

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

The authors present an experimental and theoretical investigation of a new type of multimode soliton they observe in a small diameter GRIN fiber. The paper is presented in an expository style with the main experimental result first, followed by numerical simulations and theoretical considerations. The result is new and interesting as the multimode soliton observed does not conform to the expected theory with the output pulsewidth set almost exclusively by the dispersive parameters of the fiber used.

I find this paper to be a timely addition to the rapidly growing literature on multimode solitons. The authors comments on the potential utility of this new soliton type to high-power beam delivery and ML lasers seem reasonable. I would recommend its publication in communications physics.

I do however have some comments and clarifications I would like the authors to address before publication:

(1) The paper would benefit from a longer, more detailed introduction that more fully places this work in the context of previous multimode soliton work in GRIN fibers.

(2) Figure (1) is very dense with alot information crammed into one figure. It is quite hard to follow. Perhaps the authors could consider moving some of the less important details out of this first figure.

(3) Also in Figure (1)b, do the authors have an explanation for the very sharp dip seen in the beam waist of the 1550nm (67fs) trace, when compared to the other traces. And why this sharp "resonance" is not seen in Figures 1(a) and (c)?

(4) The experimental observations are presented for a launch condition of 3 axial-symmetric LG modes.

- Does this soliton survive with launch condition that are off-axis?
- Does this soliton survive with the launch power distributed across a larger number of axial modes? In the supplementary material Figures S2 and S3 the authors talk about 15 modes, but the launched power still only appears to be in 3-4 modes?

Reviewer #2 (Remarks to the Author):

The authors report an experimental and numerical study of soliton propagation in multimode fibers.

They find experimentally that a peculiar multimode soliton can be generated. This nonlinear wave is basically single-mode, its energy and duration are quite independent on the input pulse parameters, but depend on dispersion.

The numerical simulations based on a multimode nonlinear Schrödinger equation reproduce well the experimental measurements. A simple formula is also presented, which permits to link the pulse duration to the dispersive properties of the fiber.

In general, I find this work quite interesting and the result can be considered novel, even if not groundbreaking. The style of the paper is sometimes too emphatic, starting from the title. I think that a scientific paper always solves a mystery in some sense. There is no need to stress that an open question is answered (a mystery is solved). That said, I think that the paper can be published.

Remark.

Could the author comment the following simple interpretation of the phenomenon? Most of the energy is contained in the fundamental mode, which experience the strongest Raman shift. This wavelength shift corresponds to a temporal delay with respect to the other modes which propagates almost linearly, thus inducing an extra walk-off. For long enough propagation distance, this monomode soliton is essentially a scalar soliton carried by the fundamental mode of the fiber, which interacts only weakly with higher order modes.

Reviewer #3 (Remarks to the Author):

Review of "Solving the mystery of the walk-off soliton..." by Zitelli et al.

In this paper some interesting observations of multimode pulse propagation are reported. If I understand correctly, a multimode soliton (which the authors call a spatiotemporal soliton) forms (the numerics in Fig 3 show this most clearly). This is followed by transfer of power to the fundamental mode, simultaneous with soliton self-frequency shifting.

In its current form the paper is not ready for publication. After revision to address the points below it may be suitable for publication.

- The relation of the present work to that in Ref. 14 should be stated explicitly. Exactly what does this work contribute that is not essentially contained in Ref. 14? The sentence that refers to Ref. 14 only mentions intermodal SRS.
- If multiple modes are propagating in GRIN fiber and power transfers to the fundamental mode, is this just well-known Raman beam cleanup, as described in Terry et al., Opt Exp 15, 17509 (2007)?
- The paper is written in an odd way. Rather than set the general context for the work, it starts by stating a specific surprise observed in experiments. It is claimed to be a surprise that the predictions of Refs. 10-12 are not observed. Have any prior experiments observed the predictions of Refs. 10-12? If not, why is it a surprise? So the premise of the mystery is in question.
- Early in the paper the authors very casually make a remarkable claim: "spatiotemporal solitons were observed at the fiber output..." What is the evidence for this claim? The best evidence would be monitoring of the evolution versus distance, perhaps by a cut-back experiment. That would seem to be easy to do with the parameters of the work here. Less attractive but acceptable would be a direct comparison to results of numerics with fixed fiber length and input pulse energy varying. The authors seem to take for granted that some aspect of Fig. 1 is evidence of spatiotemporal soliton formation.

Specific technical points

- The pulse duration (Fig. 1) does not vary over the range of 2-4 nJ. In contrast, the beam waist is very sensitive to the input pulse energy. Why say that they are "in correspondence?"
- I don't understand the application of Eq 1 to a multimode situation.
- Why does the temporal walkoff length have an adjustment constant? This is just the modal dispersion. What should the constant be?
- The abstract states that multimode solitons were only recently investigated experimentally and cited Refs. 6-8. Multimode solitons were experimentally investigated in Ref 3.

Minor points

A terminology point for the authors to consider: historically the expression "spatiotemporal soliton"

has largely referred to solitons that balance dispersion, diffraction, and nonlinearity. It seems strange to use it now for a situation with a waveguide or trapping potential. In this particular case there are only a few transverse modes. The term "multimode soliton" would seem to be more descriptive than "spatiotemporal soliton." I understand that terminology is not necessarily scientific and only offer this as a suggestion.

In a few places the manuscript uses atypical English. For example, "Fig. 1 resumes the experimental evidence..."

Manuscript ID: COMMSPHYS-21-0278-T

Title: Solving the mystery of the walk-off soliton

Authors: M. Zitelli, F. Mangini, M. Ferraro, O. Sidelnikov, S. Wabnitz.

Point-to-point answers to the Reviewers:

Reviewer #1 (Remarks to the Author):

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I do however have some comments and clarifications I would like the authors to address before publication:

(1) The paper would benefit from a longer, more detailed introduction that more fully places this work in the context of previous multimode soliton work in GRIN fibers.

Our response: We are grateful to the Reviewer for the positive appreciation of the innovation and quality of our work, and for the recommendation to publish our manuscript on Communications Physics. Following the Reviewer's suggestion, we have extended our introductory section by providing more details of previous multimode soliton works in GRIN fibers.

A Conclusions section has been added, with emphasis on the possible applications.

(2) Figure (1) is very dense with a lot information crammed into one figure. It is quite hard to follow. Perhaps the authors could consider moving some of the less important details out of this first figure.

Our response: in order to better illustrate the figure, it was split into three separate figures, specifically Figure 1, 2, and 3, and we added colours in order to more clearly highlight and compare the illustrative plots.

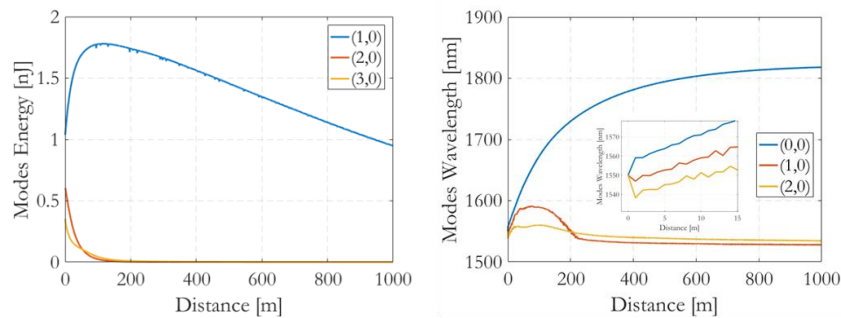
(3) Also in Figure (1)b, do the authors have an explanation for the very sharp dip seen in the beam waist of the 1550nm (67fs) trace, when compared to the other traces. And why this sharp "resonance" is not seen in Figures 1(a) and (c)?

Our response: The sharp dip (or resonance) in the beam waist that is reported in Fig. 1b (now Fig. 2) for 67 fs pulses is attributed to soliton dynamics in the presence of Raman self-frequency-shift, combined with wavelength-dependent fiber losses - as already observed in Ref. [11] (previously Ref.14)- and to the sensing properties of our InGaAs camera. As well-known from the theory of singlemode fibers, shorter pulses suffer much faster soliton self-frequency shift when their energy is increased. Once that the soliton wavelength red-shifts above 1700 nm, the soliton is progressively absorbed because of increased linear fiber losses. On the other hand, dispersive waves which remain at the input pump wavelength around 1550 (as a result of Raman

soliton fission) experience relatively low loss at the fiber output. As a result, the camera (which integrates the beam energy from all wavelengths between 1000 and 1850 nm) sees a wider beam which results from a superposition of the soliton and (a dominating contribution of) dispersive waves. In addition, it should be considered that the response of the InGaAs camera sharply drops for wavelengths above 1850 nm, which impairs its ability to record the red-shifting soliton above this wavelength. This explains why the beam width curve is so narrow for the shortest solitons.

On the other hand, the "sharp" resonance is not seen in Fig.1(a) and Fig.1(c) (now Figure 1 and Figure 3, respectively), because the autocorrelator and the optical spectrum analyzer we can always directly monitor the pulse duration and the wavelength of the Raman soliton, even if its amplitude drops with respect to the residual dispersive waves around 1550 nm.

The multimode Raman soliton dynamics is well illustrated in the following figures, already published in our Ref. [11]. Here a 67 fs pulse is simulated in GRIN fiber, distributed over 3 axial modes (0,0), (1,0), (2,0). The figures reported below show the modal energy (left) and wavelength (right) vs. distance. After 120 m of fiber, the beam energy is transferred to the fundamental mode, and its wavelength overcomes 1700 nm; correspondingly, the fundamental mode is attenuated, while residual higher-order modes conserve very small, but constant energy.



In order to explain this, we added on page 6 the sentence “The explanation of the sharper curve ... to record the red-shifting soliton above this wavelength”.

(4) The experimental observations are presented for a launch condition of 3 axial-symmetric LG modes.

- Does this soliton survive with launch condition that are off-axis?
- Does this soliton survive with the launch power distributed across a larger number of axial modes? In the supplementary material Figures S2 and S3 the authors talk about 15 modes, but the launched power still only appears to be in 3-4 modes?

Our response: On page 2, Fig. 1 of the Supplementary material (previously Fig. 2), we show the propagation of a 67 fs pulse distributed over 15 modes at the fiber input; some of the modes have same power, that is why they appear overlapped in Fig. 1 (left). The singlemode soliton is still produced after 20 m, but at higher input energy (4 nJ instead of 2).

Fig. 1 may well represent the condition of off-axis excitation; for a larger number of axial modes, similar results are obtained in our simulations.

We added on page 2 of the Supplementary: “the simulated coupling conditions are typical of non-axial excitation.”

Reviewer #2 (Remarks to the Author):

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They find experimentally that a peculiar multimode soliton can be generated. This nonlinear wave is basically single-mode, its energy and duration are quite independent on the input pulse parameters, but depend on dispersion.

The numerical simulations based on a multimode nonlinear Schrödinger equation reproduce well the experimental measurements. A simple formula is also presented, which permits to link the pulse duration to the dispersive properties of the fiber.

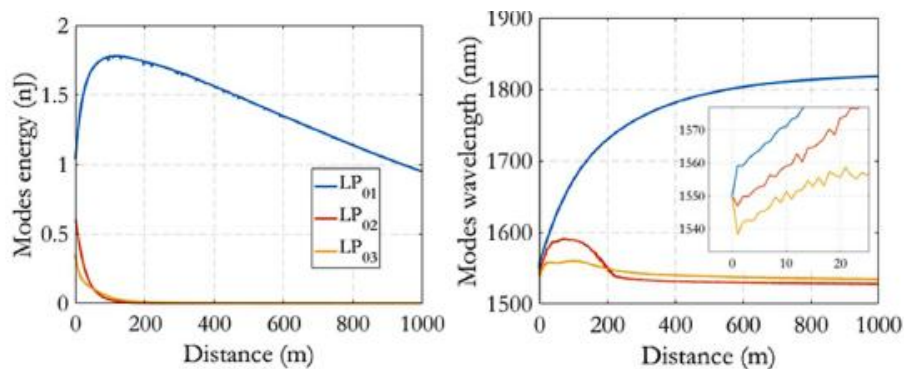
In general, I find this work quite interesting and the result can be considered novel, even if not groundbreaking. The style of the paper is sometimes too emphatic, starting from the title. I think that a scientific paper always solves a mystery in some sense. There is no need to stress that an open question is answered (a mystery is solved). That said, I think that the paper can be published.

Our response: We thank the reviewer for the very positive appreciation of the interest and novelty of our work. Concerning the remark about the paper title, we would still like to keep it as in the original manuscript, however, we are ready to follow the Editor's indication on this issue.

Remark.

Could the author comment the following simple interpretation of the phenomenon? Most of the energy is contained in the fundamental mode, which experience the strongest Raman shift. This wavelength shift corresponds to a temporal delay with respect to the other modes which propagates almost linearly, thus inducing an extra walk-off. For long enough propagation distance, this monomode soliton is essentially a scalar soliton carried by the fundamental mode of the fiber, which interacts only weakly with higher order modes.

Our response: We thank the Reviewer for suggesting a simple interpretation of our results. Unfortunately, the picture provided by the Reviewer just does not explain the numerically observed energy transfer from the higher-order modes into the fundamental mode (see left panel below, taken from Fig.1 of Ref.[11] (previously Ref.[14]). This energy transfer occurs simultaneously with the decoupling of the center wavelength of the fundamental mode with that of the higher-order modes (see right panel below), owing the Raman soliton-self-frequency shift of the soliton pulse in the fundamental mode.



Whereas in the present manuscript (see page 11), we examine the multimode soliton build-up in the initial stage of ultrashort pulse propagation in the GRIN fiber. It is this build-up process that determines the temporal duration of the multimode soliton. We show that a multimode soliton composed by multiple nondegenerate modes is formed, whenever the following conditions are verified: $L_{NL} < L_{cm}, L_{cp}$, and $L_D = L_{NL} = \text{const} \cdot L_W$ (with $\text{const}=0.87$ in our example). The

conditions are necessary for the soliton to build up, before the propagating modes separate in time; they trap each other in the first meter of fiber, and start propagating at same speed. This is better illustrated in the figure above (right), a numerical simulation of the wavelength evolution of the three modes; the wavelengths in the inset split-up, so that chromatic dispersion compensates for the modal dispersion. As a result, the three modes remain locked in time, and propagate with the same speed.

Next, for distances in the range of hundreds of dispersion lengths, power is transferred to the fundamental mode, as a consequence of inter-modal four-wave mixing (IM-FWM) and inter-mode stimulated Raman scattering. The second process becomes more efficient when the soliton wavelength shift reaches roughly 100 nm with respect to the residual dispersive wave, and contributes to the beam spatial cleaning. The spatial process is therefore a Raman cleaning, but not of a pump-Stokes type, because the pump and the “Stokes” wave are indeed provided by different wavelengths within the same soliton pulse. In this case, the residual dispersive wave at the input pump wavelength may become negligible in some conditions.

In order to better illustrate the process, on page 8 we added: “The observed process ... become negligible in some conditions”.

On page 8 we added “**Error! Reference source not found.** provides an explanation to the observed spatiotemporal effects; it is...”.

On page 9 we added “In order to provide an explanation to the observed temporal effects”.

Reviewer #3 (Remarks to the Author):

Review of “Solving the mystery of the walk-off soliton...” by Zitelli et al.

In this paper some interesting observations of multimode pulse propagation are reported. If I understand correctly, a multimode soliton (which the authors call a spatiotemporal soliton) forms (the numerics in Fig 3 show this most clearly). This is followed by transfer of power to the fundamental mode, simultaneous with soliton self-frequency shifting.

In its current form the paper is not ready for publication. After revision to address the points below it may be suitable for publication.

Our response: We thank the reviewer for considering that our paper reports interesting observations, and for recommending publication of a suitably revised version of our manuscript.

- The relation of the present work to that in Ref. 14 should be stated explicitly. Exactly what does this work contribute that is not essentially contained in Ref. 14? The sentence that refers to Ref. 14 only mentions intermodal SRS.

Our response: We thank the Reviewer for asking this question, that permits us to better clarify the relation between the present manuscript and Ref. 14 (now Ref.[11] in the revised text). In that paper, we experimentally demonstrated that a multimode (or spatiotemporal) soliton, initially including nondegenerate modes, naturally evolves into a single-mode (in the fundamental mode of the multimode fiber) soliton after a certain propagation distance. Moreover, this singlemode soliton preserves its spatial properties over long distance (thousands of dispersion lengths). In Ref.11, the input femtosecond pulse duration was fixed: as a result, the temporal duration of the multimode soliton at the point of its formation was also fixed. To summarize, in Ref.11 we studied the long-distance fate (decay) of multimode solitons, which are initially composed by nondegenerate modes, held together by fiber nonlinearity.

To the contrary, in the present work we investigate in details the dynamics of the initial formation process of those multimode solitons, by varying the input laser pulse parameters (i.e., temporal duration, wavelength, and energy) over an extended range of values. The experimental data presented in Figs. 1 to 3 (former Fig. 1) of the present paper are remarkable, as they reveal that a similar soliton pulsewidth is obtained after a 120 m distance, for similar input energies, but for different input pulse durations and for (very) different input wavelengths! Beam cleanup and soliton self-frequency shift are also illustrated in those figures.

These preliminary results, and the reference to [14], are necessary to introduce a second experimental evidence, which is illustrated in Fig. 7 (previously Fig. 4), that is the main result of our work. Figure 7 illustrates that wavelength dependence of the spatiotemporal soliton pulsewidth. Assessing this time-domain property, and associating it to the spatial soliton cleanup property, was indeed our key objective.

We added on page 9 “In order to provide an explanation to the observed temporal effects”, to split the work in new spatial and temporal properties.

- If multiple modes are propagating in GRIN fiber and power transfers to the fundamental mode, is this just well-known Raman beam cleanup, as described in Terry et al., Opt Exp 15, 17509 (2007)?

Our response: We thank the Reviewer for recalling this reference, that we believe is useful to be cited in the manuscript (it is now Ref.19). The paper from Terry et al. refers to the classical stimulated Raman scattering (SRS) process as a pump-probe configuration. There is a difference between SRS in a pump probe experiment, and the soliton self-frequency shift (SSFS) of a spatiotemporal soliton. In the first case, which is easier to observe in multimode step-index

fibers, a probe Stokes pulse builds-up at roughly 100 nm wavelength difference respect to the pump (for ultra-short pulses, the spectrum tail extends up to 100 nm); after forming, the Stokes soliton pulse experiences SSFS. In the second case, a spatiotemporal soliton forms at the pump wavelength directly, and it experiences SSFS.

We face the second case in our manuscript, although it is still correct to talk about intra-pulse, inter-modal SRS. What we observe is indeed a sort of Raman cleaning, but with pump and probe co-propagating within the same soliton. Hence the nature of the observed Raman beam cleanup is fundamentally different from the case discussed by Therry et al. Beam cleanup of a multimode femtosecond soliton under Raman SSFS was observed in several papers such as ref.[6], since the pioneering work of Gruninin et al. (ref.[3]).

In order to stress this, we added, on page 8: “The observed process of beam ... negligible in some conditions.”

A last comment: in future work, plan to experimentally investigate Raman the possibility of beam cleanup of ultrashort solitons of the pump-probe type.

- The paper is written in an odd way. Rather than set the general context for the work, it starts by stating a specific surprise observed in experiments. It is claimed to be a surprise that the predictions of Refs. 10-12 are not observed. Have any prior experiments observed the predictions of Refs. 10-12? If not, why is it a surprise? So the premise of the mystery is in question.

Our response: on page 3, we state that the predictions of the variational theory, as described in Refs. 13-18 of the revised text, are not observed. We believe the sentence is correct, because the validity of the variational theory has not been put into question so far in the literature on multimode solitons in optical fibers. The variational approach assumes the existence of spatiotemporal solitons propagating over an indefinite distance, so that the initial modal distribution is conserved, while undergoing periodical self-imaging. On the contrary, our experimental observations (Figs. 1 to 3) show that the initial modal composition of the soliton is not maintained: over long distances, the multimode soliton is attracted to the fundamental mode.

- Early in the paper the authors very casually make a remarkable claim: “spatiotemporal solitons were observed at the fiber output...” What is the evidence for this claim? The best evidence would be monitoring of the evolution versus distance, perhaps by a cut-back experiment. That would seem to be easy to do with the parameters of the work here. Less attractive but acceptable would be a direct comparison to results of numerics with fixed fiber length and input pulse energy varying. The authors seem to take for granted that some aspect of Fig. 1 is evidence of spatiotemporal soliton formation.

Our response: Fig. 1 clearly shows the formation of a temporal soliton at a proper input pulse energy (2-4 nJ); if not, dispersion would cause the pulse to broaden by hundreds of times over 120 m of fiber. Fig. 2 illustrates spatial beam condensation, which occurs at the same energy that leads to minimum pulsewidth. Finally, Fig. 3 shows a clear SSFS effect, with negligible residual spectrum at the input wavelength.

A cut-back was indeed performed, as shown in Fig. 4 (imported from the Supplementary material) and repeated at different wavelengths to produce the results illustrated in Fig. 7, which is the main result of the manuscript.

Those observations, together with the experiments and simulations in Ref. 14 (now Ref. 11), clearly prove the formation of a spatiotemporal soliton.

To better stress this point, we added on page 12: “By performing a cut-back experiment,” On pages 6 and 7, we moved the Supplementary Fig. 1 into the new Fig. 4 of the main text,

illustrating the results of the cut-back experiment, and added the text “Experimental evidence with shorter... confirmed the experimental observations.” on page 7.

Specific technical points

- The pulse duration (Fig. 1) does not vary over the range of 2-4 nJ. In contrast, the beam waist is very sensitive to the input pulse energy. Why say that they are “in correspondence?”

Our response: The strong resonance observed in Fig. 1 for 67 fs pulses is attributed to fiber losses principally, as already observed in our Ref. [11]: please refer to our response to point 3 of Reviewer 1. Shorter pulses suffer faster soliton self-frequency shift for increasing energy; as the wavelength overcomes 1700 nm, the output soliton undergoes significant differential loss with respect to dispersive waves around the input wavelength of 1550 nm. As a result of this and of the reduced detector sensitivity above 1850 nm, the camera sees a wider beam resulting from the dominant contribution of dispersive waves, which have a highly multimode content.

Please refer to the figures in answer 3 to Reviewer #1 for further evidence. The beam condensation is observed at the same energy of minimum pulsewidth, as explained in the text.

In order to better explain this point, we added on page 4 the sentence “The explanation of the sharper curve ... appears smoother vs. energy changes”.

- I don’t understand the application of Eq 1 to a multimode situation.

Our response: eq. 1 comes directly from eq. $L_D = L_{NL} = \text{const} \cdot L_W$, being L_W the modal walk-off length. The physical meaning of Eq.1 is that the formation of a spatiotemporal soliton involving non-degenerate modes is only possible if the modal walk-off length is of the same order of the nonlinearity and dispersion lengths, with *const* being a parameter close to unity 1 which depends on the specific modal distribution.

In order to stress this, we added on page 13: “in the sense that the walk-off, ... MM soliton modal distribution.” and “including non-degenerate modes”.

- Why does the temporal walk-off length have an adjustment constant? This is just the modal dispersion. What should the constant be?

Our response: Please refer to the previous response.

- The abstract states that multimode solitons were only recently investigated experimentally and cited Refs. 6-8. Multimode solitons were experimentally investigated in Ref 3.

Our response: Sorry for this typo, we added Ref. 3 (now Ref. 5) to new Refs. [5-11].

Minor points

A terminology point for the authors to consider: historically the expression “spatiotemporal soliton” has largely referred to solitons that balance dispersion, diffraction, and nonlinearity. It seems strange to use it now for a situation with a waveguide or trapping potential. In this particular case, there are only a few transverse modes. The term “multimode soliton” would seem to be more descriptive than “spatiotemporal soliton.” I understand that terminology is not necessarily scientific and only offer this as a suggestion.

In a few places, the manuscript uses atypical English. For example, “Fig. 1 resumes the experimental evidence...”

Our response: In our manuscript, we refer to “spatiotemporal” and “multimode” soliton as synonyms. We believe this is a generally accepted terminology, appearing in several recent papers

dealing with multimode optical fiber solitons. Finally, we thank the Reviewer for the English correction, and replaced “provides” with “resumes”.

REVIEWERS' COMMENTS:

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The authors have clearly addressed all my original comments. I am happy to recommend this paper is published as is.

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The authors considered in a satisfactory manner my comments as well as the issues raised by the others referees.

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Our response: We are grateful to the Reviewers for the positive appreciation of our corrections, and for the recommendation to publish our manuscript on Communications Physics.

Following the Editor's indications, we have replaced story-telling sentences with more traditional expressions, replaced the title, and checked the manuscript according to the Editorial Request Table. We believe the final version of our manuscript now deserves its publication on Communication Physics, and look forward to it.

Thanks, and best regards.

S. Wabnitz on behalf of all co-authors.