

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

Report on ``Non-hermitian topology: a unifying framework for the Andreev versus Majorana states controversy'

by

J. Avila, F. Penaranda, E. Prada, P. San-Jose, and R. Aguado.

This communication studies the appearance of quasi-like Majorana zero modes in open systems such as a superconductor with spin orbit (SO) coupling terminated by a junction with a normal metal with SO coupling in the helical phase at one end. The coupling of the superconductor to the normal lead generates a finite lifetime of the lowest energy subgap states, which become quasistates.

Using Green's function or equivalently the scattering matrix formalism, it is shown that Andreev states acquire a finite lifetime encoded in the imaginary part of the self-energy. Due to the particle-hole symmetry of the Bogolubov de Gennes equations, the eigenvalues have either non-zero real part and the same imaginary part or are both with zero real part and equal (at exceptional points only) or different imaginary parts.

After the bifurcation following

the crossing an exceptional point,
one of the eigenvalues with zero real part can migrate
towards the real axis and thus have a very small negative imaginary part,
thus mimicking a Majorana state.

These states are dubbed dark Majorana states (DMS) by the authors.

This generalizes the classification of topological versus
non-topological (trivial) in systems with non-zero coupling
to the leads (open systems). Relationships
with usual topological band theory are explained in the paper.

The authors then provide a detailed study of the zero bias peak in
the dI/dV curves, showing that a peak can be obtained even when
the superconductor is still not in a ``topological' phase.

While the scientific material presented here looks of
interest and seems right, a few objections can be raised.

1) Novelty of the results presented here as compared
to a former paper in Scientific Reports

First, the study appears to me to be somewhat redundant
with a more detailed paper by the same group,
published in 2016, entitled
``Majorana bound states from exceptional points
in non-topological superconductors'

by Pablo San-Jose, Jorge Cayao, Elsa Prada, and Ramon Aguado,
which appeared in Scientific Reports, article number 21427.

The idea developed in the manuscript by itself is not new,
as it was already present in references 22 and 23.

Ref. 22: Pikulin and Nazarov, JETP Letters 94, 693 (2012)),

Ref. 23: Pikulin And Nazarov, Phys. Rev. B 87, 235421 (2013).

In this communication, the authors focus on the importance
of the exceptional points.

In the former paper (Scientific reports) they focussed more
on showing that the dark Majorana states share respond the same way
to usual probes as conventional Majorana bound states. Namely,
the dI/dV curves are the same, so are the shapes of the wave-functions
(which can be probed by STM spectroscopy). The
 4π periodicity of the Josephson effect, and even braiding
of Majoranas can be also be achieved both by Majorana dark states
and convential Majorana bound states.

2) The authors study non-equilibrium phenomena
in an open system (coupled to leads).

Why did not they use the Nambu Keldysh formalism?

An important parameter is the coupling to the normal region (one lead).
It interpolates between closed system

(when coupling is absent or extremely small)

and an almost transparent junction.

The authors use regular Green's functions in frequency (ω) space

(Real part of frequency and imaginary part).

Generally, the formalism used to describe adequately non-equilibrium situations is the Keldysh formalism.

For example, let us take the overly simple example of Andreev reflection between a normal lead and a regular BCS superconductor.

For calculating the curve I (current) versus voltage bias V , the Keldysh formalism has to be used.

If we want to retrieve the fact that at voltage bias equal to the gap Δ , the Andreev reflection coefficient is of modulus 1, irrespective of the junction transparency (Blonder Tinkham Klapwijk), Keldysh Green-function have to be used.

In this case, to retrieve the right result, resumming all orders of perturbation theory in the hopping between the normal metal and the superconducting or calculating the self energy $\Sigma(\omega)$ due to hopping has to be performed in Keldysh.

I am sure that the authors are well aware of this; a reference using Keldysh is for example

A. Martin-Rodero, A. Levy Yeyati, and J. C. Cuevas, Phys. Rev. Lett. 80, 1066 (1998).

However, I am still confident that this will probably have no

serious impact on the physical consequences the authors draw from their calculations using simply the Green functions in complex frequency (ω) space.

Failure to use the Keldysh formalism

is generally harmful only when short time scales are involved, like in the response to high frequency probes or when looking at ultrafast phenomena.

3) The authors do not show the shape of wavefunctions at exceptional points

At exceptional points, the effective non-Hermitian Hamiltonian is no longer diagonalizable.

The decay of at least one of the Majorana like pseudo-eigenvectors of the effective non-Hermitian Hamiltonian H_{eff} is expected to be anomalous (not purely exponential even if the imaginary part Γ of the two pseudo-eigenvectors is still sizeable).

It might have been interesting to show this but maybe it is the authors's choice not to do so.

Granted, the interest comes what happens not right at the bifurcation which takes place at an exceptional point but after the bifurcation (not right after).

One eigenstate will get an almost zero imaginary part (dark MBS).

4) Minor remarks

There are very few misprints in the paper and also in the supplementary material.

They ought to be corrected.

List of misprints:

Main text:

Page 4:

``Therefore, both cases ($B > B_c$, uniform wire and $B < B_c$, ...)

The second index in B_c does not appear as an index as it should.

Page 5, at the beginning

``the typical behaviour of the dI/dV curves for the dot...'

We feel that the word ``curves' might be missing.

Supplemental information:

Page 3, left column, in the caption of Fig. 3,

last line of the caption:

``topological criterion for open NS nanowire junction'.

(instead of ``por').

Reviewer #2 (Remarks to the Author):

The paper by Avila et al. reports on a theoretical study of so called Non-Hermitian Topology with the aim of classifying Majorana nanowires in an open quantum system. An important question of current debate is the nature of observed zero bias peaks in transport by coupling such proximitized nanowires subjected to a Zeeman field to normal metal leads. One sign of Majorana bound states (MBS) (or zero modes) is an exactly quantized robust conductance resonance of $2e^2/h$ at zero bias and zero temperature. In an isolated wire, such MBS are a consequence of topology of the Hamiltonian.

Experiments have observed such resonances but such a feature could also be a sign of different physics. In this work, the authors discuss the equivalence of a topological characterization in open quantum systems that takes into account the coupling to the leads in terms of an effective Hamiltonian that is defined via the retarded Green's function of the system. The eigenvalues of this effective Hamiltonian are complex and can show the phenomenon of exceptional points (EPs). Usual Andreev bound states can always be decomposed into two MBS. In the common case, the eigenvalues associated with them come in pairs and have the same imaginary part. When the coupling of the two MBS to the leads becomes asymmetric, the real part of the energy can become zero and the imaginary parts of the two eigenvalues different. These two topologically different scenarios are connected via EPs. The authors define certain topological invariants that change over this EPs and distinguish a topological trivial from a topological non-trivial regime. The main point of the article is to examine two situations, a quantum dot and a smooth variation of the chemical potential and the superconducting gap towards the wire end. The authors show that the wire can become topologically non-trivial before the "isolated" bulk topological point as a function of the Zeeman splitting in the open quantum system sense of topology for the latter case of a smooth variation of parameters. One of the main conclusions is that the transport features (dI/dV) of the non-trivial bulk topological system and the non-trivial topological system in the open quantum system approach are indistinguishable (based on the effective Hamiltonian) which is calculated in detail within a discretized model for the nanowire and the BTK-formalism.

The work is nicely explained and illustrated with several figures and has a clear message and is certainly very relevant for the ongoing discussion about the nature of observed transport data and how they are related to MBS. Non-Hermitian Topology is in a wider sense a very active topic that recently got a lot of attention as it is also written in the introduction. I therefore would recommend the paper for publication under the condition that the authors can address satisfactorily the comments below.

One comment I have is that some of the authors have already published a paper about EPs and its consequences to stabilize MBS in the open system sense in a (bulk) topological trivial system. Since the authors do not cite that paper I wonder if it is not relevant to some of the presented work in the current manuscript? However, I see that the focus and the system under consideration is different from the already published paper. But I would like to ask the authors to comment on this point and maybe add also a small statement in the manuscript.

I have a few other suggestions:

- In Eq. (2) two Majorana wave functions "L" and "R" are introduced. What stands "L" and "R" for? Can we associate these components with wave functions shown in Fig. 1 (b) and "quasi-bound Majorana zero modes" in Fig. 1a)? I think the connection is not clear.
- About the same topic: In the Supplemental Information the Majorana wave functions have (as it should be) spin components (Eq. (4)). Later (Eq. (5)) and also in Eq. (2) in the main text, they disappear without explanation? What is the connection here?
- In the caption to Fig. 3, " $\nu > 1$ " should be " $\nu > 0$ " ?
- The transport calculation is only mentioned very briefly in the "Methods" section. It is not clear at all how the nanowire is coupled to the normal lead. In the supplemental material it is only mention that it happens via a self-energy. However, without a Hamiltonian for the coupling and the lead, you cannot calculate it. So what is the coupling (tunneling Hamiltonian) and the Hamiltonian for the leads? Even if this might be standard for the experts, it is not for the general reader. Besides, there must be some

numbers used for the coupling between lead and nanowire in order to calculate plots like in Fig. 2 (f)-(j). What are they?

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Answer: We thank the referee for his/her positive assessment of the paper. All the referee's objections are addressed below.

1) Novelty of the results presented here as compared to a former paper in Scientific Reports. First, the study appears to me to be somewhat redundant with a more detailed paper by the same group, published in 2016, entitled ``Majorana bound states from exceptional points in non-topological superconductors' by Pablo San-Jose, Jorge Cayao, Elsa Prada, and Ramon Aguado, which appeared in Scientific Reports, article number 21427.

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Answer: We thank the referee for asking this important question about the relation between our previous paper [Scientific Reports 6, 21427, 2016] as well as Pikulin&Nazarov's works [JETP Letters 94, 693, 2012, and Phys. Rev. B 87, 235421, 2013] with the present manuscript. The referee is right that we did a poor job in stressing the differences and his/her comment has helped us to explain the generality and novelty of our paper with respect to those previous works.

As the referee correctly notes, the idea of pole bifurcation was already present in Pikulin & Nazarov papers. Importantly, pole bifurcations in these papers were discussed for the standard Lutchyn-Oreg wire model (namely for nanowires with uniform chemical potentials) but not for smoothly inhomogeneous (spatially varying) chemical potentials, which is the experimentally relevant case that we discuss now. More crucially, **the important connection of pole bifurcation with Exceptional Point bifurcations and non-Hermitian topology was not understood (or at least not discussed) whatsoever in those papers**. In any case, we acknowledge these previous works and cite both papers (Refs. 22&23).

Concerning our own paper in Scientific Reports, there we presented a first example where exceptional points bifurcations help to stabilise trivial parity crossings. However, it dealt with spin-polarised reservoirs and did not include smoothly inhomogeneous chemical potentials, a key ingredient that is now believed to be a dominant factor in current experimental Majorana nanowires. Therefore, **our previous work was of less generality**. In the present manuscript, we explore the relevant scenario of inhomogeneous potentials, and also note that the reservoir is not (need not be) spin polarised, see the scheme in the supplemental material Fig. 1b). **More importantly, the full general analysis of non-hermitian topology and of the mathematical conditions for the existence of exceptional points are completely new**. This includes the full discussion of the low-energy model in Eq. 2; the sign of the dimensional parameter ν as a measure of nontrivial non-hermitian topology; the decay asymmetry γ/Γ as an experimentally accesible measure of the degree of Majorana decoupling; and the connection between the low energy model and the microscopic model through Eq. 3. This full analysis is completely novel and general (**irrespective of the microscopic mechanism leading to coupling asymmetry**). This is explicitly demonstrated with two examples: the homogenous (standard) Lutchyn-Oreg case (Fig.2) and the inhomogeneous wire case and its comparison against a trivial parity crossing (Fig.3). The new version also contains a new figure 4 that discusses the spatial shape of eigenstates for various B fields and, importantly, eigenstate coalescence at the EP (after a question from referee 2). To the best of our knowledge, this full analysis of experimentally relevant cases in proximitized nanowires coupled to reservoirs is completely new. **For further clarity, we have now explicitly mentioned the differences with our previous Scientific Reports paper about spin-polarised leads in a new paragraph towards the end of the paper.**

2) The authors study non-equilibrium phenomena in an open system (coupled to leads). Why did not they use the Nambu Keldysh formalism?

An important parameter is the coupling to the normal region (one lead). It interpolates between closed system (when coupling is absent or extremely small) and an almost transparent junction. The authors use regular Green's functions in frequency (ω) space (Real part of frequency and imaginary part). Generally, the formalism used to describe adequately non-equilibrium situations is the Keldysh formalism.

For example, let us take the overly simple example of Andreev reflection between a normal lead and a regular BCS superconductor. For calculating the curve I (current) versus voltage bias V , the Keldysh formalism has to be used. If we want to retrieve the fact that at voltage bias equal to the gap Δ , the Andreev reflection coefficient is of modulus 1, irrespective of the junction transparency (Blonder Tinkham Klapwijk), Keldysh Green-function have to be used.

In this case, to retrieve the right result, resumming all orders of perturbation theory in the hopping between the normal metal and the superconducting or calculating the self energy $\Sigma(\omega)$ due to hopping has to be performed in Keldysh.

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However, I am still confident that this will probably have no serious impact on the physical consequences the authors draw from their calculations using simply the Green functions in complex frequency (ω) space.

Failure to use the Keldysh formalism is generally harmful only when short time scales are involved, like in the response to high frequency probes or when looking at ultrafast phenomena.

Answer: We thank the referee for asking this question. Indeed, we are well aware of the non-equilibrium Keldysh formalism for dealing with transport in superconducting settings (see, for example, our paper calculating the MAR current in topological Josephson junctions in New J. Phys. 15, 075019, 2013). The referee is right that using Keldysh is of paramount importance when dealing with either high voltages and/or high frequencies. Note, however, that our study focuses on low-frequency/low-voltage properties of the junction (subgap states below the gap) and on the emergence of zero energy modes in the tunneling conductance. For describing this kind of physics the BTK formalism is equivalent to the Keldysh-Nambu technique. We have included a short paragraph in the Methods section that explain in some more detail our transport calculations and included a citation to Cuevas et al.

3) The authors do not show the shape of wavefunctions at exceptional points

At exceptional points, the effective non-Hermitian Hamiltonian is no longer diagonalizable. The decay of at least one of the Majorana like pseudo-eigenvectors of the effective non-Hermitian Hamiltonian H_{eff} is expected to be anomalous (not purely exponential even if the imaginary part Γ of the two pseudo-eigenvectors is still sizeable). It might have been interesting to show this but maybe it is the authors's choice not to do so.

Answer: We thank the referee for this insightful question. Indeed, right at the exceptional point bifurcation the effective Hamiltonian shows a very characteristic behavior where **both the eigenvalues and eigenvectors coalesce. This implies, as the referee notes, that the non-Hermitian Hamiltonian is defective (i.e. no longer diagonalizable)**. Following the referee suggestion, we have now included a new subsection about the orthogonality of the wave functions of the lowest energy eigenstates $\psi_+(x)$ and $\psi_-(x)$. This includes

a new figure (Fig.4) which shows how the inner product $\langle \psi_+ | \psi_- \rangle$ (Fig.4a) becomes 1 at the EP (namely the eigenvectors become parallel at the EP) for both the uniform and non-uniform wire cases, since the non-Hermitian hamiltonian is defective, as the referee mentions. Again, this is a direct proof that the EP bifurcation is the universal mechanism behind the formation of Majorana zero modes in proximitized nanowires irrespective of microscopic details and without the need of a bulk topological transition (a priori, coalescence of eigenstates is by no means trivial given the amount of details/parameters of the microscopic models studied in the paper). We also show the spatial shape of the wave functions at different magnetic fields before (Fig.4b) , at (Fig.4c) and after (Fig.4d) the EP. Concerning the anomalous time dependence mentioned by the referee, the study of dynamics is out of the scope of this manuscript but we have included a short paragraph about time dependence with polynomial terms beyond exponential due to coalescence at EPs.

Granted, the interest comes what happens not right at the bifurcation which takes place at an exceptional point but after the bifurcation (not right after).
One eigenstate will get an almost zero imaginary part (dark MBS).

Answer: We fully agree with the referee.

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There are very few misprints in the paper and also in the supplementary material.
They ought to be corrected.

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last line of the caption:

``topological criterion for open NS nanowire junction'.

(instead of ``por').

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Answer: we thank the referee for this question and refer him/her to our answer to Referee 1 about the same question. Also, we would like to stress that a citation to our Scientific Reports paper was included in the first versions of the manuscript (see e.g. Ref. [41] In our arXiv version <https://arxiv.org/pdf/1807.04677.pdf>) but was subsequently and inadvertently removed at some point during editing. We have now restored this citation and added a new paragraph.

I have a few other suggestions:

- In Eq. (2) two Majorana wave functions "L" and "R" are introduced. What stands "L" and "R" for? Can we associate these components with wave functions shown in Fig. 1 (b) and "quasi-bound Majorana zero modes" in Fig. 1a)? I think the connection is not clear.

Answer: we thank the referee for this comment. We have added a new sentence to clarify the notation. Indeed, L and R refer to the left and right Majorana components of the wave function.

- About the same topic: In the Supplemental Information the Majorana wave functions have (as it should be) spin components (Eq. (4)). Later (Eq. (5)) and also in Eq. (2) in the main text, they disappear without explanation? What is the connection here?

Answer: we thank the referee for this comment which has helped us to clarify the notation. We have completely rewritten these parts (both in the main text and in the supplemental). Hopefully, the notation of Majorana components, their spin components, etc is now clear. The spinorial character of the wave function is now evident in the new notation with bold characters.

- In the caption to Fig. 3, " $\nu > 1$ " should be " $\nu > 0$ " ?

Answer: The referee is right, we have corrected this misprint.

- The transport calculation is only mentioned very briefly in the "Methods" section. It is not clear at all how the nanowire is coupled to the normal lead. In the supplemental material it is only mention that it happens via a self-energy. However, without a Hamiltonian for the coupling and the lead, you cannot calculate it. So what is the coupling (tunneling Hamiltonian) and the Hamiltonian for the leads? Even if this might be standard for the experts, it is not for the general reader. Besides, there must be some numbers used for the coupling between lead and nanowire in order to calculate plots like in Fig. 2 (f)-(j). What are they?

Answer: we thank the referee for this comment, which has helped us to clarify this point about our calculations. We have now extended the method sections to discuss in some more detail how we calculate transport across the NS junction. Details about the coupling to the reservoir can be found in the methods section. As we explain, to compute the self-energy in the transport calculations we indeed use a microscopic Hamiltonian for the infinite lead (which acts as a reservoir) and the contact. The lead is a high-density nanowire, with a much greater μ than the system nanowire. The coupling to the the lead is controlled by a hopping τ . The combination $\tau^2 \rho \sum_{\alpha \in \{L,R\}} |u_{\alpha}(0)|^2$ gives the left/right couplings $\Gamma_{0_{\{L,R\}}}$, with ρ the lead density of states and a the lattice parameter. Only this quantity enters linear transport. Concerning specific parameters in the calculation, we use typical numbers relevant for experiments performed with nanowires. As can be seen in Fig. 5, typical transport widths are of the order $\Gamma_0 \sim 50\text{-}100$ mK, always in the limit $\Gamma_0 \gg k_B T$. We have included this information in the caption of Fig.5.

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

In my opinion, the authors have answered the points raised by referees in a satisfactory fashion and I would recommend the new version for publication.

Reviewer #2 (Remarks to the Author):

This is my second report on the manuscript. The authors have addressed all points of my first report satisfactorily. I therefore can recommend now the manuscript for publication.

I have one technical remark:

In the "Methods" section, the position basis Nambu spinor $\{\hat{\Psi}(x)\}$ on page 8, above Eq. (7) is not consistent with the Hamiltonian in Eq. (6), where the pair potential Δ comes with the τ_x Pauli matrix. In the Nambu basis used, Δ should be $-i\sigma_y\Delta\tau_z$, or I might misunderstand something.