**Supplementary material for *Scalable Reservoir Computing on Coherent Linear Photonic Processor***
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**Supplementary note 1: Operation principle of input mask circuits**

Figure S1 explains the detailed operation of our optical mask. As described in eq. (1) in the main article, the signals from our input circuit is described by the following equation.

$$s_l(t) = \int_{-\infty}^{t} h_l(\tau) u'(t-\tau) d\tau, \tag{S1}$$

where $h_l(\tau)$ is an impulse response of the input port to the $l$-th output port. For the comparison with the digital mask operation, we consider a discretized time $t(n,i)$ corresponding to each interval of duration $\theta$ and $T$: $t'(n, i) = nT+i\theta$, where $n \in Z$ and $i \in [1; N]$. Then, the discretized expression is described as following convolution operation formula:

$$s_l(n,i) = \mathbf{h'}_\mathbf{l} \cdot u'(n,i) + \mathbf{h''}_\mathbf{l} \cdot u'(n-1,i) \tag{S2}$$

where $\mathbf{h'}_\mathbf{l}$ and $\mathbf{h''}_\mathbf{l}$ are the discretized response of input mask circuit, which can be expressed as

$$\mathbf{h'}_\mathbf{l} = \begin{pmatrix} h_{l,1} & h_{l,2} & h_{l,3} & & \cdots & h_{l,K-1} & h_{l,K} \\ 0 & h_{l,1} & h_{l,2} & & \cdots & h_{l,K-2} & h_{l,K-1} \\ 0 & 0 & h_{l,1} & & \cdots & h_{l,K-3} & h_{l,K-2} \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & \cdots & & 0 & h_{l,1} & h_{l,2} \\ 0 & 0 & \cdots & & 0 & 0 & h_{l,1} \end{pmatrix}, \tag{S3}$$

$$\mathbf{h''}_\mathbf{l} = \begin{pmatrix} 0 & 0 & 0 & & \cdots & 0 & 0 \\ h_{l,K} & 0 & 0 & & \cdots & 0 & 0 \\ h_{l,K-1} & h_{l,K} & 0 & & \cdots & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ h_{l,3} & h_{l,4} & \cdots & & h_{l,K} & 0 & 0 \\ h_{l,2} & h_{l,3} & \cdots & & h_{l,K-1} & h_{l,K} & 0 \end{pmatrix}, \tag{S4}$$

$$h_{l,i} = m_{l,i} \exp(j\Psi_{l,i}), \tag{S5}$$

where $m_{l,i} \in [0; 1]$ and $\Psi_{l,i} \in [0; 2\pi]$ are the amplitude and phase delay from $l$-th delay line. By considering the ideal optical impulse with repeating time of $T$, the $\delta(n,i)$ is described as

$$\delta(n,i) = \begin{cases} 1 & (\mathrm{i} = 1) \\ 0 & (\mathrm{i} \neq 1) \end{cases}, \tag{S6}$$

Then, we can derive the following filtered outputs for the case of *M*=1

$$s_l(n,i) = \begin{pmatrix} s_l(n,1) \\ s_l(n,2) \\ \vdots \\ s_l(n,N) \end{pmatrix} = \begin{pmatrix} h_{l,1} & 0 & \cdots & 0 \\ h_{l,2} & h_{l,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{l,N} & h_{l,N-1} & \cdots & h_{l,1} \end{pmatrix} \begin{pmatrix} u'(n) \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{S7}$$

which can be simply described as

$$s_l(n,i) = h_{l,i} u'(n). \tag{S8}$$

Thus, the input optical circuit acts as all-optical mask for the reservoir computer. Note that the virtual node interval $\theta$ is determined by delay difference in the optical convolutional circuits. As can be seen in Fig. S1, this operation up-convert the bandwidth of transmitted signals beyond the one for the RF-input. Equation (S8) requires the $6NL$ times multiplication for the operation. Thus MAC/s in the input mask circuit for $M$=1 is estimated as $6NL/\theta$.

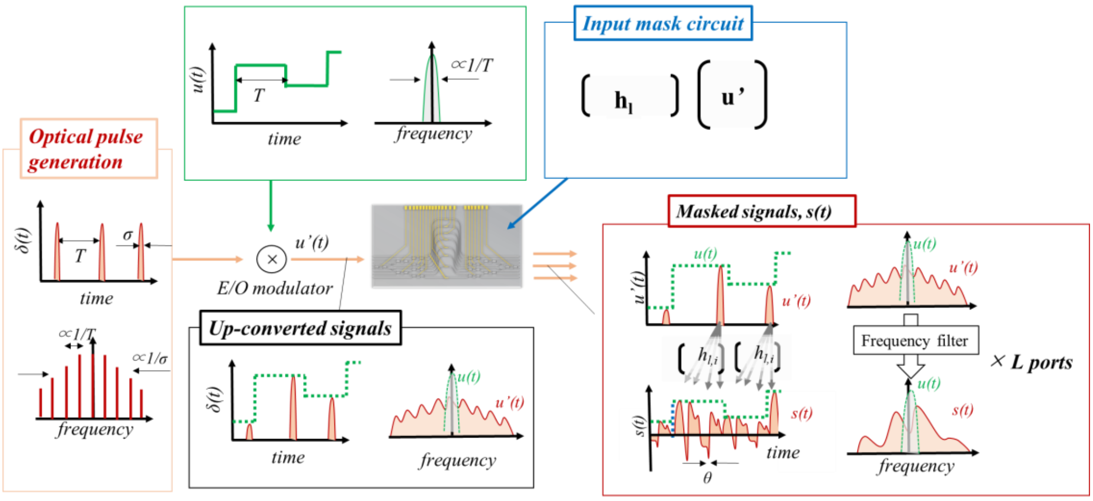
For $M$-dimensional input case, the repeating time of optical pulse is set to T' as described in method part. Then, the $\delta(n,i)$ is described as follows;

$$\delta(n,i) = \begin{cases} 1 & (i = N/M) \\ 0 & (i \neq \mathrm{N/M}) \end{cases}, \tag{S9}$$

Then, the modulated optical pulse $u'(n,i)$ for discretized time $t(n,i)$ can be described as the orthonormal basis of input vector

$$\begin{pmatrix} u'(n,1) \\ \vdots \\ u'\left(n,\frac{N}{M}\right) \\ \vdots \\ u'\left(n,\frac{2N}{M}\right) \\ \vdots \\ u'(n,N) \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ u_1(n) \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ u_2(n) \\ \vdots \\ 0 \end{pmatrix} + \cdots + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ u_M(n) \end{pmatrix} \tag{S10}$$

To satisfy the synchronization between RF-input and optical pulse, $N/M$ should be set to integer value. The masked signal can be calculated by using eqs. (S2) and (S10). For this operation, 2(4M-1) $\sigma NL$ times MAC operation is required.




**Supplementary Figure 1. Processing flow in the input mask circuit.** In this system, the radio-frequency (RF) input signal *u(t)* with modulation time interval *T* is coded to the amplitude of the electromagnetic field of optical pulse, *δ(t)*, by an optical electro-optic (E/O) modulator. Then, the ultrafast optical pulse up-converts the RF-signal to the optical frequency, and this electro-optic (E/O) conversion results in a sinusoidal nonlinearity. The up-converted signals *u'(t)* are weighted along spatiotemporal domain by transmitting the input mask circuit.