)$$

$$\frac{da_{\text{if}}}{dt} = (i\Delta - \Gamma_{\text{ti}}/2)a_{\text{if}} + i\beta_{\text{i}}a_{\text{ib}} - iga_{\text{sf}}^\dagger a_{\text{pf}}^2 + \zeta_{\text{if}}(t), \quad (23)$$

$$\frac{da_{\text{ib}}}{dt} = (i\Delta - \Gamma_{\text{ti}}/2)a_{\text{ib}} + i\beta_{\text{i}}^*a_{\text{if}} - iga_{\text{sb}}^\dagger a_{\text{pb}}^2 + \zeta_{\text{ib}}(t), \quad (24)$$

where $\Delta \equiv \omega_{\text{p}} - \omega_{0\text{p}}$ is the detuning between the carrier frequency of the input pump waves and the

resonance frequency of the pump cavity mode. The coupled pump equations (Eq. 19 & 20) may now be trivially solved due to the removal of any explicit time dependence. The remaining equations (Eq. 21-24) are conveniently solved in the frequency domain by applying Fourier transforms, yielding

$$\tilde{\zeta}_{\text{sf}}(\omega) = -\{i(\omega + \Delta) - \Gamma_{\text{ts}}/2\}\tilde{a}_{\text{sf}}(\omega) - i\beta_{\text{s}}\tilde{a}_{\text{sb}}(\omega) + ig a_{\text{pf}}^2 \tilde{a}_{\text{if}}^\dagger(-\omega), \quad (25)$$

$$\tilde{\zeta}_{\text{sb}}(\omega) = -\{i(\omega + \Delta) - \Gamma_{\text{ts}}/2\}\tilde{a}_{\text{sb}}(\omega) - i\beta_{\text{s}}^* \tilde{a}_{\text{sf}}(\omega) + ig a_{\text{pb}}^2 \tilde{a}_{\text{ib}}^\dagger(-\omega), \quad (26)$$

$$\tilde{\zeta}_{\text{if}}^\dagger(-\omega) = -\{i(\omega + \Delta) - \Gamma_{\text{ti}}/2\}\tilde{a}_{\text{if}}^\dagger(-\omega) + i\beta_{\text{i}}^* \tilde{a}_{\text{ib}}^\dagger(-\omega) - ig(a_{\text{pf}}^*)^2 \tilde{a}_{\text{sf}}(\omega), \quad (27)$$

$$\tilde{\zeta}_{\text{ib}}^\dagger(-\omega) = -\{i(\omega + \Delta) - \Gamma_{\text{ti}}/2\}\tilde{a}_{\text{ib}}^\dagger(-\omega) + i\beta_{\text{i}} \tilde{a}_{\text{if}}^\dagger(-\omega) - ig(a_{\text{pb}}^*)^2 \tilde{a}_{\text{sb}}(\omega), \quad (28)$$

where $\Gamma_{\text{tm}} = \Gamma_{0\text{m}} + \Gamma_{\text{em}}$ ($\text{m} = \text{s}, \text{i}$) are the total photon decay rates for the cavity modes. Eq. 25-28 may now be written using matrix formalism as, $\vec{\zeta}(\omega) = -\mathbf{M}\vec{a}(\omega)$, with the following vector (matrix) components (elements)

$$\begin{pmatrix} \zeta_{\text{sf}}(\omega) \\ \zeta_{\text{sb}}(\omega) \\ \zeta_{\text{if}}^\dagger(-\omega) \\ \zeta_{\text{ib}}^\dagger(-\omega) \end{pmatrix} = - \begin{pmatrix} i(\omega + \Delta) - \Gamma_{\text{ts}}/2 & i\beta_{\text{s}}e^{i\phi_{\beta_{\text{s}}}} & -iga_{\text{pf}}^2 & 0 \\ i\beta_{\text{s}}e^{-i\phi_{\beta_{\text{s}}}} & i(\omega + \Delta) - \Gamma_{\text{ts}}/2 & 0 & -iga_{\text{pb}}^2 \\ ig^*(a_{\text{pf}}^*)^2 & 0 & i(\omega + \Delta) - \Gamma_{\text{ti}}/2 & -i\beta_{\text{i}}e^{-i\phi_{\beta_{\text{i}}}} \\ 0 & ig^*(a_{\text{pb}}^*)^2 & -i\beta_{\text{i}}e^{i\phi_{\beta_{\text{i}}}} & i(\omega + \Delta) - \Gamma_{\text{ti}}/2 \end{pmatrix} \begin{pmatrix} a_{\text{sf}}(\omega) \\ a_{\text{sb}}(\omega) \\ a_{\text{if}}^\dagger(-\omega) \\ a_{\text{ib}}^\dagger(-\omega) \end{pmatrix}, \quad (29)$$

where we have dropped the tilde notation. Additionally, we have written the coupling terms as $\beta_{\text{m}} = \beta_{\text{m}}e^{i\phi_{\beta_{\text{m}}}}$ to explicitly display their spatial phase dependence which results from the orientation of the standing-wave mode patterns to the point of coupling at the waveguide. Now, we can solve for the intracavity fields, noting that $\vec{a}(\omega) = \mathbf{T}\vec{\zeta}(\omega)$, where $\mathbf{T} \equiv -\mathbf{M}^{-1}$. Finally, we relate the transmitted fields, c_{mj} , to the input and intracavity fields through the standard input-output relations [3], $c_{\text{mj}} = b_{\text{mj}} + i\sqrt{\Gamma_{\text{em}}}a_{\text{mj}}$.

The device presented in Supp. Fig. 1 couples photon pairs generated in the forward and backward directions, resulting in correlations between the following four configurations: signal forward - idler forward (SF-IF), signal forward - idler backward (SF-IB), signal backward - idler forward (SB-IF), and signal backward - idler backward (SB-IB). The correlations may be theoretically described using second-order temporal correlation functions [4], $p_{c,jk}(t_{\text{sj}}, t_{\text{ik}})$, with

$$\begin{aligned} p_{c,jk}(t_{\text{sj}}, t_{\text{ik}}) &\equiv \langle c_{\text{ik}}^\dagger(t)c_{\text{sj}}^\dagger(t + \tau_{\text{jk}})c_{\text{sj}}(t + \tau_{\text{jk}})c_{\text{ik}}(t) \rangle - \langle c_{\text{sj}}^\dagger(t)c_{\text{sj}}(t) \rangle \langle c_{\text{ik}}^\dagger(t)c_{\text{ik}}(t) \rangle \\ &= \Gamma_{\text{es}}\Gamma_{\text{ei}} \frac{1}{2\pi} \int d\omega K_{\text{jk}}(\omega) e^{-i\omega\tau_{\text{jk}}} |^2, \end{aligned} \quad (30)$$

where $\tau_{\text{jk}} \equiv t_{\text{sj}} - t_{\text{ik}}$ ($\text{j} = \text{f}, \text{b}$ and $\text{k} = \text{f}, \text{b}$) denotes the delay time between the emission of signals and idlers from the microresonator into the waveguide (see Supp. Fig. 1), and $K_{\text{jk}}(\omega)$ are Kernel functions which depend upon the path configuration. The Kernel functions, associated with the four path configurations, are defined as

$$K_{\text{ff}} \equiv T_{31}^*(\omega)[\Gamma_{\text{ts}}T_{11}(\omega) - 1] + \Gamma_{\text{ts}}T_{12}(\omega)T_{32}^*(\omega), \quad (31)$$

$$K_{\text{fb}} \equiv T_{41}^*(\omega)[\Gamma_{\text{ts}}T_{11}(\omega) - 1] + \Gamma_{\text{ts}}T_{12}(\omega)T_{42}^*(\omega), \quad (32)$$

$$K_{\text{bf}} \equiv T_{32}^*(\omega)[\Gamma_{\text{ts}}T_{22}(\omega) - 1] + \Gamma_{\text{ts}}T_{21}(\omega)T_{31}^*(\omega), \quad (33)$$

$$K_{\text{bb}} \equiv T_{42}^*(\omega)[\Gamma_{\text{ts}}T_{22}(\omega) - 1] + \Gamma_{\text{ts}}T_{21}(\omega)T_{41}^*(\omega), \quad (34)$$

where T_{nl} are elements of the matrix $\mathbf{T}$. After computing the matrix $\mathbf{T}$ and performing the inverse Fourier transform in Eq. 30 (both of these steps are accomplished in Mathematica), we arrive at the pair correlation functions for the four path configurations

$$p_{c,ff}(\tau) = \begin{cases} Ne^{\Gamma_{\text{ti}}\tau} |\{c_0 f e^{-i\phi} - c_1 b\} \cos(\beta_i \tau) + \{c_2 f e^{-i\phi} + c_3 b\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ Ne^{-\Gamma_{\text{ts}}\tau} |\{c_0 f e^{-i\phi} - c_1 b\} \cos(\beta_s \tau) - \{c_3 f e^{-i\phi} + c_2 b\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (35)$$

$$p_{c,fb}(\tau) = \begin{cases} Ne^{\Gamma_{\text{ti}}\tau} |\{c_2 f e^{-i\phi} + c_3 b\} \cos(\beta_i \tau) - \{c_0 f e^{-i\phi} - c_1 b\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ Ne^{-\Gamma_{\text{ts}}\tau} |\{c_2 f e^{-i\phi} + c_3 b\} \cos(\beta_s \tau) - \{c_1 f e^{-i\phi} - c_0 b\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (36)$$

$$p_{c,bf}(\tau) = \begin{cases} Ne^{\Gamma_{\text{ti}}\tau} |\{c_3 f e^{-i\phi} + c_2 b\} \cos(\beta_i \tau) + \{c_1 f e^{-i\phi} - c_0 b\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ Ne^{-\Gamma_{\text{ts}}\tau} |\{c_3 f e^{-i\phi} + c_2 b\} \cos(\beta_s \tau) + \{c_0 f e^{-i\phi} - c_1 b\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (37)$$

$$p_{c,bb}(\tau) = \begin{cases} Ne^{\Gamma_{\text{ti}}\tau} |\{-c_1 f e^{-i\phi} + c_0 b\} \cos(\beta_i \tau) + \{c_3 f e^{-i\phi} + c_2 b\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ Ne^{-\Gamma_{\text{ts}}\tau} |\{-c_1 f e^{-i\phi} + c_0 b\} \cos(\beta_s \tau) - \{c_2 f e^{-i\phi} + c_3 b\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (38)$$

where $f \equiv |a_{pf}(\Delta)|^2$ and $b \equiv |a_{pb}(\Delta)|^2$ are the optical energies contained in the forward and backward pump cavity modes, respectively, and $\phi \equiv 2\phi_p + \phi_\beta$ defines the phase relationship between them. In particular, $\phi_p \equiv \phi_{pf} - \phi_{pb}$ and $\phi_\beta \equiv 2\phi_{\beta p} - \phi_{\beta s} - \phi_{\beta i}$. We note that in the process of reaching these solutions we have made use of the fact that our system operates in the weak interaction limit ($gf \ll \Gamma_{\text{tp}}$ and $gb \ll \Gamma_{\text{tp}}$), such that terms beyond first order in f and b may be dropped. In Eq. 35-38, the constants c_n ($n = 1$ to 4) and N depend upon properties of the device in the following manner

$$N = \frac{4\Gamma_{es}\Gamma_{ei}\Gamma_t^4 g^2}{((c_0 - c_1)(c_0 + c_1))^2}, \quad (39)$$

$$c_0 = (4\beta_s^2 + 4\beta_i^2 + \Gamma_t^2)\Gamma_t, \quad (40)$$

$$c_1 = 8\beta_s\beta_i\Gamma_t, \quad (41)$$

$$c_2 = 8\beta_i^3 - 8\beta_s^2\beta_i + 2\beta_i\Gamma_t^2, \quad (42)$$

$$c_3 = 8\beta_s^3 - 8\beta_s\beta_i^2 + 2\beta_s\Gamma_t^2, \quad (43)$$

where $\Gamma_t \equiv \Gamma_{\text{ts}} + \Gamma_{\text{ti}}$. Equations 35-38, along with the constants defined in Eq. 39-43, formulate the expanded version of **equation (2)** that appears in the main text.

As is evident in Eq. 35-38, the intracavity pump fields play a crucial role in controlling the structure of the pair correlations. Equations 19 & 20 describe the coupled intracavity pump modes, and after assuming a steady state solution, may be written as,

$$0 = (i\Delta - \Gamma_{\text{tp}}/2)a_{\text{pf}} + i\beta_{\text{p}}a_{\text{pb}} + i\sqrt{\Gamma_{\text{ep}}}b_{\text{pf}}, \quad (44)$$

$$0 = (i\Delta - \Gamma_{\text{tp}}/2)a_{\text{pb}} + i\beta_{\text{p}}^*a_{\text{pf}} + i\sqrt{\Gamma_{\text{ep}}}b_{\text{pb}}, \quad (45)$$

or alternatively in the following matrix form,

$$\begin{pmatrix} i\Delta - \Gamma_{\text{tp}}/2 & i\beta_{\text{p}}e^{i\phi_{\beta_{\text{p}}}} \\ i\beta_{\text{p}}e^{-i\phi_{\beta_{\text{p}}}} & i\Delta - \Gamma_{\text{tp}}/2 \end{pmatrix} \begin{pmatrix} a_{\text{pf}} \\ a_{\text{pb}} \end{pmatrix} = -i\sqrt{\Gamma_{\text{ep}}} \begin{pmatrix} b_{\text{pf}} \\ b_{\text{pb}} \end{pmatrix}. \quad (46)$$

Upon inverting the matrix in Eq. 46, we arrive at detuning-dependent solutions for the intracavity pump fields, which are given as

$$a_{\text{pf}}(\Delta) = \frac{-i\sqrt{\Gamma_{\text{ep}}}}{(i\Delta - \Gamma_{\text{tp}}/2)^2 + \beta_{\text{p}}^2} \{(i\Delta - \Gamma_{\text{tp}}/2)b_{\text{pf}} - i\beta_{\text{p}}e^{i\phi_{\beta_{\text{p}}}}b_{\text{pb}}\}, \quad (47)$$

$$a_{\text{pb}}(\Delta) = \frac{-i\sqrt{\Gamma_{\text{ep}}}}{(i\Delta - \Gamma_{\text{tp}}/2)^2 + \beta_{\text{p}}^2} \{-i\beta_{\text{p}}e^{-i\phi_{\beta_{\text{p}}}}b_{\text{pf}} + (i\Delta - \Gamma_{\text{tp}}/2)b_{\text{pb}}\}. \quad (48)$$

Equations 47 & 48 describe the intracavity pump fields for the general case of two counter-propagating input pump waves (see Supp. Fig. 1) entering the device. However, all results presented in the main text are obtained with a single forward-propagating input pump wave ($b_{\text{pb}} = 0$), such that the intracavity pump fields may be expressed in the following simpler forms,

$$a_{\text{pf}}(\Delta) = \frac{-i\sqrt{\Gamma_{\text{ep}}}}{(i\Delta - \Gamma_{\text{tp}}/2)^2 + \beta_{\text{p}}^2} \{(i\Delta - \Gamma_{\text{tp}}/2)b_{\text{pf}}\}, \quad (49)$$

$$a_{\text{pb}}(\Delta) = \frac{-i\sqrt{\Gamma_{\text{ep}}}}{(i\Delta - \Gamma_{\text{tp}}/2)^2 + \beta_{\text{p}}^2} \{-i\beta_{\text{p}}e^{-i\phi_{\beta_{\text{p}}}}b_{\text{pf}}\}. \quad (50)$$

Supplementary Note 5: Bell test implementation

Here, we present the methodology by which we computed a Bell parameter of $S = 2.80 \pm 0.07$, violating the classical Clauser-Horne-Shimony-Holt (CHSH) inequality $|S| \leq 2$ [5]. Typically, a Bell test entails varying two projective measurement angles, each between two different analyzer settings [6, 7]. In the microresonator system, however, the test may be implemented by analogously varying the difference in arrival time between signal and idler photons $\tau = t_s - t_i$, as long as the photon coherence times are much larger than the temporal resolution of the detectors [8].

We generate a Bell correlation coefficient $E(\tau)$ in terms of second-order correlation functions $G_{jk}^{(2)}(\tau)$, where j and k denote the signal and idler path, respectively. Let f indicate the forward path and b the backward path. $E(\tau)$ is then

$$E(\tau) = \frac{G_{ff}^{(2)}(\tau) + G_{bb}^{(2)}(\tau) - G_{fb}^{(2)}(\tau) - G_{bf}^{(2)}(\tau)}{G_{ff}^{(2)}(\tau) + G_{bb}^{(2)}(\tau) + G_{fb}^{(2)}(\tau) + G_{bf}^{(2)}(\tau)}, \quad (51)$$

where $G_{jk}^{(2)}(\tau)$ is proportional to $p(t_{sj}, t_{ik})$, given in Eq. 2 of the main text. This implies that the only time-dependence in $E(\tau)$ is in the form of sines and cosines of $\beta_m \tau$, where β_m is the modal coupling, and m refers to either the signal s or idler i . We note that the exponential time-dependence in $p(t_{sj}, t_{ik})$ cancels in $E(\tau)$.

Now we allow τ to vary as a function of two angles θ_a and θ_b . This will enable us to vary τ in a manner equivalent to the usual Bell test angle variation. To mimic the angle-dependence of the typical Bell test, $\tau(\theta_a, \theta_b)$ must have the form [8]

$$\tau(\theta_a, \theta_b) = \begin{cases} \frac{1}{\beta_i(\theta_a - \theta_b)} & \text{if } \theta_a - \theta_b < 0, \\ \frac{1}{\beta_s(\theta_a - \theta_b)} & \text{if } \theta_a - \theta_b \geq 0. \end{cases} \quad (52)$$

The Bell parameter can now be written as

$$S = |E(\tau(\theta_1, \theta_2)) - E(\tau(\theta_1, \theta'_2))| + |E(\tau(\theta'_1, \theta_2)) + E(\tau(\theta'_1, \theta'_2))|. \quad (53)$$

Here θ_1 and θ'_1 are the measurement settings for θ_a while θ_2 and θ'_2 are the measurement settings for θ_b . The settings are defined such that

$$\theta'_n = \theta_n - \pi/4 \quad (54)$$

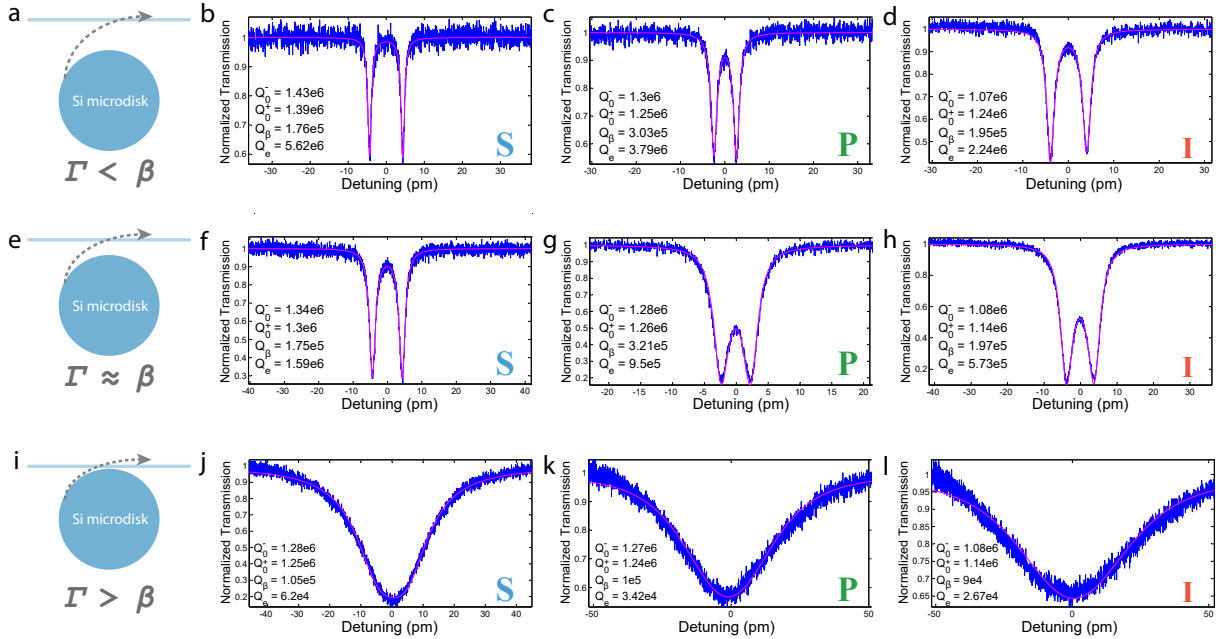
for $n = 1, 2$. We achieved our greatest violation of the CHSH inequality for $\theta_1 = 4.13$, $\theta'_1 = 3.34$, $\theta_2 = 0$, $\theta'_2 = -0.79$. The resulting Bell parameter is $S = 2.80$, well above the classical limit of 2 but still within the quantum bound of $2\sqrt{2}$.

To determine error in our Bell parameter, we simulated 2500 data sets under the assumption of Poissonian counting statistics, using the number of real counts in each 4 ps time bin as mean. For each simulated data set, the Bell parameter producing the largest CHSH violation was computed. The standard deviation of these simulated Bell parameters was 0.07, giving the result $S = 2.80 \pm 0.07$ that appears in the main text. We note that the values of β_m ($m = s, i$), appearing in Eq. 52, were measured using a calibrated

Mach-Zehnder interferometer (MZI). The uncertainty associated with these measurements was so small in comparison to the photon counting uncertainty that it had no bearing on the overall error of the Bell parameter.

Supplementary Note 6: Theory plots for different waveguide-microdisk coupling rates

In this section, we describe the method used to generate the theoretically predicted pair correlation waveforms that appear in Fig. 5 of the main text. In Fig. 5a-c of the main text, we plot the experimental response of the pair correlations under three coupling regimes, $\Gamma < \beta$, $\Gamma \approx \beta$, and $\Gamma > \beta$, respectively, along with their corresponding theory plots in Fig. 5d-f. Here, Γ and β represent the total photon decay rates and modal coupling rates, averaged between the signal and idler cavity modes.

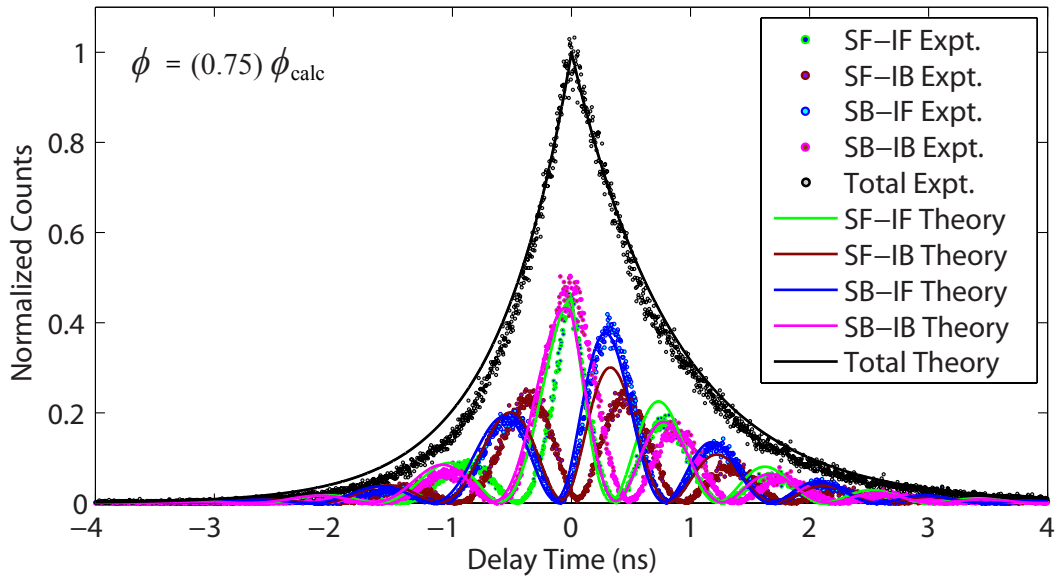


Supplementary Figure 4. Fits of the measured cavity transmission spectra for different coupling configurations. Optical fitting is used to determine the intrinsic photon decay rates ($\Gamma_0 = \omega_0/Q_0$), external coupling rates ($\Gamma_e = \omega_0/Q_e$), and modal coupling rates ($\beta = \omega_0/2Q_\beta$) for the three coupling regimes. (a) Illustration depicting the gap between the optical waveguide and microdisk in the $\Gamma < \beta$ case, along with corresponding fits of the signal (b), pump (c), and idler (d) transmission profiles. (e) Illustration depicting the waveguide-microdisk gap for the $\Gamma \approx \beta$ case, along with corresponding fits of the signal (f), pump (g), and idler (h) transmission profiles. (i) Illustration depicting the waveguide-microdisk gap for the $\Gamma > \beta$ case, along with corresponding fits of the signal (j), pump (k), and idler (l) transmission profiles.

The three coupling regimes are experimentally accessed by varying the waveguide-microdisk gap, as is schematically shown in Supp. Fig. 4. For each coupling case, we perform fits (shown in magenta) of the signal (s), pump (p), and idler (i) transmission profiles (shown in blue) in order to extract the intrinsic photon decay rates ($\Gamma_{0m} = \omega_{0m}/Q_{0m}$, where $m = p, s, i$), external coupling rates ($\Gamma_{em} = \omega_{0m}/Q_{em}$), and modal coupling rates ($\beta_m = \omega_{0m}/2Q_{\beta m}$). The measured cavity quantities are then inserted into Eq. 39-43, which defines all of the constants relevant to the pair correlation theory (see Eq. 35-38). By incorporating the measured cavity quantities and laser-cavity detuning into Eq. 49 & 50, we compute the relative phase and amplitude of the intracavity pump fields, such that the theory describing the pair correlations would be completely defined (no free parameters) with knowledge of the spatial phases associated with the orientation

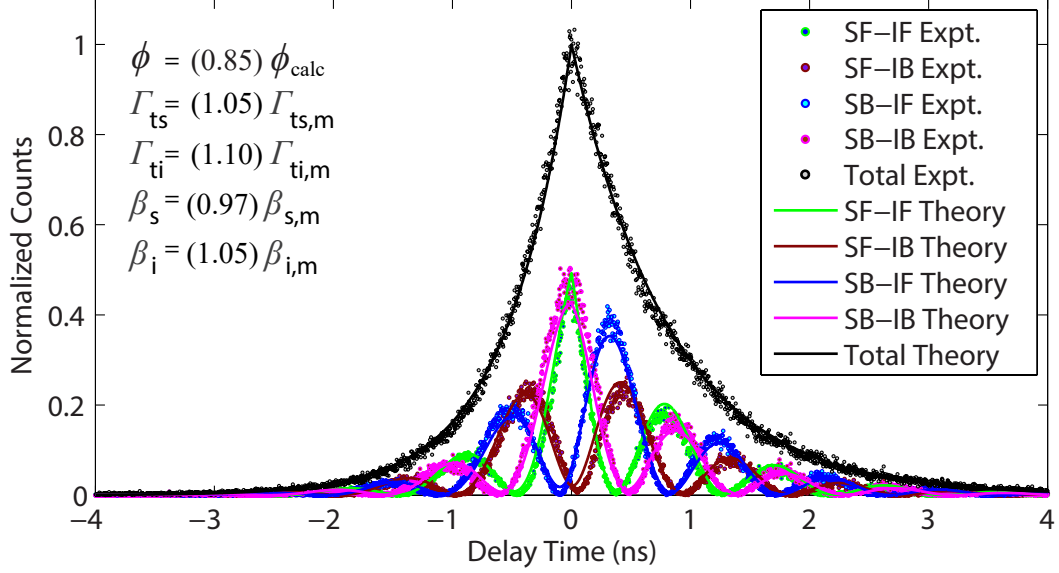
of the standing-wave mode patterns. As we do not possess the necessary experimental apparatus to measure the spatial phases that determine ϕ_β , we consequently leave it as a free parameter in our theory.

In Supp. Fig. 5, we directly compare the pair correlation theory (shown as solid lines) to the experimental results (shown as dots) that appear in Fig. 5a of the main text. The theoretical curves are generated by incorporating the measured photon decay and coupling properties of the microdisk (see Supp. Fig. 4b-d) into Eq. 35-38, and then varying the relative phase, ϕ . We start with the relative phase computed from the intracavity pump fields (Eq. 49 & 50), ϕ_{calc} , and then vary the phase until the best qualitative match among all the waveforms is reached. Here, it turns out that the best match results from using $\phi = 0.75\phi_{\text{calc}}$. Consequently, these are the theoretical curves presented in Fig. 5d of the main text, which provide a qualitative comparison to the experimental data contained in Fig. 5a ($\Gamma < \beta$ case).



Supplementary Figure 5. Comparison between pair correlation theory and experimental results, with phase as the only free parameter. The cross-correlation waveforms (without background subtraction), measured between signal forward - idler forward (SF-IF) (green dots), signal forward - idler backward (SF-IB) (red dots), signal backward - idler forward (SB-IF) (blue dots), and signal backward - idler backward (SB-IB) (magenta dots) are plotted, along with their corresponding theoretically predicted pair correlation waveforms (solid lines). Inset denotes adjustment from theoretically calculated relative phase between intracavity pump modes.

The theory curves in Supp. Fig. 5 capture the essential behavior of the experimental results and have reasonable agreement. However, we note that some deviation is to be expected, given that our theory lacks certain details. For instance, the theory assumes that all the doublets are symmetric, which is not exactly the case for our device (see Supp. Fig. 4). The doublet asymmetry implies that there will be some discrepancy between the intracavity amplitudes and phases when comparing against the perfectly symmetric case. Additionally, we have not included effects due to two-photon absorption (TPA)/free-carrier absorption (FCA), which may have a small contribution to the deviation seen in Supp. Fig. 5. In particular, TPA/FCA may explain why the theoretically predicted photon decay envelope (black solid line) in Supp. Fig. 5 is slightly broader than the corresponding experimental result (black dots). Given these non-idealities, we found it informative to test how good the match could be if certain measured parameters



Supplementary Figure 6. Comparison between the pair correlation theory and experimental results, with the best fit achieved by small adjustments to multiple cavity-dependent quantities. The cross-correlation waveforms (without background subtraction), measured between signal forward - idler forward (SF-IF) (green dots), signal forward - idler backward (SF-IB) (red dots), signal backward - idler forward (SB-IF) (blue dots), and signal backward - idler backward (SB-IB) (magenta dots) are plotted, along with their corresponding theoretically predicted pair correlation waveforms (solid lines). Inset denotes adjustments from theoretically calculated relative phase between intracavity pump modes and measured (m) cavity properties.

were allowed to be tuned by a small amount in the theory. In doing so, we have produced the theoretical curves shown in Supp. Fig. 6, where the changes to measured values are listed in the inset. Here, the agreement between theory and experiment is extremely good, further strengthening the assertion that our theory describes the predominant physics of the system. However, we stress that these theoretical curves *have not* been included in the main text, because they incorporate small changes to measured values.

Thus far, we have described the manner in which the theory plots were obtained for the $\Gamma < \beta$ case. The same procedure was applied to the $\Gamma \approx \beta$ and $\Gamma > \beta$ cases, for which we used the measured values from Supp. Fig. 4f-h and Supp. Fig. 4j-l, respectively. Additionally, in both of these cases the relative phase values that gave the best match between theory and experiment were ϕ_{calc} , the value computed directly from the intracavity pump fields (Eq. 49 & 50), without any additional tuning.

Supplementary Note 7: Balanced optical energy between intracavity pump modes

In this section we consider the implications of balancing the optical energy stored between the counter-propagating intracavity pump modes. In general, the intracavity pump modes will have unique values of stored optical energy ($f \neq b$, where $f \equiv |a_{\text{pf}}(\Delta)|^2$ and $b \equiv |a_{\text{pb}}(\Delta)|^2$), as determined by the photon decay rates and modal coupling rates of the cavity, along with the laser-cavity detuning (see Eq. 49 & 50). This is reflected in the pair correlation theory (see Eq. 35-38) by allowing f and b to assume any arbitrary values. However, in the special case that $f = b$, and after some minor algebra, the pair correlation functions become,

$$p_{c,ff}(\tau) = \begin{cases} |f|^2 N e^{\Gamma_{\text{ti}}\tau} |\{c_0 e^{-i\phi} - c_1\} \cos(\beta_i \tau) + \{c_2 e^{-i\phi} + c_3\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ |f|^2 N e^{-\Gamma_{\text{ts}}\tau} |\{c_0 e^{-i\phi} - c_1\} \cos(\beta_s \tau) - \{c_3 e^{-i\phi} + c_2\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (55)$$

$$p_{c,fb}(\tau) = \begin{cases} |f|^2 N e^{\Gamma_{\text{ti}}\tau} |\{c_2 e^{-i\phi} + c_3\} \cos(\beta_i \tau) - \{c_0 e^{-i\phi} - c_1\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ |f|^2 N e^{-\Gamma_{\text{ts}}\tau} |\{c_2 e^{-i\phi} + c_3\} \cos(\beta_s \tau) - \{c_1 e^{-i\phi} - c_0\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (56)$$

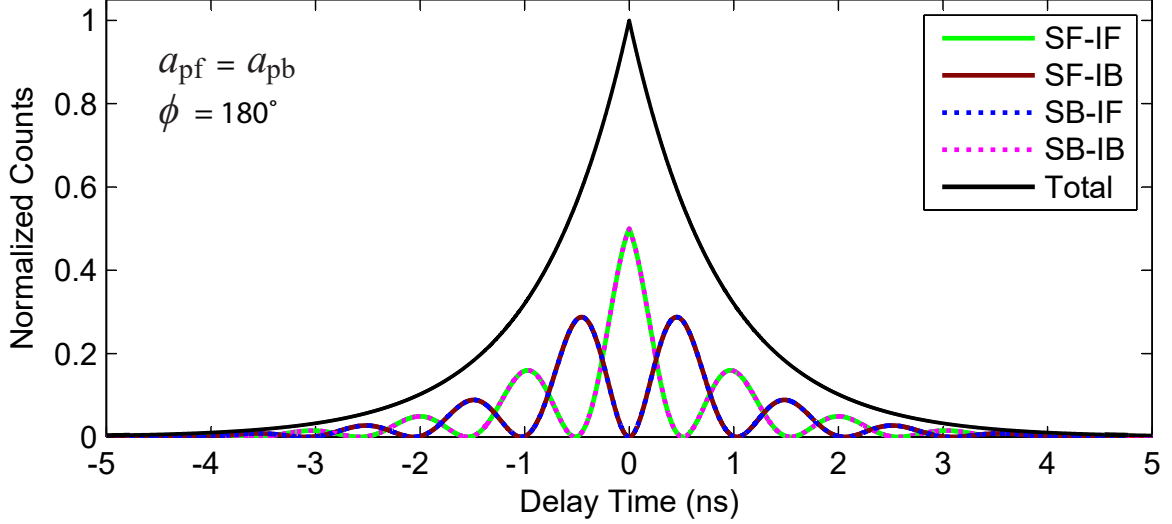
$$p_{c,bf}(\tau) = \begin{cases} |f|^2 N e^{\Gamma_{\text{ti}}\tau} |\{c_2 e^{-i\phi} + c_3\} \cos(\beta_i \tau) - \{c_0 e^{-i\phi} - c_1\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ |f|^2 N e^{-\Gamma_{\text{ts}}\tau} |\{c_2 e^{-i\phi} + c_3\} \cos(\beta_s \tau) - \{c_1 e^{-i\phi} - c_0\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (57)$$

$$p_{c,bb}(\tau) = \begin{cases} |f|^2 N e^{\Gamma_{\text{ti}}\tau} |\{c_0 e^{-i\phi} - c_1\} \cos(\beta_i \tau) + \{c_2 e^{-i\phi} + c_3\} \sin(\beta_i \tau)|^2 & (\tau < 0), \\ |f|^2 N e^{-\Gamma_{\text{ts}}\tau} |\{c_0 e^{i\phi} - c_1 b\} \cos(\beta_s \tau) - \{c_3 e^{-i\phi} + c_2\} \sin(\beta_s \tau)|^2 & (\tau \geq 0), \end{cases} \quad (58)$$

where we have made use of the identity $|n_1 \pm n_2 e^{i\phi}|^2 = |n_1 \pm n_2 e^{-i\phi}|^2$ (n_1 and n_2 are constants). Thus, we see that when the optical energy stored between the intracavity pump modes is exactly balanced, the pair correlation waveforms from the same path classification (co-propagating vs. counter-propagating) become indistinguishable. A plot of the theory describing this phenomenon may be seen in Supp. Fig. 7, where we have set the photon decay rates and modal coupling rates between the pump, signal and idler cavity modes to be equivalent.

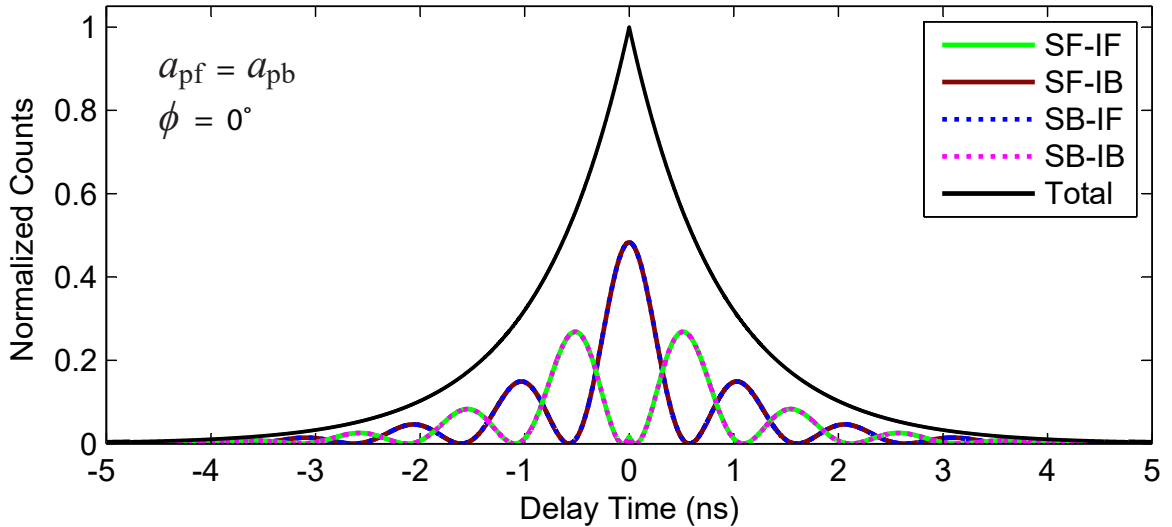
Here, we clearly see that the pair correlations for the forward-forward (SF-IF) and backward-backward (SB-IB) co-propagating path configurations are mutually indistinguishable, as well as the pair correlations associated with the forward-backward (SF-IB) and backward-forward (SB-IF) counter-propagating path configurations. As mentioned in the main text, this phenomenon is a consequence of the system possessing an internal mirror symmetry (with respect to the photon generation processes) upon exact balancing of the intracavity pump energies.

In Supp. Fig. 7, we see that the correlations belonging to the co-propagating state classification are maximally correlated at zero delay-time, whereas the opposite occurs for the correlations associated with the counter-propagating state classification. As discussed in the main text, by varying the relative pump phase, we can control the quantum interference within the device, such that this behavior is flipped between state classifications. With $\phi = 0^\circ$, as shown in Supp. Fig. 8, the correlations waveforms from the



Supplementary Figure 7. Theoretical prediction of pair correlation waveforms with balanced intracavity pump energies and $\phi = 180^\circ$. For this special case, the correlation waveforms from the same path classification (co-propagating vs. counter-propagating) become indistinguishable. For simplicity, we have chosen to set the photon decay rates and modal coupling rates for the pump, signal, and idler cavity modes to be the same.

same state classification are now perfectly transposed in comparison to Supp. Fig. 7. We note that the small hump appearing in the correlation waveforms corresponding to the co-propagating states (see Supp. Fig. 8) is a consequence of the coupling vs total photon decay rate. This feature can be made arbitrarily small by increasing the rate of coupling compared to the rate of decay (as we have verified in theoretical investigations not presented here). In fact, the presence or absence of this feature is a manifestation of the

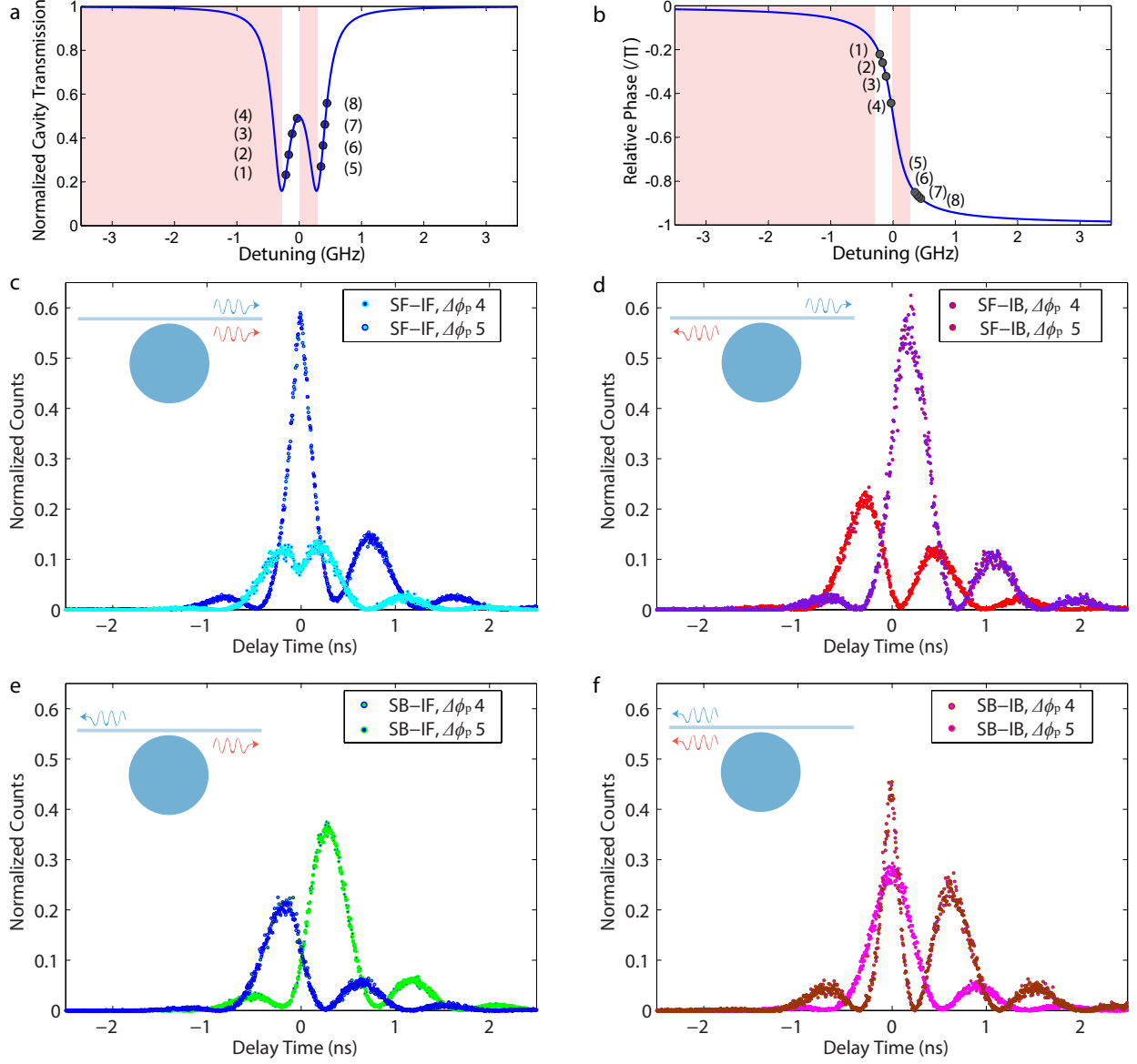


Supplementary Figure 8. Theoretical prediction of pair correlation waveforms with intracavity pump energies and $\phi = 0^\circ$. The pair correlations are flipped with respect to the $\phi = 180^\circ$ case. For simplicity, we have chosen to set the photon decay rates and modal coupling rates for the pump, signal, and idler cavity modes to be the same.

distinguishability between alternative two-photon pathways within the system. When the coupling rate is comparable to the photon decay rate, we gain some knowledge about whether a photon originally came from the forward or backward traveling mode. However, when the coupling rate is much greater than the photon decay rate, then the ‘which-path’ information is totally removed from the system, allowing for perfect interference visibility and complete removal of the hump.

Supplementary Note 8: Response of pair correlations to laser-cavity detuning around resonance

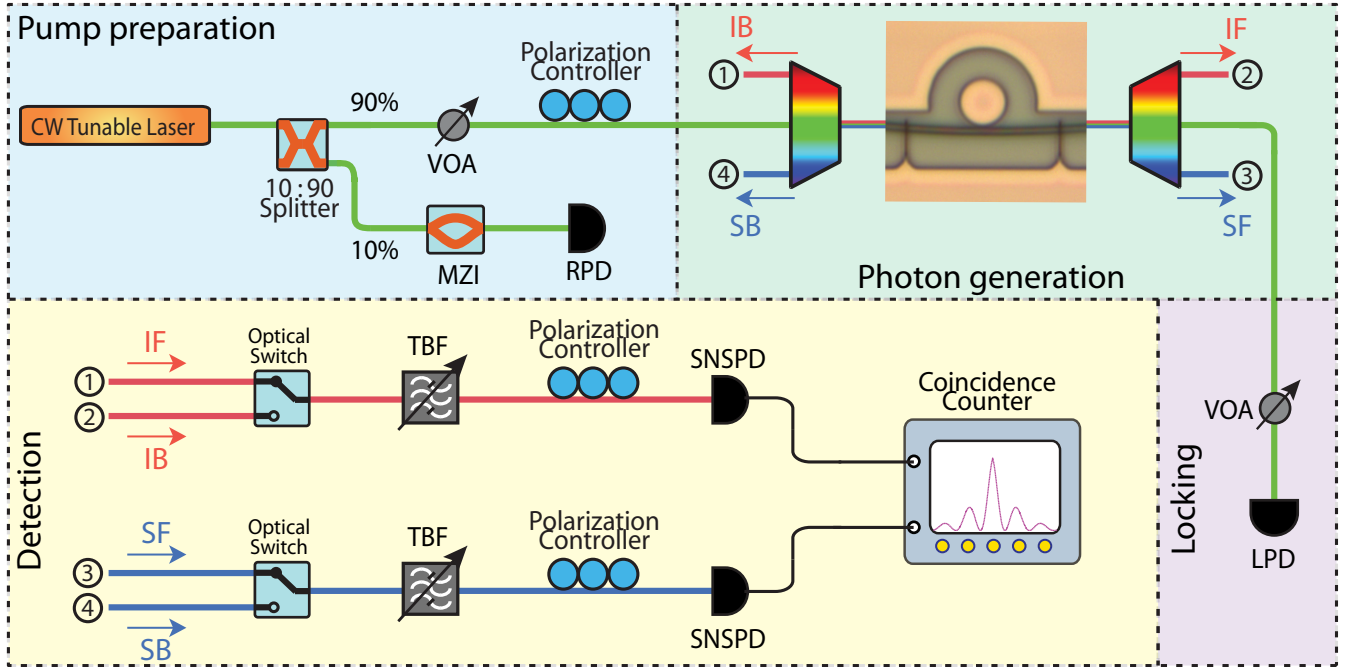
In this section, we provide detailed versions of a subset of the pair correlations appearing in Fig. 6 of the main text. The experimental results in that figure were obtained by varying the laser-cavity detuning about resonance (as indicated by the laser locking points in Supp. Fig. 9a, which was copied from Fig. 6a in the main text), which induces a large relative phase shift between the counter-propagating intracavity pump modes, as shown in Supp. Fig. 9b (which was copied from Fig. 6b of the main text). Although all eight points are important in characterizing the pump-induced interference phenomena, the most pronounced changes occur between the detuning values labeled as (4) and (5) (see Supp. Fig. 9a,b), because this transition includes tuning through the resonance frequency of the pump cavity mode. Here, we provide detailed versions of the response of the pair correlations for each path configuration as the carrier frequency of the pump laser is swept from negative to positive detuning.



Supplementary Figure 9. Response of the pair correlations to laser-cavity detuning across the pump resonance. Theoretical plots of (a) the pump transmission profile and (b) relative phase between counter-propagating pump modes with the labels, (1)-(8) denoting the detuning values $\Delta/2\pi = (-0.21, -0.16, -0.11, -0.06, 0.35, 0.38, 0.41, 0.44)$ GHz, respectively. The light-red shading indicates the unstable locking regions. Cross-correlation waveforms (without background subtraction), measured between (c) signal forward - idler forward (SF-IF), (d) signal forward - idler backward (SF-IB), (e) signal backward - idler forward (SB-IF), and (f) signal backward - idler backward (SB-IB) for detuning values (4) and (5) only. As seen in (b), the transition from (4) to (5) tunes the system through resonance and consequently induces the largest phase shift. Insets depict path configuration.

Supplementary Note 9: Experimental setup

In this section, we provide a schematic of the experimental setup (see Supp. Fig. 10) used to obtain all of the results appearing in the main text. A discussion of the experimental techniques and procedures may be found in the Methods section of the main text; an abridged version of which may be found in the caption of Supp. Fig. 10.



Supplementary Figure 10. Schematic of the experimental setup used to obtain all results presented in the main text. A forward-propagating pump wave is launched into optical fiber and is split using a 10/90 directional coupler. A small portion (10%) of the pump light is passed through a Mach-Zehnder interferometer (MZI) and detected with a fast reference photodiode (RPD) in order to accurately calibrate the fitting of optical resonances. The primary portion (90%) of the pump light passes through a variable optical attenuator (VOA) to control the input optical power, a fiber polarization controller (FPC) to align the polarization state to that of the quasi-transverse magnetic mode family within the microdisk, and a course-wavelength division multiplexing (CWDM) multiplexer (MUX) before evanescently coupling into the microdisk. The transmitted pump light is separated from the single photon channels by a demultiplexer (DEMUX) and detected with a fast locking photodiode (LPD), in order to lock the laser to the cavity. Within the microdisk, signal (s) and idler (i) photon pairs are created between the coherently coupled forward (f) and backward (b) traveling-wave modes via spontaneous four-wave mixing, such that four single photon pathways are established: signal-forward (SF), signal-backward (SB), idler-forward (IF), and idler-backward (IB). The four single photon pathways are separated by demultiplexing and sent to optical switches. Photons exiting the switches are then passed through narrowband tunable bandpass filters (TBF) which remove the Raman noise generated in the delivery optical fiber. The photon pairs are ultimately detected using superconducting nanowire single photon detectors (SNSPD) with state-of-the-art timing resolution. The photon arrivals are then processed with a coincidence counter in the time-tagged configuration, such that the absolute arrival time of every photon detection event is recorded.

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