Reviewers' comments:

Reviewer #1 (Remarks to the Author):

Please see attached file.

Reviewer #2 (Remarks to the Author):

see attached document.

Editors Note: please specifically comment in both the revised manuscript and the response to the referee, on the similarity/differences with the bibliographic reference mentioned by the reviewer.

Reviewer #3 (Remarks to the Author):

The paper by Shukla et al. reports experimental results on the quantum tunneling of a Bose condensate out of an optical dipole trap. The subject matter of the paper is certainly of general interest to the physics community, and the paper does contain some apparently sound results. However, in the present form in cannot recommend the paper for publication. While the experimental procedure and results are reasonably well presented, the conclusions of the paper are not at all clear to me. In my view, the entire final section (second half of page 10) of the paper needs to be revised in order to state more clearly what the authors actually deduce from their experimental results.

My detailed observations on the paper are as follows:

p. 3, line 4: "... was used for the first time to demonstrate...": I think this is a very strong and rather general claim that should be toned down (or qualified) a bit. For instance, apart from ref [22], there are experiments that explore the dynamics of Landau-Zener tunneling in momentum space (PRA 82, 013633, 2010). Or do the authors only refer to tunneling in real space? If so, they should say so.

p. 3, line 19: "weakest part of the potential..." I'm not sure what "weak" means in this context - maybe the lowest potential barrier? This should be rephrased.

p. 4, line 7 of main text: "in comparison with kicking": This is not clear, do the authors mean to say something like "In a control experiment, the ODT power.... " ?

p. 5, line 1 of main text: "no signature of condensate..." This is not completely clear. Is this referred to the transverse width of the outcoupled atomic beam, or is there another measurable quantity that could be used to strengthen this claim?

p. 6, line 6: "tunneling efficiency": Maybe "tunneling amplitude?" is a more standard expression?

p. 6, line 19: "energy of the pulses" It is not clear to me what is meant by this.

p. 6, line 24: "It implies that 10% of the entire BEC was not excited" It is not clear to me what this means, and how this could come about physically. Could the authors expand on this?

p. 8, Fig. 4: The vertical axis in part (b) of the figure is labelled "tunnel out atom %", which is difficult to understand - maybe "Fraction of tunneled atoms". Moreover, the in the main text the terms " tunneling / output transmission / coupling efficiency / transmission efficiency / tunneling efficiency " appear to be used as synonyms - the authors should decide on one single term and then use that consistently in order to avoid confusion.

p. 8, line 8: "... that is lower than the trap depth": what is the trap depth here?

p. 8, line 9: "It is worthy to note that..." Why? Is anything deduced from those values?

There are also a number of linguistic problems with the paper that make it unnecessarily hard to read. A few examples are:

p. 3: "The leakages tunneling out of trap"

p. 4: "...nor collective mode excitation is expected to give"

Reviewer 1 Review Attachment

In this manuscript the authors report on an experiment aimed at studying the process of quantum tunnelling across a potential barrier. A Bose-Einstein condensate is created in a hybrid (optical + magnetic) trap, and it is then out-coupled to the continuum via non adiabatic modifications (kicks) of the trapping potential. The condensate is then able to escape from the trap via quantum tunnelling. The authors characterise this mechanism in several respects. The topic is definitely interesting, but the manuscript presents several issues that do not not allow me to make a final recommendation at this stage. I kindly recommend the authors to consider the following points, and in view of that to carefully revise the whole manuscript.

1) As a general comment, I notice that the English is often not fluent, and this somehow affects the clarity of the paper.

2) The authors insist several times on their BEC being a "quantum simulator" of the time-dependent Schrödinger equation, an electron wave packet, quantum phenomena, or the dynamics of single particle. I find these claims somehow trivial, confusing, and unnecessary. The topic at study, quantum tunnelling, stands on its own. There's no need to appeal to the fact that it is simulating something else. The authors want to investigate quantum tunnelling, and the way they do it by employing a BEC is a way to realise it. Moreover, it is misleading to say that the BEC allows to simulate the (nonlinear) Schrödinger equation (or any equation), rather it is the equation that governs the behaviour of the system. Possibly, one would be interested in simulating the behaviour of another physical system, but this is not the case here, the authors are aimed at investigating a general phenomenon (quantum tunnelling).

3) In the Abstract, the authors talk about "optically trapped BEC in a hybrid trap"… is it just optical or hybrid?

4) The sentence "the weakest part of the potential well is on the bottom of the trap" (page 3) looks tautological. Please recast it in a meaningful form.

5) In the section "Tunneling matter wave pulses" the authors introduce two kick protocols (step-kick and pulse-kick), which are also shown in Fig. 1. They start by discussing the step-kick case, and suddenly they write "In comparison with kicking, the ODT power was ramped down adiabatically in 2 sec to the same final trap depth as the step-kick." - I find this sentence confusing in several respects: i) it is not grammatically correct for expressing what (I understand) the authors want to say; ii) at this point one would expect to read about the other kick protocol (which appears only much later on), whereas here the authors talk about an

additional protocol that has not been introduced before and, iii) it is not shown in Fig. 1.

6) The green arrow in Fig. 1a (I guess) represents the effect of the step-kick protocol, that is instead indicated by blue arrows in Fig. 1b. I would suggest to use the same color code. Then, what about the pulse-kick? The effect of the potential is more complicated and cannot be simplified by an arrow in a single direction.

7) Fig. 1c: it looks that the horizontal axis (the direction of gravity) is reversed with respect to that in Fig. 1a; this is confusing.

8) Page 4: "Our observation is in good agreement with the numerical simulation under a similar scenario [23]." - Can the author comment more about this agreement? Is it just a qualitative similarity? In what respect? Have the authors performed any numerical simulation for their specific experimental setup?

9) The authors say that their kick protocol excites a rotation of the condensate. This explains why the wave packets that tunnel across the barrier do not fall along the direction of gravity, see Fig. 2. However, it is not at all clear why it is so, nor this very important point is clearly discussed in the paper. Since the way the excitation is induced is needed to be known with accuracy in order to make reasonable claims about the behaviour of the condensate and its implications on quantum tunnelling, this issue has to be carefully addressed.

10) Related to the above point, the authors employ a description which does not take into account of the rotation. The trapping potential is always written as a function on the coordinate z alone [see e.g. Eqs. (1) and (3)], suggesting that the other spatial directions do not matter. Clearly, this is not so.

11) Page 11: "the nonlinear term \psi^2 of the GPE" - the term that appears in the GPE has is proportional to | \psi|^2.

12) Eq. (3): is \beta positive? then there should be a minus in front of the term \beta*z, otherwise the resulting potential would not be consistent with that shown in Fig. 1a.

13) Page 13: "The sudden change in the ODT power by ΔU" - changing the power of the (lasers that produce the) optical dipole trap affects the value of U_t, so it would be more precise to write "ΔU_t".

Reviewer 2 Review Attachment

The paper deals with an experimental realization of quantum tunneling using Bose-Einstein condensates (BEC). The study is motivated by attosecond ionization experiments and the puzzle of tunneling time.

The present study and its results, based on a BEC experiment seem to be sound. Especially, the instantaneous nature of the tunneling event is reasonable.
There are, however, previous experimental works from some years ago, on the phenomenon of coherent destruction of tunneling (CDT), that use a similar (BEC) setup (Lignier et al, PRL, 99 220403 (2007)), which have not been referenced to. Furthermore, the English needs some improvement. If the two issues raised are dealt with satisfactorily, I recommend the publication of the manuscript.

Response to the Referee

All the comments from reviewers are responded one-by-one. The responses are formatted by colours as: Grey-reviewer's comment, Purple-our response, Blue-corresponding revision.

Reviewer #1 (Remarks to the Author):

In this manuscript the authors report on an experiment aimed at studying the process of quantum tunnelling across a potential barrier. A Bose-Einstein condensate is created in a hybrid (optical + magnetic) trap, and it is then outcoupled to the continuum via non adiabatic modifications (kicks) of the trapping potential. The condensate is then able to escape from the trap via quantum tunnelling. The authors characterise this mechanism in several respects. The topic is definitely interesting, but the manuscript presents several issues that do not allow me to make a final recommendation at this stage. I kindly recommend the authors to consider the following points, and in view of that to carefully revise the whole manuscript.

(1) *As a general comment, I notice that the English is often not fluent, and this somehow affects the clarity of the paper.*

Response: The reviewer comment is very appreciated. We have carefully gone through and revised the manuscript to make it more fluent and clearer. The result should be satisfactory now.

Revision: (see the revised manuscript.)

(2) *The authors insist several times on their BEC being a "quantum simulator" of the time-dependent Schrödinger equation, an electron wave packet, quantum phenomena, or the dynamics of single particle. I find these claims somehow trivial, confusing, and unnecessary. The topic at study, quantum tunnelling, stands on its own. There's no need to appeal to the fact that it is simulating something else. The authors want to investigate quantum tunnelling, and the way they do it by employing a BEC is a way to realise it. Moreover, it is misleading to say that the BEC allows to simulate the (nonlinear) Schrödinger equation (or any equation), rather it is the equation that governs the behaviour of the system. Possibly, one would be interested in simulating the behaviour of another physical system, but this is not the case here, the authors are aimed at investigating a general phenomenon (quantum tunnelling).*

Response: We agree with the reviewer that the quantum tunnelling of BEC is interesting and important on its own. Hence, we have reduced the statement on quantum simulator and related claims from the manuscript. However, we would like to keep some sentences to point out our report as a more general study on the quantum tunnelling and a analogue to other systems. Such a link would bring more interest into our report.

Revision:

Page 2 abstract;
"The evolution of the BEC…...time dependent Schrodinger equation."
Revised :
(the sentence is removed)

Page 2 para. 2;
"The tunability of the……such as Josephson effect"

Revised:
"Especially the laser cooled atomic BEC with tunability of the relevant parameters, such as the barrier height, the kinetic energy and the number of atoms in the tunnelling process, is advantageous for experimental realization."

Page 3 para. 1:
"The BEC act as simulator……attosecond streaking technique."
Revised:
"Our experiment is analogue to the strong field tunnelling ionization experiment where the condensate acts as a bound electron wave packet. The potential formed by the optical dipole trap (ODT), the magnetic trap and gravity acts as the Coulomb potential of nucleus distorted by strong light pulse in the attosecond streaking technique."

Page 10 para. 1;
"…. object to simulate quantum phenomenon."
Revised:
"…. object to study quantum tunnelling."

Page 12 para.1;
"The BEC is a quantum simulator……dynamics of single particle"
Revised: (moved to page 11 para.1)
"In such cases, the BEC is an ideal platform, not only for many-body quantum physics, but as a giant wave function to study the quantum dynamics of single particle."

(3) In the Abstract, the authors talk about "optically trapped BEC in a hybrid trap"… is it just optical or hybrid?

Response: It is indeed a hybrid trapped BEC, the trap consists of crossed beam optical dipole trap and quadrupole trap.

Revision:

Page 2 abstract;
"…tunneling of optically trapped BEC in a hybrid trap."
Revised:
"…tunnelling of Bose Einstein Condensate in a hybrid trap."

(4) The sentence "the weakest part of the potential well is on the bottom of the trap" (page 3) looks tautological. Please recast it in a meaningful form.

Response: We meant here the lowest barrier height on the bottom of the trap.

Revision:

page 3 para. 3;
"…the weakest part of the potential well is on the bottom of the trap."
Revised:
"Due to the gravity, the potential barrier height becomes lowest on the bottom of the trap."

(5) In the section "Tunneling matter wave pulses" the authors introduce two kick protocols (step-kick and pulse-kick), which are also shown in Fig. 1. They start by discussing the step-kick case, and suddenly they write "In comparison with kicking, the ODT power was ramped down adiabatically in 2 sec to the same final trap depth as the step-kick." - I find this sentence confusing in several respects: i) it is not grammatically correct for expressing what (I understand) the authors want to say; ii) at this point one would expect to read about the other

kick protocol (which appears only much later on), whereas here the authors talk about an additional protocol that has not been introduced before and, iii) it is not shown in Fig. 1.

Response:
(i) Here we were trying to emphasise on the difference between the non-adiabatic and the adiabatic change in the ODT power with respect to tunnelling. The additional procedure to produce a continuous thermal atomic beam has been now introduced in the 'Method' section. The three protocols are now clearly described (step-kick, Pulse-kick and adiabatic ramp).

(ii) To be more clear, in the revised manuscript, three protocols (step-kick, pulse-kick, and adiabatic ramp) are brought up in the beginning of the "TUNNELLING MATTER WAVE PULSES". The detailed description on these protocols are all included in "METHOD". We still have later discussed the pulse type kick with the tunnelling efficiency section.

(iii) We have added a pulse type protocol in fig. 1.

Revision:

Page 3 para. 3:

"….A "kick", which is a non-adiabatic change in the potential well, was then exerted on the BEC. Two types of kick were employed in our experiment: step-kick and pulse-kick (see Method) …."

Revised:
"….BEC is then perturbed by varying the potential well. The protocols of the time-varying potential include: an adiabatic ramping and two types of kick (step-kick and pulse-kick), which are non-adiabatic changes in the potential well (see Method)."

Page 4 para. 2:
"In comparison with kicking……expected to give."
Revised:
"On the other hand, when an adiabatic ramping was employed in comparison with a kick, a thermal atomic beam was produced. The ODT power was ramped down adiabatically in 2 sec to the same final trap depth as the step-kick. In such a case, neither additional kinetic energy, nor collective mode excitation was given to the condensate reservoir."

Page 13 para. 3: (new paragraph)
"Additionally, to compare with non-adiabatic kicks, once BEC was produced, then the ODT power was further ramped down adiabatically in two seconds to the trap depth same as the final potential barrier of a typical kick."

Page 4
see 'Fig. 1 (a)'

(6) *The green arrow in Fig. 1a (I guess) represents the effect of the step-kick protocol, that is instead indicated by blue arrows in Fig. 1b. I would suggest to use the same color code. Then, what about the pulse-kick? The effect of the potential is more complicated and cannot be simplified by an arrow in a single direction.*

Response: We have incorporated the same color scheme for the step kick and we have added schematic representation of pulse-kick in the fig.1

Revision:

Page 4
see 'Fig. 1'

(7) Fig. 1c: it looks that the horizontal axis (the direction of gravity) is reversed with respect to that in Fig. 1a; this is confusing.

Response: We have rotated the image to the correct direction.

Revision:

Page 4
see 'Fig. 1(c)'

(8) Page 4: "Our observation is in good agreement with the numerical simulation under a similar scenario [23]." - Can the author comment more about this agreement? Is it just a qualitative similarity? In what respect? Have the authors performed any numerical simulation for their specific experimental setup

Response: Our experimental results agree qualitatively with the findings done in the [23] in several aspects, (i) pulse shape (ii) periodicity of the pulse train. We have more explicitly expressed these points in our revised manuscript. We have not performed any numerical simulation for our specific experimental setup.

Revision:
Page 4 para. 2:
"Our observation……similar scenario [23]."
Revised:
"Our experimental results agree qualitatively with the previous numerical simulation in several aspects, such as periodicity and pulse shape [23]."

(9) The authors say that their kick protocol excites a rotation of the condensate. This explains why the wave packets that tunnel across the barrier do not fall along the direction of gravity, see Fig. 2. However, it is not at all clear why it is so, nor this very important point is clearly discussed in the paper. Since the way the excitation is induced is needed to be known with accuracy in order to make reasonable claims about the behaviour of the condensate and its implications on quantum tunnelling, this issue has to be carefully addressed.

Response:
Indeed, by applying a kick protocol many possible modes of the condensate can be excited. However, in our experiment, we were interested in an excitation that was related to CM motion. Our trap potential is elliptical, but close to spherical symmetry, where the pure CM rotational motion ($\omega=\omega_0$, l=1) is the lowest excitation mode. Therefore, by applying a small perturbation of the trap (kick), we observed the rotational motion of BEC. It was confirmed by our TOF momentum space measurement (Fig. 5). The "INTERNAKL DYNAMICS" section was revised to address this.

Revision:
Page 9:
"With the TOF measurement, the observed trajectory, ….an energy lower than the potential barrier….."

Revised:
"In a three-dimensional trap, the pure CM rotation mode, which is the lowest collective excitation mode with the rotation frequency $\omega=\omega_0$ and the angular momentum $l=1$ [26,27] is most pronouncing by applying a small perturbation. Using the TOF measurement, the observed trajectory and the distribution of atom number confirmed that the kick excites a collective rotational mode in the condensate with an energy lower than the potential barrier…."

(10) *Related to the above point, the authors employ a description which does not take into account of the rotation. The trapping potential is always written as a function on the coordinate z alone [see e.g. Eqs. (1) and (3)], suggesting that the other spatial directions do not matter. Clearly, this is not so.*

Response: As our earlier response, the rotation is excited by the kick. It makes the trajectory of the tunnelling output atoms oscillating, and is important to our analysis. In our revised manuscript, all dimensions (x,y,z) are considered in the related equations, as suggested by the reviewer.

Revision:

equation (1) and (3) are revised.

(11) *Page 11: "the nonlinear term \psi^2 of the GPE" - the term that appears in the GPE has is proportional to | \psi|^2.*

Response: we have revised the segment.

Revision:

page 10 para. 1
"…. term \psi^2 of the GPE, is negligible……"
Revised:
"… term |\psi|^2 of the GPE…"

(12) *Eq. (3): is \beta positive? then there should be a minus in front of the term \beta*z, otherwise the resulting potential would not be consistent with that shown in Fig. 1a.*

Response: We have now defined \beta (x,y,z) as resulting potential formed by quadrupole field and gravity. The correct form of the potential written as in revised equation (3)

Revision: revised definition of \beta and equation (3).

(13) *Page 13: "The sudden change in the ODT power by ΔU" - changing the power of the (lasers that produce the) optical dipole trap affects the value of U_t, so it would be more precise to write "ΔU_t".*

Response: ΔU corresponds to the change in the ODT power. Indeed, it would be more appropriate to write it as ΔU_t.

Revision:

page 13 para. 2
Revised text: ('ΔU' is replaced by 'ΔU_t')

Reviewer #2 (Remarks to the Author):

The paper deals with an experimental realization of quantum tunneling using Bose-Einstein condensates (BEC). The study is motivated by attosecond ionization experiments and the puzzle of tunneling time.

(1) *The present study and its results, based on a BEC experiment seem to be sound. Especially, the instantaneous nature of the tunneling event is reasonable. There are, however, previous experimental works from some years ago, on the phenomenon of coherent destruction of tunneling (CDT), that use a similar (BEC) setup (Lignier et al, PRL, 99 220403 (2007)), which have not been referenced to. Furthermore, the English needs some improvement. If the two issues raised are dealt with satisfactorily, I recommend the publication of the manuscript.*

Editor's Note:

Please specifically comment in both the revised manuscript and the response to the referee, on the similarity/differences with the bibliographic reference mentioned by the reviewer.

Response: We thank the reviewer for positive response. We have addressed the issues raised by the reviewer and incorporated in the revised manuscript In our system, the BEC itself is excited via external perturbation resulting in quantum tunnelling to "*open space*" and observed in "*real space*", while [18] and [19] studied the "interwell" tunnelling of BEC in "momentum space". Both of our work and theirs are for the dynamics of BEC, but the difference is still significant.

Revision:

page 2 para. 2;
"…multi-particle has been studied experimentally [15–17]. Such experiments have provided…"

Revised:
"…has been studied experimentally [15–17]. The tunnelling efficiency of BEC in a accelerating [18] or shaking [19] periodic potential well has also been studied in momentum space. Such experiments have provided…"

Page 3, para 1:
"Here, the matter wave packet was used for the first time to demonstrate the tunneling dynamics, which has only been experimentally studied with optical soliton [22]. Our experiment is analogue…"

Revised:

"The tunnelling dynamics to open space has been demonstrated experimentally only with optical soliton in [23]. In this report, the matter wave packet was used for the first time to exhibit the tunnelling dynamics to open space and to be observed in real space. Our experiment is analogue…"

Reviewer #3 (Remarks to the Author):

The paper by Shukla et al. reports experimental results on the quantum tunneling of a Bose condensate out of an optical dipole trap. The subject matter of the paper is certainly of general interest to the physics community, and the paper does contain some apparently sound results. However, in the present form I cannot recommend the paper for publication. While the experimental procedure and results are reasonably well presented, the conclusions of the paper are not at all clear to me. In my view, the entire final section (second half of page 10) of the paper needs to be revised in order to state more clearly what the authors actually deduce from their experimental results.

We thank the reviewer for thorough reading of manuscript. To response reviewer's criticism on the "DISCUSSION" section (the final section, page 10), we have revised the entire section to point out more clearly the result of our report: By exploring the tunnelling dynamics, the tunnelling time should considered with the wave mechanics.

My detailed observations on the paper are as follows:

***(1)** p. 3, line 4: "... was used for the first time to demonstrate...": I think this is a very strong and rather general claim that should be toned down (or qualified) a bit. For instance, apart from ref [22], there are experiments that explore the dynamics of Landau-Zener tunneling in momentum space (PRA 82, 013633, 2010). Or do the authors only refer to tunneling in real space? If so, they should say so.*

Response: In our revised manuscript, we emphasised that our experiment is "tunnelling to open space" and "observation in real space" to be distinguished from the mentioned reference and other related works. Under such a condition, it should be eligible to claim "for the first time".

Revision:

See the response to the reviewer #2.

***(2)** p. 3, line 19: "weakest part of the potential..." I'm not sure what "weak" means in this context - maybe the lowest potential barrier? This should be rephrased.*

Response: We have rephrased it as "lowest". See response to reviewer #1's comment (4)

***(3)** p. 4, line 7 of main text: "in comparison with kicking": This is not clear, do the authors mean to say something like "In a control experiment, the ODT power.... " ?*

Response: It is to compare the non-adiabatic (kick) and adiabatic ramping of ODT. Please see the response to reviewer #1's comment (5).

***(4)** p. 5, line 1 of main text: "no signature of condensate..." This is not completely clear. Is this referred to the transverse width of the outcoupled atomic beam, or is there another measurable quantity that could be used to strengthen this claim?*

Response: In the context of "no signature of the condensate…" we refer to the phase transition of outcoupled atomic beam from BEC to thermal cloud of atoms. To make it more clear, we have revised the sentence.

Revision:

Page 5 para. 2;

"…. no signature of the condensate…."
Revised:
"The diffusing leakage goes even above the position of BEC reservoir, and shows as thermal atomic cloud, although it is still confined by the transverse trapping force."

(5) p. 6, line 6: "tunneling efficiency": Maybe "tunneling amplitude?" is a more standard expression?

Response: 'Transmission amplitude' is should be preserved for referring to the "probability density" of the tunnelling process. Because the quantity we measured is the ratio of out-coupled atoms to the initial atoms presented in the trap, we would like to keep the term 'tunnelling efficiency' to avoid confusion.

(6) *p. 6, line 19: "energy of the pulses" It is not clear to me what is meant by this.*

Response: Here we refer the kinetic energy of atoms. The revised term is "…kinetic energy of the pulses"

Revision: All the term "energy" was revised as "kinetic energy".

(7) *p. 6, line 24: "It implies that 10% of the entire BEC was not excited" It is not clear to me what this means, and how this could come about physically. Could the authors expand on this?*

Response: For a 4-pulse tunnelling output for a certain outcoupling time, the extrapolated atom number is smaller than the remaining atoms in the BEC reservoir which implies that the exerted kick was not able to excite enough. More pulse could come out physically only with a very energetic kick (high kick strength).

Revision:

Page 6 para. 2
"However, the observed atoms (the red point) was much more than this expectation. it implies that 10% of the entire BEC was not exited, and may transfer from the rotational mode to the vibrational mode at a later time, as described in [25]."
Revised:
"Meanwhile, using Eq. (2), the total number of atoms remaining in the BEC reservoir after 4 pulse output is extrapolated shown as the grey point connected by a dashed line in Fig. 3. However, the observed atoms (the red point) was more than the extrapolated atom number."

(8) *p. 8, Fig. 4: The vertical axis in part (b) of the figure is labelled "tunnel out atom %", which is difficult to understand - maybe "Fraction of tunneled atoms". Moreover, the in the main text the terms " tunneling / output transmission / coupling efficiency / transmission efficiency / tunneling efficiency " appear to be used as synonyms - the authors should decide on one single term and then use that consistently in order to avoid confusion.*

Response: We have corrected the figure label. We have also removed the all synonyms to one standard expression (tunnelling efficiency) in the revised manuscript.

Revision:

page 8 (see Fig. 4)

(9) p. 8, line 8: "... that is lower than the trap depth": what is the trap depth here?

Response: The trap depth here is 38 nK.

Revision:

page 8 para. 1:
"…that is lower than the trap depth."
Revised:
"…that is lower than the trap depth (38 nk)."

(10) p. 8, line 9: "It is worthy to note that..." Why? Is anything deduced from those values?

Response: The estimated sound velocity and chemical potential of our BEC give the energy scale of the GPE nonlinear interaction term. In comparison with the CM velocity and kinetic energy of BEC, it is relevant to the validity of the approximation of GPE as a single particle Schrodinger equation.

(11) There are also a number of linguistic problems with the paper that make it unnecessarily hard to read. A few examples are:

Response: We have revised whole manuscript in order to eliminate all possible linguistic errors.

(12) p. 3: "The leakages tunneling out of trap"

Response: Here we meant to say the ejected pulses from the BEC reservoir. We have replaced the above term by 'tunnelling output atoms'

Revision:

Page 4 para. 2; page 3 para. 3
Revised text:
"…, tunnelling output atoms…"

(13) p. 4: "...nor collective mode excitation is expected to give"

Response: The entire paragraph was largely revised. Please see the response to comment (5) of the reviewer #1.

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

I appreciate the work done by the authors for delivering this revised version of the manuscript. In my opinion, they have seriously considered all the Referee comments, and they have satisfactory addressed the various concerns. The manuscript and the figures have been revised and amended accordingly. I am pleased to recommend this paper for publication.

Reviewer #3 (Remarks to the Author):

The revised manuscript by Shukla et al. takes into account the observtions by two referees and myself (which, in many cases, coincide). While the revised version is an improvement over the original manucscript, I still find it somewhat unclear in a number of places. Overall, it could certainly still do with a linguistic overhaul as it contains numerous grammatical errors.

Specifically, I would like the authors to take another look at the following points:

- page 2, line 11 of the introduction: although I appreciate that the term "tunneling efficiency" is used in the literature, I think it may less familiar than "tunneling amplitude" or "tunneling probability", so it would be helpful if the authors could clearly define early in the manuscript what they mean by this term, and how it relates to the other terms.

- page 3, line 7 from bottom: "varying the potential well": it should be stated clearly here *what* is varied, i.e., in this case the depth of the well.

- page 5, line 7: "...and shows as thermal atomic cloud, although it is still confined...": it is not clear what "it" refers to here, making it difficult to understand the sentence. Please clarify.

- page 5, line 7 from bottom: "... constant tunneling efficiency eta, the tunneling probability...": Could the authors explicitly state how tunneling efficiency and tunneling probability are related, so that this statement becomes easier to understand?

In conclusion, although if find the (certainly improved) revised version still less than satisfactory, I can recommend the manuscript for publication if the authors address the above criticism.

Response to the Referee

This response is composed of two parts, including our response to the two reviewers and to the comments given by editors. All the comments from reviewers are responded one-by-one. The responses are formatted by colours as: Grey-reviewer's comment, Purple-our response, Blue-corresponding revision. To editor's comment, our response are direct replies in the in-line sticky note box.

Reviewer #1 (Remarks to the Author):

I appreciate the work done by the authors for delivering this revised version of the manuscript. In my opinion, they have seriously considered all the Referee comments, and they have satisfactory addressed the various concerns. The manuscript and the figures have been revised and amended accordingly. I am pleased to recommend this paper for publication.

Response: We are pleased.

Reviewer #3 (Remarks to the Author):

The revised manuscript by Shukla et al. takes into account the observations by two referees and myself (which, in many cases, coincide). While the revised version is an improvement over the original manuscript, I still find it somewhat unclear in a number of places. Overall, it could certainly still do with a linguistic overhaul as it contains numerous grammatical errors.

We thank the reviewer for careful reading of manuscript. We have revised the manuscript in order to eliminate grammatical errors.

Specifically, I would like the authors to take another look at the following points:

- page 2, line 11 of the introduction: although I appreciate that the term "tunneling efficiency" is used in the literature, I think it may less familiar than "tunneling amplitude" or "tunneling probability", so it would be helpful if the authors could clearly define early in the manuscript what they mean by this term, and how it relates to the other terms.

Response: In the context of single particle transmission the term 'Tunnelling probability' is more commonly used but in case of many particles, the 'tunnelling efficiency' term is much preferable. As we didn't measure the amplitude of wavefunction of incident/transmitted atoms. We measured the ratio of number of atoms travelled across the barrier, so it is more appropriate to use tunnelling efficiency to avoid confusion.

Revision:

Page 6 para. 3 line 1;

“…In quantum tunnelling, the tunnelling efficiency \eta only depends…”

Revised:

“…the tunnelling efficiency \eta, the fraction of tunnelled atoms, only depends. …”

*- page 3, line 7 from bottom: "varying the potential well": it should be stated clearly here *what* is varied, i.e., in this case the depth of the well.*

Response: In this case, the trap depth of ODT is varied in order to apply perturbation to the BEC.

Revision:

Page 3 para. 4 line 7

“….varying the potential well….”

Revised:

“….varying the trap depth of ODT potential.”

- page 5, line 7: "...and shows as thermal atomic cloud, although it is still confined...": it is not clear what "it" refers to here, making it difficult to understand the sentence. Please clarify.

Response: As we mentioned in the main text that for long observation time, the quadrupole magnetic field is kept on during the outcoupling of atoms through the trap. That is why the thermal atomic cloud was still confined by transverse trapping force.

Revision:

Page 5 para. 1 line 7

"...and shows as thermal atomic cloud, although it is still confined..."

Revised:

“…and shows as a thermal atomic cloud, although this thermal cloud is still confined by the transverse trapping force.”

- page 5, line 7 from bottom: "... constant tunneling efficiency eta, the tunneling probability...": Could the authors explicitly state how tunneling efficiency and tunneling probability are related, so that this statement becomes easier to understand?

Response: Here we correct the sentence. With a constant tunnelling efficiency \eta, number of tunnelled atoms decays exponentially.

Revision:

Page 5 para. 3 line 5:

"…constant tunneling efficiency eta, the tunneling probability …"

Revised:

"With a constant tunnelling efficiency \eta, number of tunnelled atoms decays exponentially."

In conclusion, although if find the (certainly improved) revised version still less than satisfactory, I can recommend the manuscript for publication if the authors address the above criticism.

Macroscopic Matter Wave Quantum tunnelling

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Abstract

Quantum tunnelling is a phenomenon of non-equilibrium quantum dynamics and its detailed process is largely unexplored. We report the experimental observation of macroscopic quantum tunnelling of Bose Einstein Condensate in a hybrid trap. By exerting a non-adiabatic kick to excite a collective rotation mode of the trapped condensate, a periodic pulse train, which remains as condensate, is then out-coupled by quantum tunnelling. This non-equilibrium dynamics is analogue to tunnelling ionization. The imaged tunnelling process shows the splitting of matter wave packet by the potential barrier. The controversial "tunnelling time" question is found inadequate, from the point of view of wave propagation. The realized matter wave pulse train can also be a passive pulsed atom laser for atom interferometer applications.

INTRODUCTION

Quantum tunnelling, as a non-equilibrium process, is one of the most interesting problem among modern quantum physics. It has been investigated in microscopic systems such as alpha decay of nucleus [1], in quantum cosmology [2], tunnelling in Josephson's junction [3], and in biology [4, 5]. Recently, the development of ultrafast laser allows the attosecond angular streaking and attoclock techniques to investigate the temporal momentum distributions of electron, based on the tunnelling ionization [6–11]. However, quantum tunnelling has no counterpart in the classical physics and its process remains mysterious. Particularly, the controversial tunnelling time problem is still unsettled [10, 12–14]. Because of the realization of Bose-Einstein condensate (BEC), the macroscopic quantum tunnelling (MQT) is now experimentally feasible. The interaction-assisted tunnelling of single- and multi-particle has been studied experimentally [15–17]. The tunnelling efficiency of BEC in an accelerating [18] or shaking [19] periodic potential well has also been studied in momentum space. Such experiments have provided an unique approach to perform many-body physics experiments [20] in an isolated environment, using controllable potential and inter-atomic interactions [21]. Furthermore, to study the time evolution of the tunnelling process, the condensate, degenerate as a single macroscopic wave function and directly observable in spatial and temporal scales, is an excellent platform to study such non-equilibrium dynamics [17, 22].

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Especially the laser cooled atomic BEC with tunability of the relevant parameters, such as the barrier height, the kinetic energy and the number of atoms in the tunnelling process, is advantageous for experimental realization.

The tunnelling dynamics to open space has been demonstrated experimentally only with optical soliton by [23]. In this report, the matter wave packet was used for the first time to exhibit the tunnelling dynamics to open space and to be observed in real space. Our experiment is analogue to the strong field tunnelling ionization experiment, where the condensate acts as a bound electron wave packet. The potential formed by the optical dipole trap (ODT), the magnetic trap and gravity acts as the Coulomb potential of nucleus distorted by strong light pulse in the attosecond streaking technique. Using the macroscopic matter wave packet of BEC, the quantum tunnelling "ionization" then can be studied in details. The distribution of tunnelling atoms through the trapping potential is a chronology of the temporal evolution of a collectively excited condensate inside the trap, as temporal imaging of valence electron motion [7–9].

On the other hand, as demonstrated in the report, tunnelling output atoms remained as condensates and were converted to quasi-one-dimensional matter wave pulses as a "passive" pulsed atom laser, which has no active output coupling mechanism and can be utilized for atom interferometer applications.

TUNNELLING MATTER WAVE PLUSES

The BEC was produced in a hybrid trap with a crossed ODT configuration, accompanied by a weak magnetic quadrupole field. Due to the gravity, the potential barrier height becomes lowest at the bottom of the trap. The field gradient of the magnetic quadrupole on the X-Y plane provides a transverse confinement, as shown in Fig. 1(c). The BEC was then perturbed by varying the potential well. The protocols of the time-varying potential include: an adiabatic ramping and two types of kick (step-kick and pulse-kick), which are non-adiabatic changes in the potential well, as shown in Fig. 1(b) (see Method).

By exerting a step-kick, a series of periodic matter wave pulses was generated by quantum tunnelling as Fig. 2, which shows a typical time evolution of the BEC dynamics. Using a step-kick, 4-5 pulses were ejected from the BEC reservoir in 90 ms. In some cases with a larger condensate, the output pulsing can last for 200 ms to generate more than 10 mat-

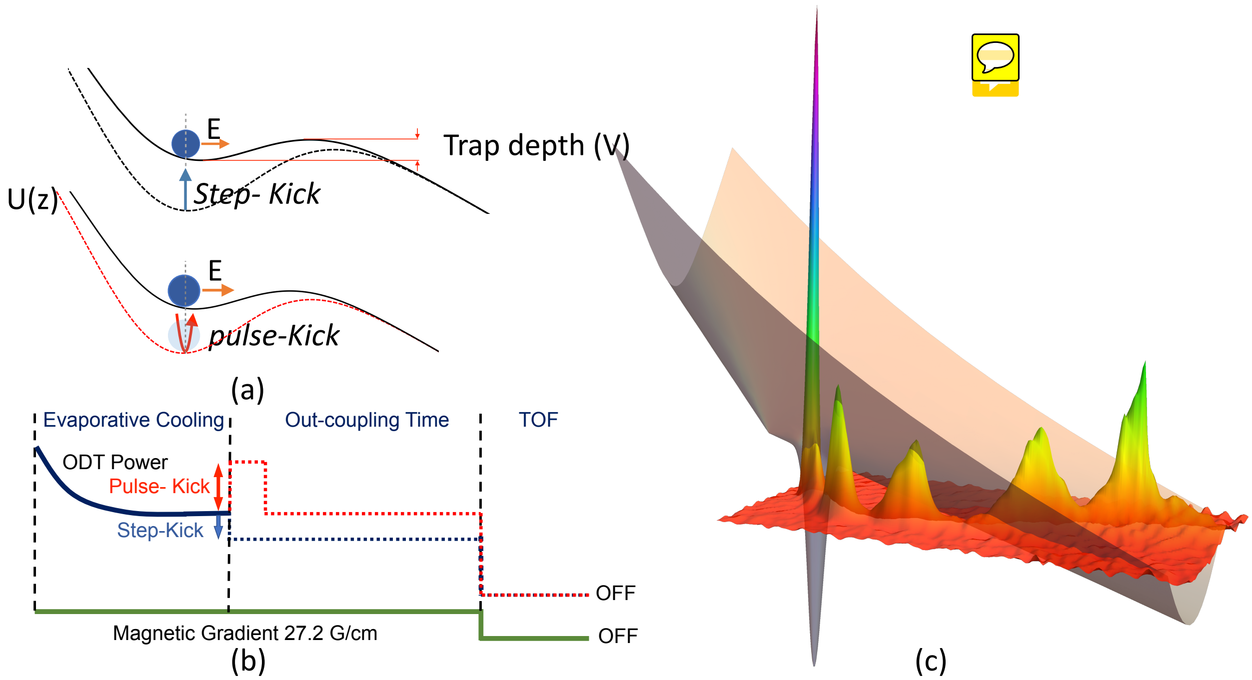

FIG. 1. (a) The schematic representation of the potential well in z-direction for the non-adiabatic kicks: step-kick and pulse-kick. The change in the trap potential is done by varying the ODT power. (b) The operation time sequence is illustrated. The red dash line is the time-varying ODT power for the pulse -kick and the blue dash line is that for the step-kick. These kicks are implemented after creating BEC. The out-coupling time is the duration in which atoms are allowed to escape from the ODT. (c) The illustration for quantum tunnelling through the trapping potential (the grey shadow surface). The out-coupled matter wave pulses (the false color peaks) decay exponentially as described by the quantum tunnelling theory.

ter wave pulses. The first pulse came out in ∼10 ms after exertion of kick. A complete pulse was ejected from the BEC reservoir in ∼ 30 ms with peak flux 6.34×10^6 atom/s as shown in Fig. 2 (d). The output pulses are with a pulse duration of 20 ms defined by full-width-half-maximum, and a repetition rate of ∼50 Hz. The duty cycle is 50%, which implies a symmetrical internal oscillating motion of the BEC reservoir. Considering the X-Y confinement by the magnetic quadrupole trap, the sinusoidal trajectory of the pulses also suggests an accompanied initial transverse momentum. Our experimental results agree qualitatively with the previous numerical simulation in several aspects, such as periodicity and pulse shape [24].

On the other hand, when an adiabatic ramping was employed in comparison with a kick, a thermal atomic beam was produced. The ODT power was ramped down adiabatically in 2 sec to the same final trap depth as the step-kick. In such a case, neither additional kinetic energy, nor collective mode excitation was given to the condensate reservoir. A straight continuous diffusing leakage from the BEC reservoir was observed as Fig. 2 (k). It clearly showed that the final trap strength of ODT was still capable of holding the reservoir against the effective gravitational force (see METHOD). The diffusing leakage goes even above the position of BEC reservoir, and shows as thermal atomic cloud, although it is still confined by the transverse trapping force. Therefore, we conclude that these are thermalized atoms energized by the ODT light assisted heating, and the condensate can tunnel through the trap only by applying a kick, even with a such low final potential barrier.

To characterise the output pulses, the time of flight (TOF) measurement was performed for the three pulses generated in 60 ms of out-coupling time. The absorption images were taken after 20 ms free fall under the gravity only, without the magnetic field, as shown in Fig. 3 (a). There were certain amount of thermal atoms observed in the very beginning of the first output pulse. The rest of the sub-sequential pulses shows no thermal expansion that indicates them preserved as condensate. Fig. 3 (c) is a plot of pulse separation expansion with the free fall time. Its linearity implies that the pulses ejected from the BEC reservoir are periodic, and the pulse rate shows no slowing down.

In quantum tunnelling, the tunnelling efficiency η only depends on the potential barrier $U(\mathbf{r})$ and the kinetic energy of the matter wave E_k. Using the WKB approximation, it can be expressed as [25]:

$$\eta = \exp\left(-2\int_{\boldsymbol{r_1}}^{\boldsymbol{r_2}} \sqrt{\frac{2m}{\hbar^2}(U(\mathbf{r}) - E_k)}d\mathbf{r}\right) \tag{1}$$

where $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ are the classical turning points on the both sides of the barrier. With a constant tunnelling efficiency η, the tunnelling probability decays exponentially. It is an important characteristic of the quantum tunnelling. In our pulsed BEC experiment, the out-coupled atom number in the i^{th} pulse is related to the tunnelling efficiency and given by:

$$N_i = N_0\eta(1-\eta)^{i-1} \tag{2}$$

where N_i is the number of atoms in i^{th} pulse, N_0 is total number of the condensate. The exponential decay then exhibits as a discrete geometric progression. The Fig. 3(b) shows

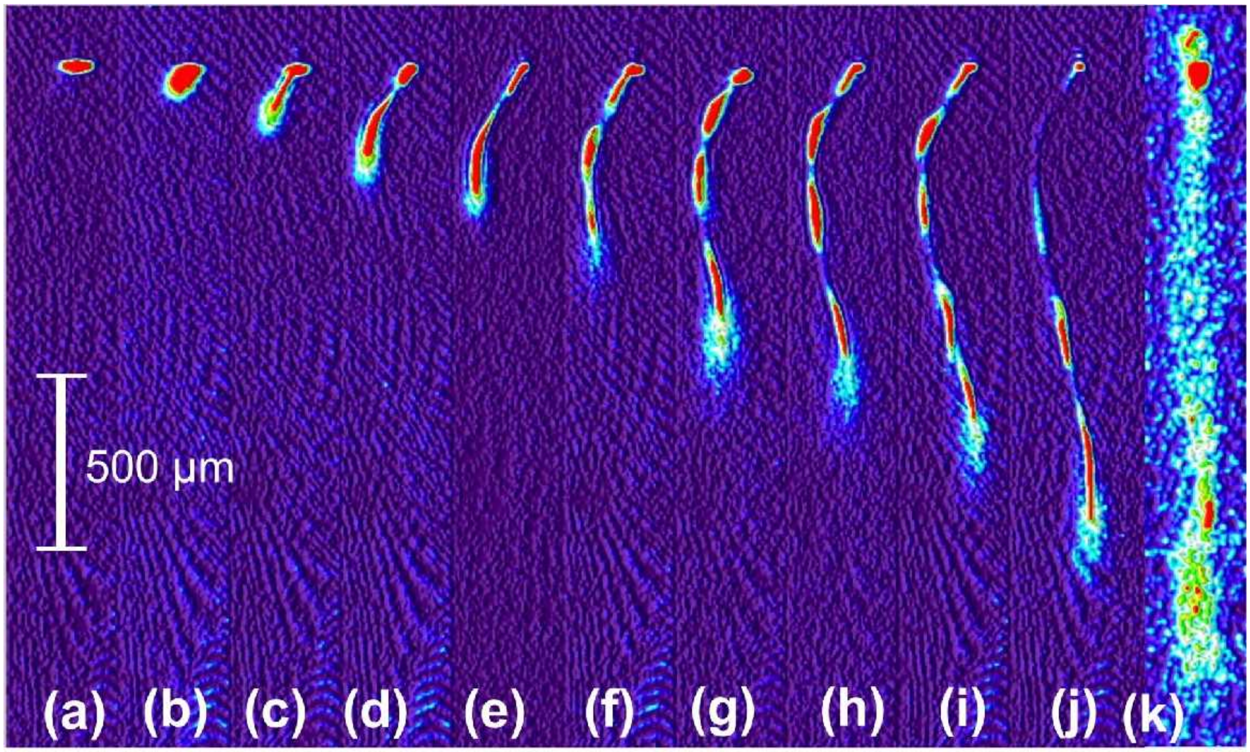

FIG. 2. (a) The absorption images of the BEC created in the hybrid trap. Figures (b), (c), (d), (e), (f), (g), (h), (i) and (j) are the absorption images taken during the periodic tunnelling by applying a step-kick. In Fig. (b) and (c), the atoms started to eject from the BEC reservoir at 10 ms and 20 ms after the kick, respectively. In Fig. (d), the first complete pulse with peak flux of 6.34×10^6 atom/s out-coupled at 30 ms after the kick. Figures (e), (f), (g), (h), (i) and (j) are periodic pulses out-coupled at 40 ms, 50 ms, 60 ms, 70 ms, 80 ms and 90 ms after the kick, respectively. In (k), after creating the BEC, the ODT power was reduced adiabatically and no kick was applied. It shows that the BEC was still held in the original position, and a continuous thermal atomic beam diffused out in both upward and downward directions.

that the output pulses decay as described by Eq.(2), i.e., a linear line in logarithmic scale. Thus, the tunnelling efficiency η is a constant, and we conclude that the kinetic energy of the pulses E_k, remains constant during the course of the 80 ms out-coupling time. The kinetic energy of atoms did not dissipate and the oscillation was not damped. Meanwhile, using Eq.(2), the expected total number of atoms remaining in the BEC reservoir after 4 pulse output is extrapolated and shown as the grey point connected by a dashed line in Fig. 3. However, the observed number of atoms remained in the BEC reservoir (the red point)

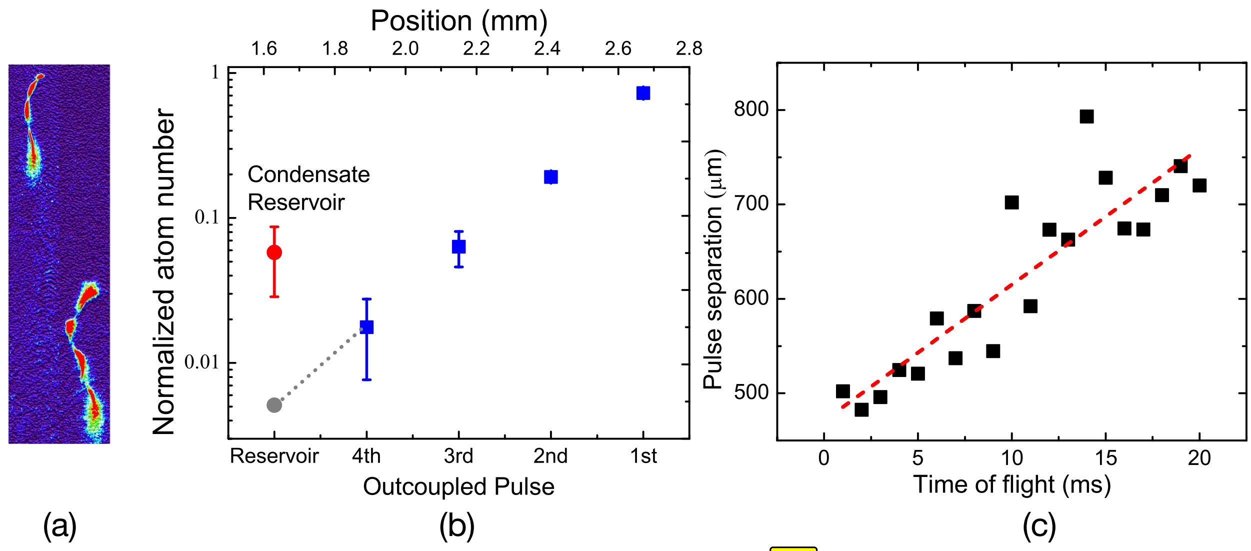

FIG. 3. (a) The comparison between the 1 ms and 20 ms TOF images of the matter wave pulses after 60 ms out-coupling time. Both the reservoir and the out-tunnelling pulses remain as quantum degeneracy. No thermal expansion was observed. (b)The decay of the out-coupled pulses. The atom numbers of each pulses are normalized to the total out-coupled atom numbers. The bottom axis represents order of the out-coupled pulses and the top axis denotes the vertical position of pulses. The geometric progression decay, Eq. (2), exhibits as a linear line in the logarithmic scale. To compare with the measured residual atom in the condensate reservoir, the grey-dotted point is the extrapolated projection from the out-coupled pulses using Eq. (2). The unexpected large residual in the reservoir indicates that 10% of the entire condensate will eventually not tunnel. (c) The pulse separations versus the time of TOF.

was more than the extrapolated atom number (the grey point). It implies that 10% of the entire BEC was not excited, or may transfer from the rotational mode to the vibrational mode at a later time, as described in [26].

TUNNELLING EFFICIENCY

The pulse-kicks were applied to control the tunnelling efficiency η (Eq. 2). It allows us to set various final barrier height $U(\mathbf{r})$, and to tune the kinetic energy E_k by varying the pulse height, the kick strength. The tunnelling efficiency decays exponentially as the barrier height increases with a fixed kick strength, as shown in Fig. 4(a). The kick strength,

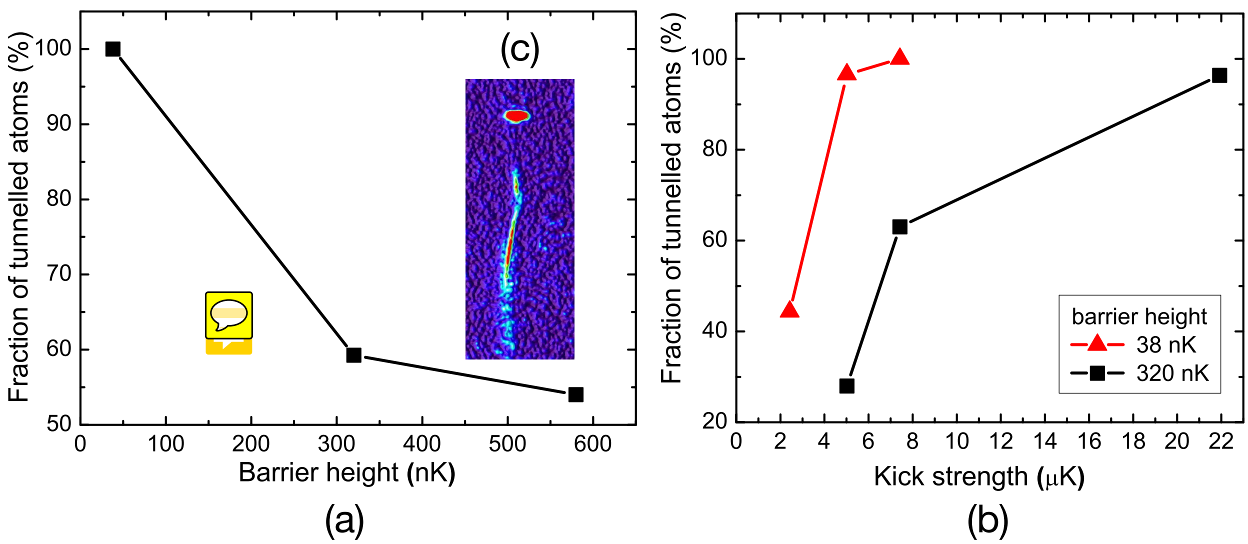

FIG. 4. The tunnelling efficiency (fraction of tunnelled atoms) depends on barrier height and kick strength. Atoms are out-coupled with different efficiencies by applying a pulse-kick with various final barrier heights and kick strengths. The kick strength corresponds to the 2 ms sudden increment of the trap depth, and the barrier heights are the trap depth before and after the pulse kick. (a) By varying the barrier height with a constant kick strength of 7.42 μK, the fraction of tunnelled atoms is plotted. At lower barrier height, large number of atoms can be out-coupled. (b) the kick strength was varied with the barrier heights of 38 nK and 320 nK. (c) is a typical out-coupled pulse by a pulse-kick.

expressed as the increment of the trap depth, is related to the kinetic energy E_k given to the the condensate. The kick strength v.s. the tunnelling efficiency is shown in Fig. 4(b), where the final potential barrier height was kept as a constant. While the pulse-kick gives an excitation energy higher than the barrier height, the output reach to 100%. It shows an exponential saturation that qualitatively agrees with Eq. (1).

INTERNAL DYNAMICS

Although the trajectory of the output pulse in Fig. 2 has mapped out the internal dynamics of the condensate reservoir, the conventional TOF method was utilized for further confirmation. The momentum oscillation inside the ODT, the excited collective mode, was mapped as the CM (center of mass) spatial oscillation after TOF. The result is as Fig. 5, which shows sinusoidal oscillations for both the velocities in y and z directions (V_y and V_z), but with a $\pi/2$ relative phase shift. The time period of oscillation is 20 ms that is in good agreement with the trap frequency. The amplitudes of the velocity oscillation are $V_{z,y} = 1.5$ μm/ms and the kinetic energy per atom is calculated to be 23.5 nK that is lower than the trap depth (38 nk). It is worthy to note that the created ^{87}Rb BEC is with a sound velocity of 0.60 μm/ms, and a chemical potential of 3.8 nK.

In a three-dimensional trap, the pure CM rotation mode, which is the lowest collective excitation mode with the rotation frequency $\omega = \omega_0$ and the angular momentum $l = 1$ [27, 28], is the most pronouncing mode by applying a weak perturbation. Using the TOF measurement, the observed trajectory and the distribution of atom number confirmed that the kick excites a collective rotational mode in the condensate with an energy lower than the potential barrier. The output matter wave pulse from the reservoir are due to *quantum tunnelling*. In this process, the output pulses and the reservoir itself remain in the phase of quantum degeneracy.

TUNNELLING PROCESS

The dynamics of BEC can be well described by the time-dependent Gross-Pitaevski equation (GPE) that is a mean-field approximation. While the non-linear interaction term in GPE, $a_s|\psi(\boldsymbol{r}, t)|^2\psi(\boldsymbol{r}, t)$, becomes sufficiently small and negligible in comparison with the kinetic and potential energy terms, then it becomes a standard time-dependent Schrödinger equation. The BEC, which has a length scale of 'μm' and an evolution time scale of 'ms', is an excellent object to study quantum tunneling. Unlike a single particle wavefunction that can only be fully explored by statistical measurement, the distribution of the BEC that exhibits the probability density by its number density can be imaged using a CCD camera by a single image. To resolve the controversial "*tunnelling time*" problem [29], we hence performed quantum tunnelling using BEC, which is considered as a giant wave function with slow motion dynamics.

A series of images was taken with high time resolution from 0 ms to 40 ms, while the first output pulse is completely coming out by exerting a step-kick. On the right side of Fig. 6. the typical images at 5 ms, 27 ms and 37 ms show the BEC distributions for the initial, tunnelling and final stages, respectively. To observe the time evolution of tunnelling,

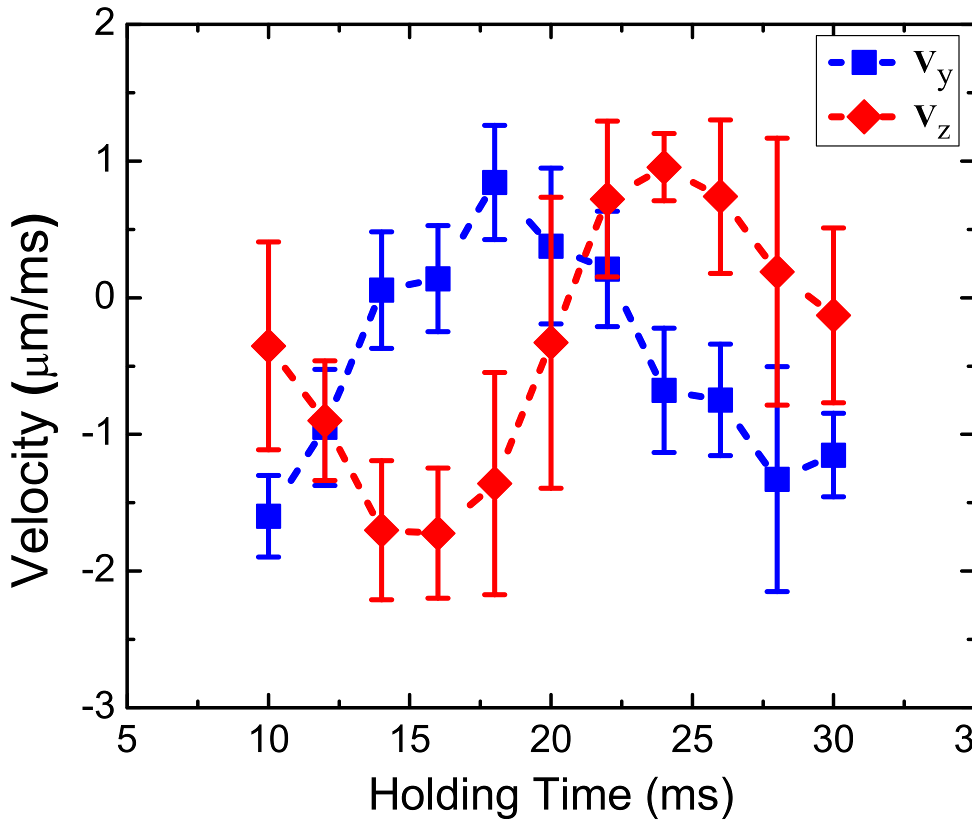

FIG. 5. The internal dynamics of BEC by execution of the step-kick. The horizontal velocity V_y and vertical velocity V_z are the initial velocities of the condensate in the trap caused by kick. The velocities are extracted from the 10 ms TOF measurement of the atoms with various holding time after giving a step-kick, but without generating matter wave pulse.

the images with 2 ms time interval were taken and processed by summing each pixel in y-direction to convert them as an one-dimensional (z-direction) distribution, as shown in the left of Fig. 6. The whole process can be categorised in three stages: the initial stage where the condensate acquires some initial momentum, the tunnelling stage where the tunnelling of atoms occurred, and the final stage where a complete BEC pulse is produced. A steep change in the velocity of the front (lower) end of the tunnelling wave packet appears in the tunnelling stage between 20-30 ms. After the completion of tunnelling, 30-40 ms, the velocity is then slowed down. In contrast to the classical physics, where the potential barrier slows down the tunnelling particle's velocity, the wave packet "suddenly" emerges from the

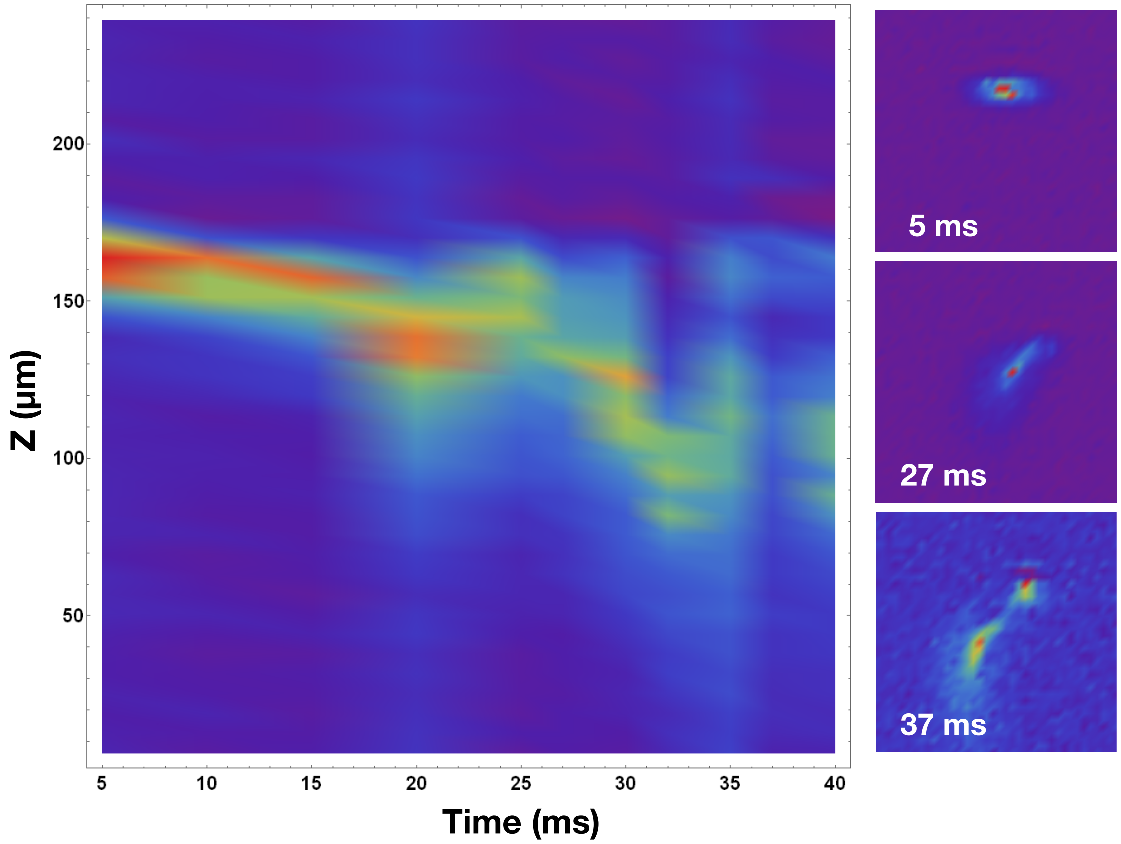

FIG. The sum of the number density along horizontal y-direction presented by false color for all images before and after the tunnelling. The horizontal axis represents out-coupling time of atoms from the reservoir after applying a step-kick. The vertical axis denotes the downward position (along z-direction). It shows the three stages of the tunnelling process: the initial stage, tunnelling and final stages.

other side of the barrier, as a wave packet splitting in the quantum tunnelling. Although the tunnelling wave packet grows up within a finite time period, it may look "instantaneous" and causes the no-tunnling-time-delay measurement in [10]. However, as shown in our experiment, quantum tunnelling has no classical analogue and its entire process can not be properly described by particle transmission [14].

DISCUSSION

The quantum tunnelling was demonstrated using a hybrid cross optical dipole trap and exerting a non-adiabatic kick, which results in a collective rotational motion in the condensate. Unlike the previous work [21], the density dependent chemical potential, the nonlinear term $|\psi|^2$ of the GPE, is negligible under the given energies of the barrier and the kicks. In such cases, the BEC is an ideal platform, not only for many-body quantum physics, but as a giant wave function to study the quantum dynamics of single particle. In our experiment, the detailed process of quantum tunnelling to open space was recorded as a sequence of images in real space. The time evolution of this non-equilibrium dynamics, which was induced by the collective rotational mode of BEC, was then studied. The tunnelling efficiency was found to remain constant, and there is no energy loss of the internal rotational motion during more than 80 ms output time. The tunnelling output pulses remains in BEC phase as indicated by the TOF measurement, and its trace is a temporal map to the internal evolution, as a chronology of its dynamics. The similar methodology has been used to investigate the spatiotemporal motion of valence electron in molecules [8, 9]. This result sheds light on the study of tunnelling ionization, which is currently under intensive study using the ultrafast technology, particularly on the "tunnelling time" problem. As shown in our observation, due to the nature of matter wave in quantum regime, the "tunnelling time" that can only be well-defined from the point of view of particle, is inadequate [14]. Instead, the quantum tunnelling process should be described by wavepacket distribution using wave mechanics.

METHOD

BEC and Trap potential

Our experimental setup is a hybrid trap [30] together with magnetic transport via a linear motor. Initially, ^{87}Rb atoms were loaded to a 3D MOT and then magnetically transported to the UHV science cell through a differential pump of 3 mm diameter. In the science cell, $\sim 2.1 \times 10^8$ atoms were successfully transported with a temperature of 41 μK. Then the atomic cloud was pre-cooled to 8.2 μK using the RF-knife-cut cooling method. The atoms were subsequently loaded to a hybrid trap that consists of a crossed ODT and a weak magnetic trap to provide additional confinement in the horizontal direction, and partially compensate the gravity. The final stage to reach the quantum degeneracy is the forced-evaporative cooling that was performed by ramping down the ODT power to a effective trap depth of 580 nK which yielded a Bose-Einstein condensate of 1×10^5 atoms with a peak number density 1.02×10^{13} cm^{-3}. The final trapping frequencies were estimated to be: the radial frequencies $\omega_r = 2\pi \times 251$ Hz and the axial frequency $\omega_x = 2\pi \times 82$ Hz and $\omega_y = 2\pi \times 31$ Hz. In our experiment, the location of ODT relative to the center of magnetic quadrupole trap is $(x, y, z) = (0, 0, z_0 = 90$. With the gravity on the z direction, the relevant barrier potential, as shown in Fig. 1 (a), can be expressed as:

$$U(x,y,z) = -U_t \left\{ \exp\left(-2\frac{x^2 + (z-z_0)^2}{w_0^2}\right) + \exp\left(-2\frac{y^2 + (z-z_0)^2}{w_0^2}\right) \right\} + \beta(x,y,z) \quad (3)$$

where the total effective downward force is $\beta(x,y,z) = -m_{Rb} g z + \mu_{Rb} \frac{dB}{dz} \sqrt{\frac{x^2}{4} + \frac{y^2}{4} + z^2}$, w_0 is the ODT beam waist, and U_t is the trap depth. The center of atomic cloud corresponding to $(0, 0, z_0)$ is then located at the stable point $dU/dz = 0$, where $z = z_0 - w_0^2 \beta / 8U_t$ by the approximation of the potential well as a quadratic function. The trapping force in the horizontal X-Y plane given by the magnetic quadrupole provides an extra confinement, while the atoms move out of ODT. The 3D illustration of the potential shape is as Fig. 1 (c).

Kick

After creating BEC in the hybrid trap, a '*kick*' was then applied to the condensate to perform quantum tunnelling. The '*kick*' is a sudden change in the ODT power. In our system, two different types of kick were employed. The first type is defined as a 'step' kick in which ODT power was reduced to a certain value and maintained during out-coupling time as shown in Fig. 1 (b). The second type is a 'pulse' kick, in which ODT power was increased suddenly for 2 ms and turned back to the previous value. In order to partially compensate gravity, a magnetic gradient was provided by the compensation quadrupole field to reduce the downward acceleration as low as 0.77 m/s^2. The advantage of reduced effective gravitational acceleration is to have a longer observation time by avoiding atom's quick escape out of the field of view. This quadrupole magnetic field also provided a confinement in X-Y plane.

The sudden change in the ODT power by ΔU_t is equivalent to shift the stable point (the bottom of the potential well) by $\sim \Delta U_t \frac{w_0^2 \beta}{8U_t^2}$. Because the atomic cloud cannot follow such quick change, it sits on the original position and gains an energy of $\gamma^2 \frac{\beta^2 w_0^2}{8U_t}$, where $\gamma = \Delta U_t / U_t$ is the ODT power change ratio. The kicks shifted the location of potential minima and effectively moves the condensate away from the potential minima. Then, a collective mode of BEC was excited due to such a non-adiabatic process [27, 28]. The step-kick also opens a tunnelling channel on the bottom of the hybrid trap, where the potential barrier is lower because of the gravitational field. Typically, the step-kick is to reduce the trap depth by 30-40%.

Additionally, to compare with non-adiabatic kicks, once BEC was produced, then the ODT power was further ramped down adiabatically in two seconds to the trap depth same as the final potential barrier of a typical kick.

Internal dynamics TOF measurement

A step-kick was applied with a relatively low kick strength, just below the tunnelling threshold, to prevent any observable tunnelling. After a holding time, the BEC was then released from ODT. The position of CM of the atomic cloud was measured with a 10 ms TOF. This was performed for various holding times, from 10 ms to 30 ms with 2 ms time resolution. The periodic oscillation of the CM displacements was found and the variation of the initial velocity after releasing from trap was derived.

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