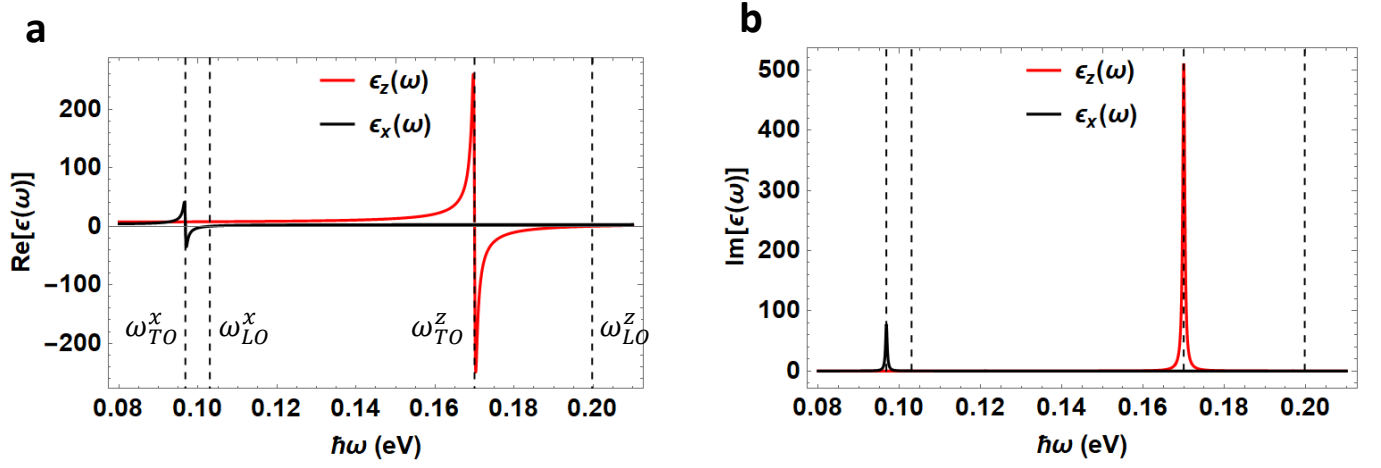
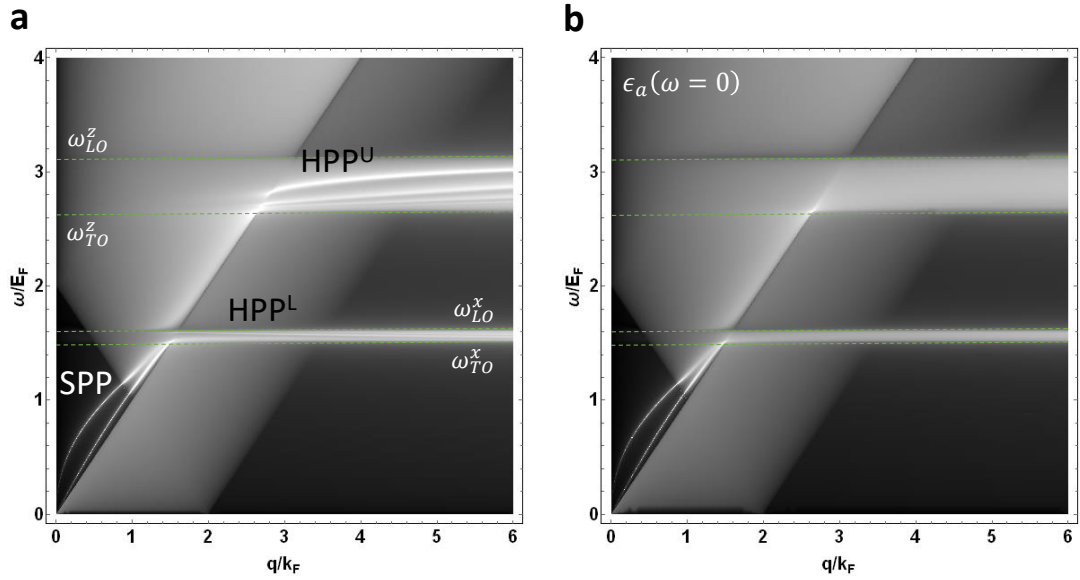
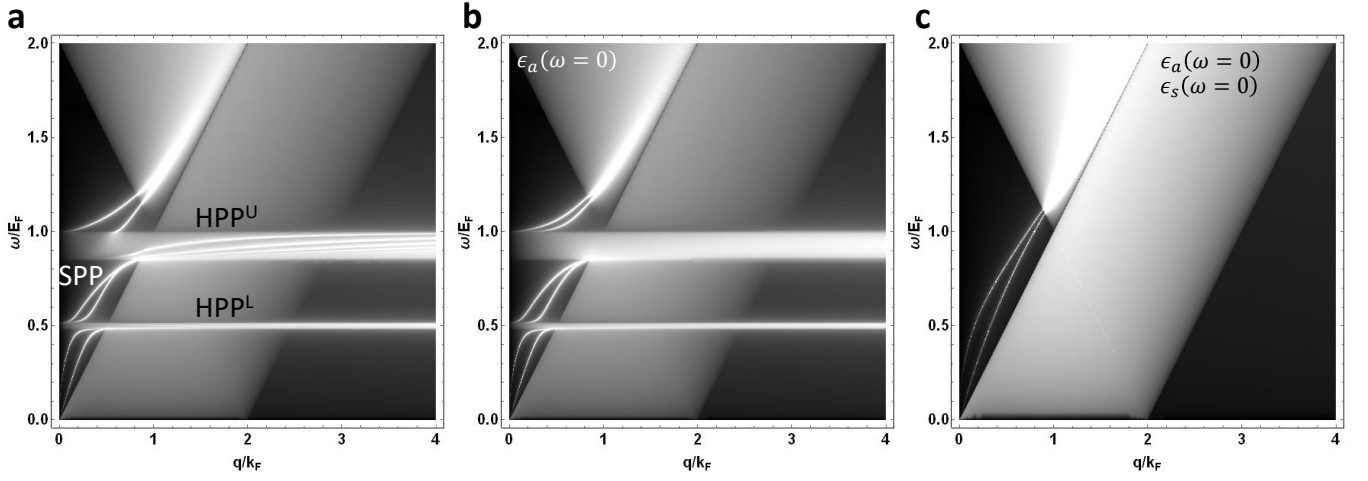




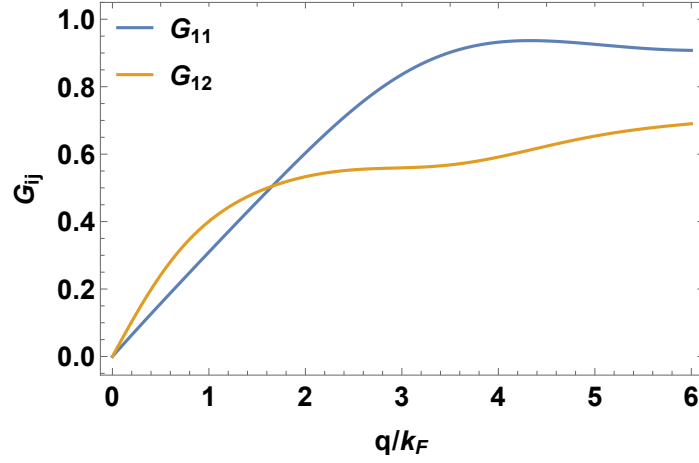
Supplementary Figure 1: (a) Real and (b) imaginary part of the h-BN dielectric function, where $\epsilon_{x(z)}(\omega)$ is the in- (out-of-) plane relative permittivity. The vertical dashed lines $\omega_{TO(LO)}^{x(z)}$ are the in- (out-of-) plane transverse (longitudinal) optical phonon frequencies of h-BN.



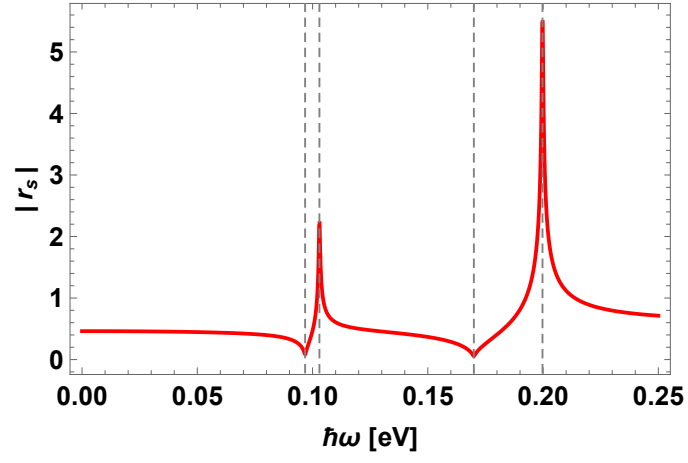
Supplementary Figure 2: Density plot of the loss function, $L(\mathbf{q}, \omega, T)$, calculated for the h-BN/G/h-BN/G/h-BN system at $T = 0$ K for $E_F = E_{F1} = E_{F2} = 0.064$ eV and $d = 6$ nm for (a) $\epsilon_a(\omega)$ and $\epsilon_s(\omega)$ and (b) $\epsilon_a(\omega = 0)$ and $\epsilon_s(\omega)$. The horizontal dashed lines are the phonon frequencies of h-BN. The hybridized surface and hyperbolic plasmon-phonon modes are labeled as SPP and HPP, respectively, where the superscript in HPP denotes the upper (U) and lower (L) reststrahlen bands.



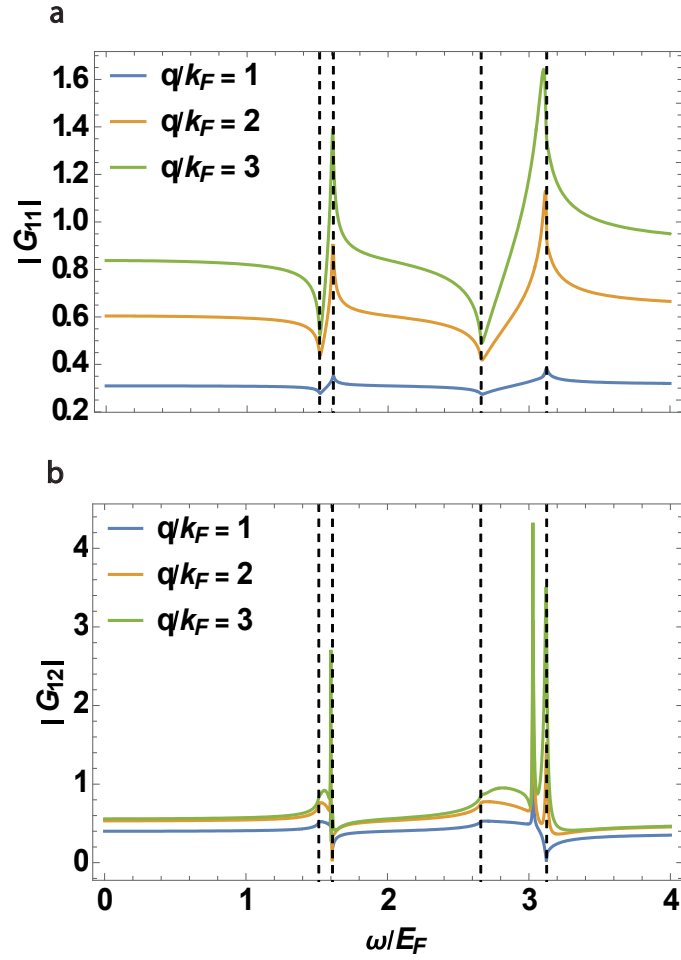
Supplementary Figure 3: Density plot of the loss function, $L(\mathbf{q}, \omega, T)$, calculated for the h-BN/G/h-BN/G/h-BN system at $T = 0$ K for $E_F = E_{F1} = E_{F2} = 0.2$ eV and $d = 6$ nm for (a) $\epsilon_a(\omega)$ and $\epsilon_s(\omega)$, (b) $\epsilon_a(\omega = 0)$ and $\epsilon_s(\omega)$, and (c) $\epsilon_a(\omega = 0)$ and $\epsilon_s(\omega = 0)$. The horizontal dashed lines are the phonon frequencies of h-BN. The hybridized surface and hyperbolic plasmon-phonon modes are labeled as SPP and HPP, respectively, where the superscript in HPP denotes the upper (U) and lower (L) reststrahlen bands.



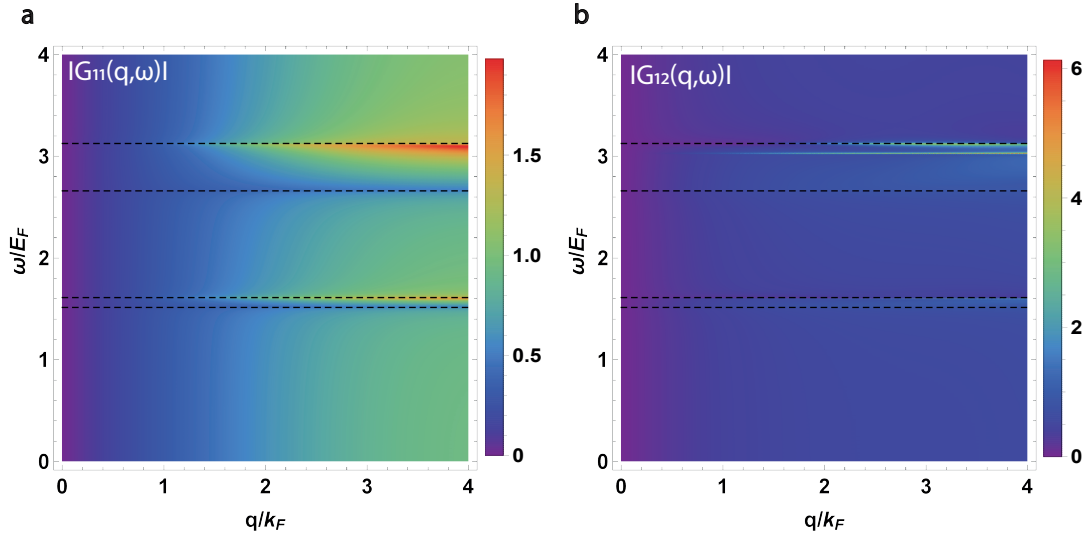
Supplementary Figure 4: Static interlayer, $G_{12}(\mathbf{q})$, and intralayer, $G_{11}(\mathbf{q})$, LFFs calculated for a interaction parameter of $r_s = 0.46$.



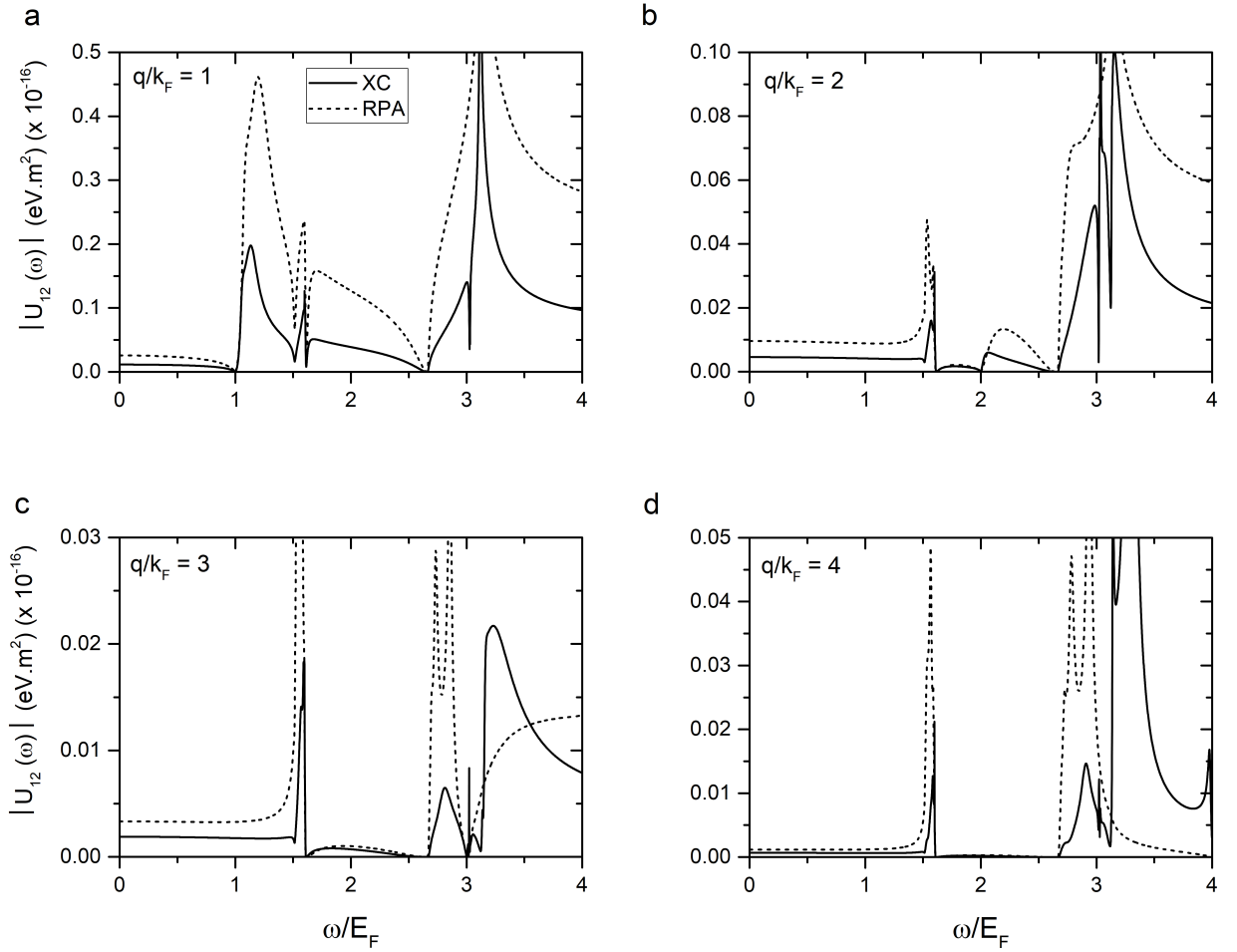
Supplementary Figure 5: Absolute value of the interaction parameter, $|r_s(\omega)|$, as a function of the frequency. The vertical dashed lines are the phonon frequencies of h-BN.



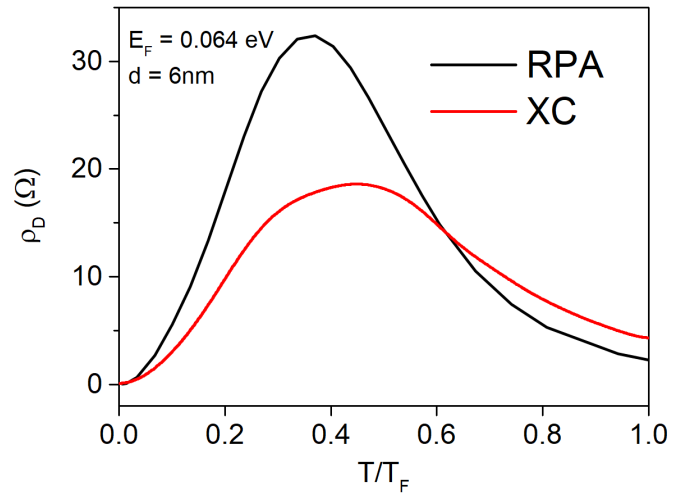
Supplementary Figure 6: Absolute value of the (a) intralayer, $|G_{11}(\mathbf{q}, \omega)|$, and (b) interlayer, $|G_{12}(\mathbf{q}, \omega)|$, LFF as a function of the normalized frequency ω for various q values and for $E_F = E_{F1} = E_{F2} = 0.064$ eV. The vertical dashed lines are the phonon frequencies of h-BN.



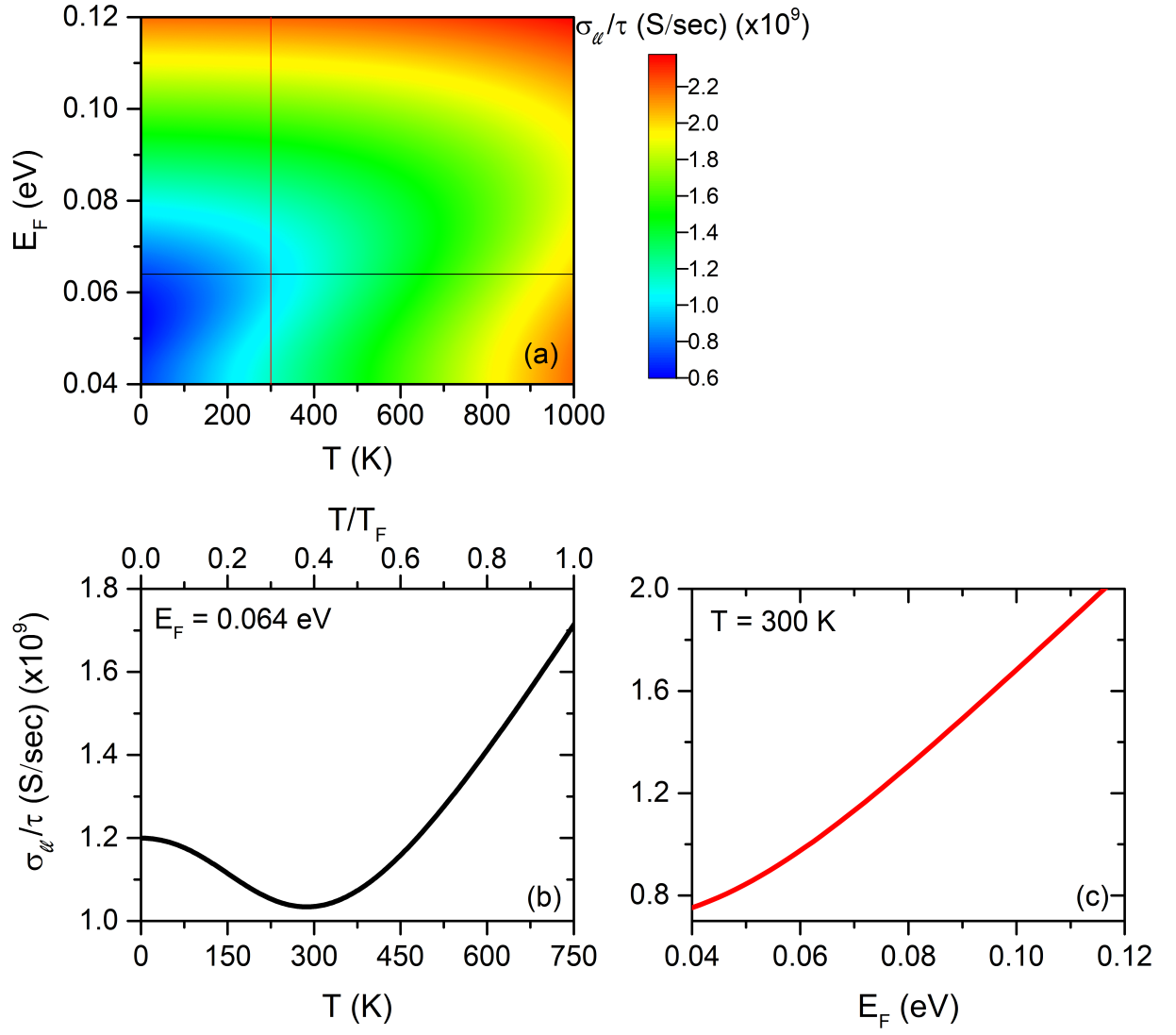
Supplementary Figure 7: Density plot of the absolute value of the (a) intralayer, $|G_{11}(\mathbf{q}, \omega)|$, and (b) interlayer, $|G_{12}(\mathbf{q}, \omega)|$, LFFs for $E_F = E_{F1} = E_{F2} = 0.064$ eV. The horizontal dashed lines are the phonon frequencies of h-BN.



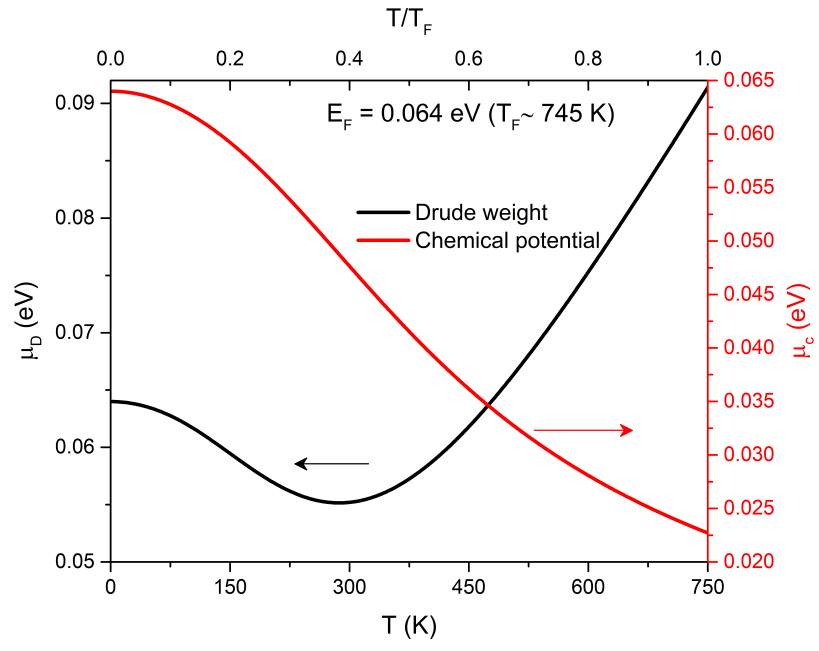
Supplementary Figure 8: $|U_{12}(\omega)|$ for (a) $q/k_F = 1$, (b) 2, (c) 3, and (d) 4 calculated both within RPA (dashed curves) and XC (solid curves). Parameters are: $T = 300$ K, $d = 6$ nm, and $E_F = E_{F1} = E_{F2} = 0.064$ eV.



Supplementary Figure 9: Drag resistivity, ρ_D , as function of the temperature calculated for the RPA and XC cases. Parameters are: $d = 6 \text{ nm}$ and $E_F = E_{F1} = E_{F2} = 0.064 \text{ eV}$.



Supplementary Figure 10: (a) Density plot of the temperature and Fermi energy dependent graphene conductivity normalized by the carrier transport time, $\sigma_u(T, E_F)/\tau$. The horizontal black and vertical red lines are the cross section of graphene conductivity for a fixed Fermi energy ($E_F = 0.064$ eV) (carrier density, $n = 3 \times 10^{11} \text{ cm}^{-2}$) and fixed temperature, $T = 300$ K, and are plotted in (b) and (c) as function of temperature and Fermi energy respectively.



Supplementary Figure 11: Graphene Drude weight, μ_D , (black curve) and its chemical potential, μ_c , (red curve) as a function of temperature for $E_F = 0.064$ eV.

SUPPLEMENTARY NOTE 1: DIELECTRIC FUNCTION OF h-BN

The frequency-dependent dielectric function or relative permittivity of h-BN is given by [1, 2]

$$\epsilon_m(\omega) = \epsilon_{\infty,m} \frac{(\omega_{LO}^m)^2 - (\omega_{TO}^m)^2}{(\omega_{TO}^m)^2 - i\gamma_m\omega - \omega^2}, \quad (1)$$

where $m = x, z$ and $\epsilon_{\infty,x} = 2.95$ and $\epsilon_{\infty,z} = 4.87$ are the high-frequency dielectric constants, $\omega_{TO}^x = 0.096$ eV, $\omega_{LO}^x = 0.102$ eV, $\omega_{TO}^z = 0.169$ eV, and $\omega_{LO}^z = 0.199$ eV are the phonon frequencies, and $\gamma_x = 0.49$ meV and $\gamma_z = 0.62$ meV are the damping frequencies. The real and imaginary parts of $\epsilon_m(\omega)$ are plotted in Supplementary Figures 1(a) and 1(b), respectively. The real part of $\epsilon_m(\omega)$ becomes negative in the restrahlen bands ($\omega_{TO}^m < \omega < \omega_{LO}^m$). The expressions $\epsilon_s(\omega)$ and $\epsilon_a(\omega)$ defined in the main text and plotted in Figure 1 of the main text are responsible for the plasmon-phonon coupling and the appearance of the hyperbolic phonons. The plasmon dispersion of the h-BN/G/h-BN system is shown in Supplementary Figure 2(a), where the density plot of the loss function, $L(\mathbf{q}, \omega, T)$, calculated at $T = 0$ K is depicted. This graph is calculated using the full $\epsilon_s(\omega)$ and $\epsilon_a(\omega)$ expressions and is similar to those in Figures 3(a) and 3(d) of the main text calculated at $T = 50$ K and 300 K, respectively. Supplementary Figure 2(b) plots the same plasmon dispersion curve as in Supplementary Figure 2(a) but setting $\epsilon_a(\omega = 0)$, thus eliminating the hyperbolic phonons. In addition, setting $\epsilon_s(\omega = 0)$ will also lead to the cancellation of the plasmon-phonon coupling. However, this latter case is not plotted as this effect will not be apparent for the low Fermi energy considered (0.064 eV), since the phonon frequencies of h-BN lie within the particle hole continuum. In order to illustrate this effect, the plasmon dispersion for a higher Fermi energy (0.2 eV) is plotted in Supplementary Figures 3(a)-(c) for the three cases mentioned above. Thus, in Supplementary Figure 3(b), same as Supplementary Figure 3(a) but setting $\epsilon_a(\omega = 0)$, it can be seen that the hyperbolic phonons in the two restrahlen bands are completely eliminated. And in Supplementary Figure 3(c), same as Supplementary Figure 3(a) but setting both $\epsilon_a(\omega = 0)$ and $\epsilon_s(\omega = 0)$, it can be noticed that the contribution from both hyperbolic and bulk phonons is eliminated, leaving behind the two intrinsic graphene plasmon branches, optical and acoustic.

SUPPLEMENTARY NOTE 2: LOCAL FIELD FACTORS FOR EXCHANGE AND CORRELATION EFFECTS

The exchange and correlation (XC) effects enter in the system via local field factors (LFFs) as a correction to the inter- and intralayer Coulomb interactions (see Eq. 2 of the main text). The simplest form of the LFFs is given by the Hubbard approximation, but Hubbard form is only for the intralayer LFF, while it is zero for the interlayer. Another approach for obtaining both inter- and intralayer LFFs are the self-consistent Singwi-Tosi-Land-Sjölander (STLS) [3] and the Fermi hypernetted-chain (FHNC) [4] methods. However, both STLS and FHNC methods are cumbersome. Therefore, in this work, quantum Monte Carlo (QMC) simulation data for LFFs [5, 6] have been used instead.

The LFFs are typically a function of q and of the interaction parameter r_s . In the case of graphene, $r_s = \alpha/\kappa$, where α is the graphene fine structure constant and κ is the background dielectric constant. The static inter- and intralayer LFFs, $G_{ij}(\mathbf{q})$, obtained for $r_s = 0.46$ are plotted in Supplementary Figure 4. The frequency dependence in the LFFs, $G_{ij}(\mathbf{q}, \omega)$, has been introduced by making r_s frequency dependent. This can be done by making use of a frequency-dependent κ term. A proper way of doing this is to substitute κ with $\epsilon_T(\mathbf{q}, \omega, T)$ (see eq. 3 of the main text). However, since $\epsilon_T(\mathbf{q}, \omega, T)$ itself depends on $G_{ij}(\mathbf{q}, \omega)$, the LFFs need to be solved self-consistently making the calculation difficult and thus beyond the scope of this paper. A simpler way, used here, to introduce the frequency dependence is to substitute κ with $\epsilon_s(\omega) = \sqrt{\epsilon_x(\omega)\epsilon_z(\omega)}$. A plot of the frequency-dependent $|r_s(\omega)|$ is shown in Supplementary Figure 5. As $\epsilon_s(\omega)$ peaks near ω_{TO} (see Figure 1(a) of the main text), $|r_s(\omega)|$ takes the minimum value, while $r_s(\omega)$ takes its maximum value for ω_{LO} around the two restrahlen bands. Thus, comparing with the static case where $r_s = 0.46$ (Supplementary Figure 4), the graphene is a weakly interacting system, while once encapsulated in h-BN, it can have $|r_s(\omega)| > 2$ around the two restrahlen bands of h-BN. A more detailed discussion on this will be the subject for further investigation. The intra- and interlayer frequency-dependent LFFs, $|G_{11}(\mathbf{q}, \omega)|$ and $|G_{12}(\mathbf{q}, \omega)|$, are plotted in Supplementary Figures 6(a) and 6(b), respectively, for various q values. It can be observed that $|G_{ij}(\mathbf{q}, \omega)|$, similarly to $|r_s(\omega)|$, attains a maximum value around the two restrahlen bands of h-BN. The intensity of these maxima increases with increasing q/k_F , as shown in the density plots of $|G_{11}(\mathbf{q}, \omega)|$ and $|G_{12}(\mathbf{q}, \omega)|$ depicted, respectively, in Supplementary Figures 7(a) and 7(b).

As mentioned in the main text, the inclusion of the LFFs leads to an increase in the magnitude of $\epsilon_T(\mathbf{q}, \omega, T)$ and a decrease in the interlayer Coulomb interaction, which all together leads to a decrease in the magnitude of the dressed interlayer Coulomb interaction, $U_{12}(\mathbf{q}, \omega, T)$, given by eq. 5 of the main text. In Supplementary Figure 8, the $|U_{12}(\mathbf{q}, \omega, T)|$ is plotted for $T = 300$ K within RPA and with XC effects for different q/k_F values, where the reduction

of $|U_{12}(\mathbf{q}, \omega, T)|$ induced by the LFFs is observed, which in turn leads to decrement in the drag resistivity ρ_D as shown in Supplementary Figure 9.

SUPPLEMENTARY NOTE 3: GRAPHENE CONDUCTIVITY, DRUDE WEIGHT, AND CHEMICAL POTENTIAL

The intralayer conductivity of graphene has the following form

$$\sigma_{ll} = e^2 \mu_D \tau / (\pi \hbar^2), \quad (2)$$

where τ is the carrier transport time in graphene, which is assumed to be energy independent, and μ_D is the effective Drude weight, which depends on the carrier density and the temperature T , and is given by

$$\mu_D = \mu_c + 2k_B T \log[1 + \exp(-\mu_c/k_B T)], \quad (3)$$

where μ_c is the chemical potential. The latter can be calculated solving the following relation for the Fermi energy:

$$E_F = k_B T \sqrt{2 \int_0^\infty (f_- - f_+) x dx}, \quad (4)$$

where

$$f_\pm = \frac{1}{1 + \exp(x \pm \mu_c/k_B T)}. \quad (5)$$

Equation 4 can be simplified to

$$E_F = k_B T \sqrt{2 [Li_2[-\exp(-\mu_c/k_B T)] - Li_2[-\exp(\mu_c/k_B T)]]}, \quad (6)$$

where

$$Li_2(x) = \int_x^0 \frac{\log(1-t)}{t} dt \quad (7)$$

is a Spence function. The density plot of the graphene conductivity normalized by the carrier transport time, $\sigma_{ll}(T, E_F)/\tau$, is shown in Supplementary Figure 10(a). Supplementary Figures 10(b) and 10(c) present, respectively, the temperature dependence of this function for $E_F = 0.064$ eV and the Fermi energy dependence of this function for $T = 300$ K, corresponding to the horizontal and vertical cross section lines indicated in Supplementary Figure 10(a). The chemical potential and the Drude weight as a function of the temperature for a fixed Fermi energy of $E_F = 0.064$ eV are plotted in Supplementary Figure 11.

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