Supplementary Information: Coexistence of solid and liquid phases in shear jammed colloidal drops

Phalguni Shah[1,†], Srishti Arora[1,†], Michelle M. Driscoll [1,*]

[1] *Department of Physics and Astronomy, Northwestern University*

†*These authors contributed equally to this work.*

**corresponding author*

Supplementary Note 1: Contact angle of the colloidal suspensions

As the contact angle of our suspensions is very small, it is challenging to measure it via imaging data taken from the side. Therefore, we gently place a 5 μL drop on a hydrophilic glass plate (prepared in a manner identical to the slides used for our experiments), and measure its spreading size. We use the spherical cap approximation to estimate the contact angle from the spreading size. [Inset in Figure S1] 5 μL drops at $\phi = 0.01$, $\phi = 0.15$, and $\phi = 0.35$ (left to right) are placed on a hydrophilic glass slide. All the drops spread to the same extent (average 9.0 mm), shown quantitatively in the plot [Figure S1]. Using the spherical cap approximation, we can solve for the drop height h:

$$V_{cap} = \frac{1}{6}\pi(3a^2 + h^2), \tag{1}$$

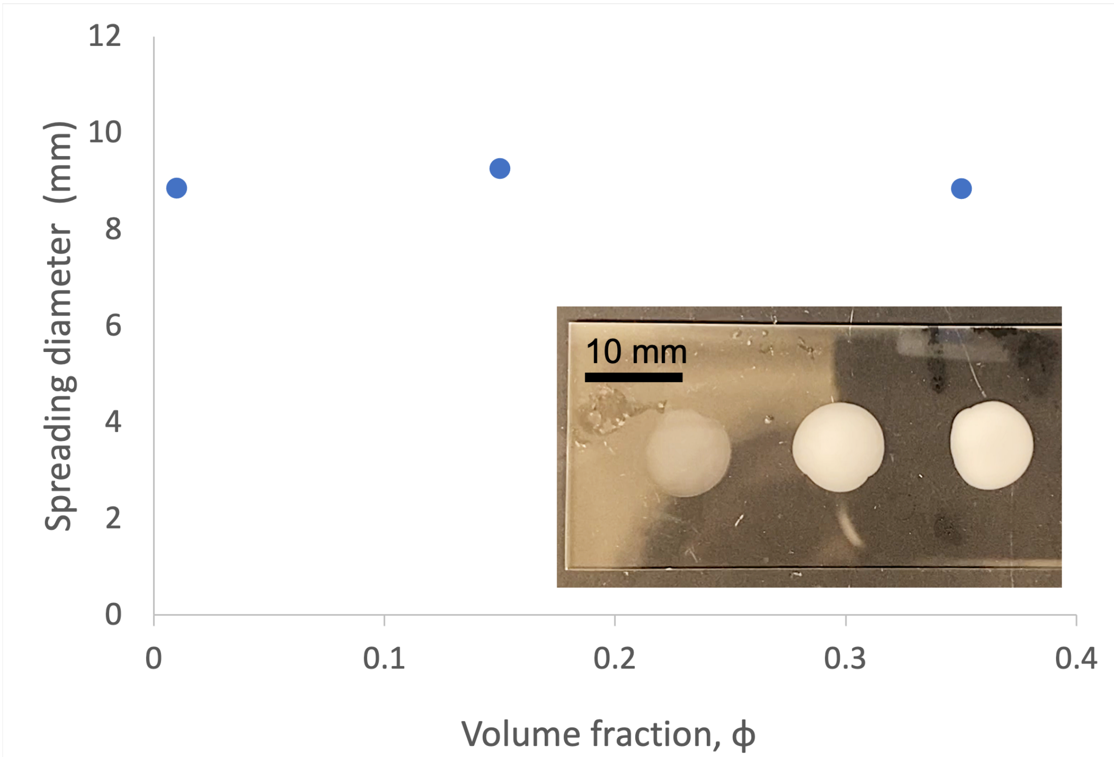


Figure S1: Contact angle estimation for colloidal suspensions. Inset: Gently placed drops of volume 5 µL of silica colloidal suspensions of $\phi = 0.01$, $\phi = 0.15$, and $\phi = 0.35$ respectively, on a hydrophilic glass slide. All three drops spread to the same extent. Main plot: The spreading diameter of the colloidal suspensions remains constant with increasing ϕ, and using the spherical cap approximation the contact angle can be estimated as $\theta = 3.8°$.

where $V_{cap} = 5$ µL is the volume of the spherical cap, and $a = 4.5$ mm is the radius of the spherical cap.. The contact angle can then be estimated as:

$$\theta = \sin^{-1}(\frac{2ah}{a^2 + h^2}). \tag{2}$$

The above equations give the contact angle of $\theta = 3.8°$. Therefore, the contact angle is constant over ϕ, and it is very low, as expected on hydrophilic substrates.

Supplementary Note 2: Viscosity vs. shear rate for colloidal suspensions

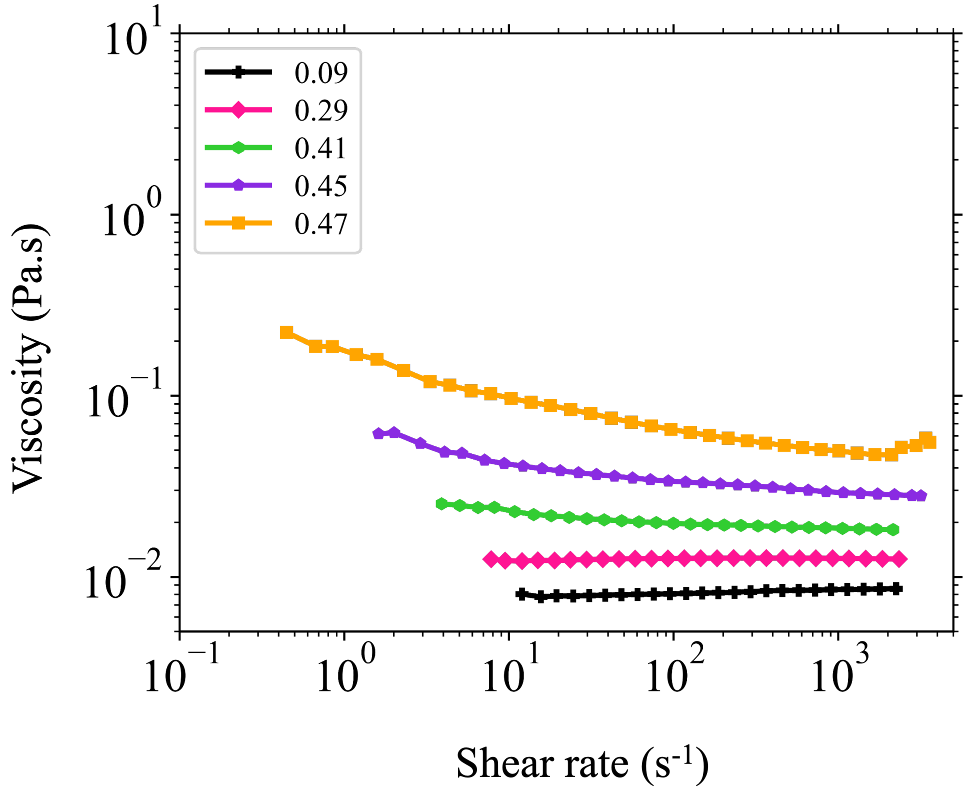


Figure S2: Viscosity of colloidal suspensions ($\phi \leq 0.47$) plotted against shear rate. Rheology data for colloidal suspensions, $\phi \leq 0.47$ (from the same dataset as Fig. 1b), plotted against shear rate.

Figure S2 shows viscosity of our colloidal suspensions plotted against shear rate (the same data is plotted against shear stress in Fig. 1b). We use the viscosity at the highest shear rate as the effective suspension viscosity in the spreading phase.

Supplementary Note 3: Scaling laws for maximum spreading after impact

Scheller et al.[1] reported a $ReWe^{1/2}$ empirical scaling for spreading viscous fluids. Figure S3a shows this scaling fit to our data (dashed black line). The exponent of our fit (0.165 ± 0.002) shows an impressive agreement with that by Scheller et al. (0.166) More recently, scalings based on models that consider the balance between inertial, viscous, and capillary effects have been reported. Laan et al.[2] considered the relation

$$\beta = Re^{1/5} f(WeRe^{-2/5}) \tag{3}$$

and used the first-order Padé approximation to fit experimental data of $\beta/Re^{1/5}$ vs. $WeRe^{-2/5}$:

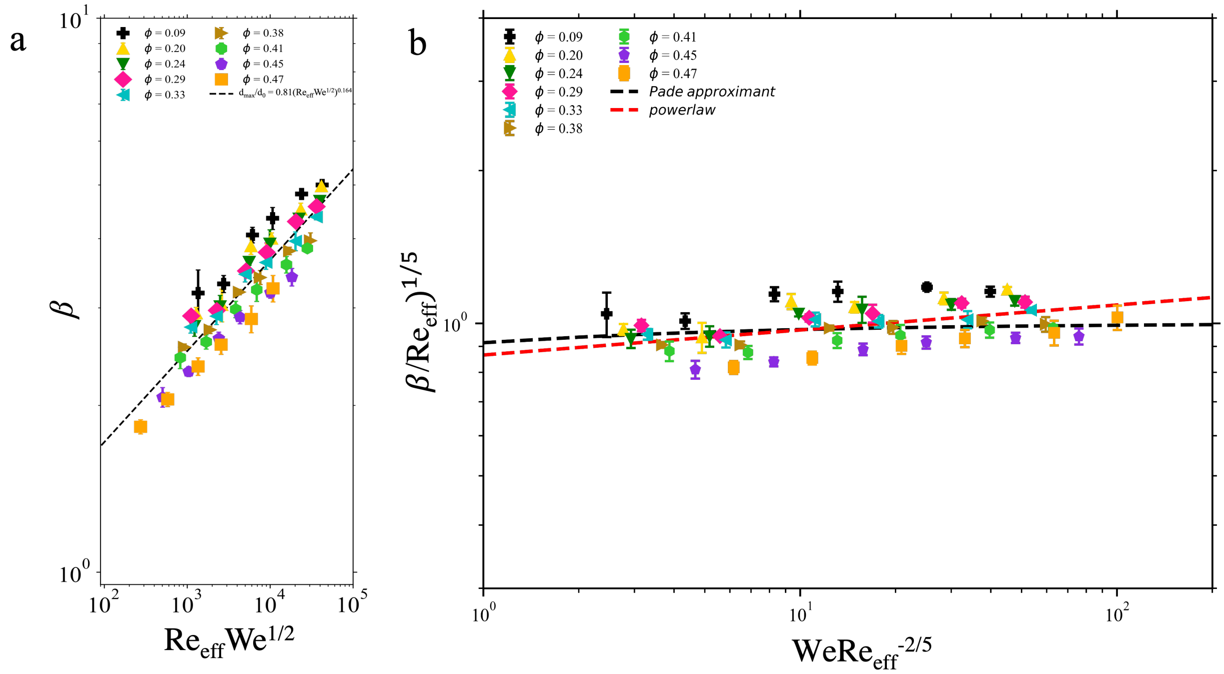
$$\frac{\beta}{Re^{1/5}} = \frac{(WeRe^{-2/5})^{1/2}}{A + (WeRe^{-2/5})^{1/2}} \tag{4}$$

This equation interpolates between the viscous regime (linear in $WeRe^{-2/5}$) and the inertial regime (constant with respect to in $WeRe^{-2/5}$), and consists of a fitting parameter. As discussed in the review by Josserand and Thorodssen [3], the Scheller et al. scaling can be rewritten, so that it has the same functional form as Equation 3, where

$$\beta = (ReWe^{1/2})^{1/6} = Re^{1/5}(WeRe^{-2/5})^{1/12}. \tag{5}$$

Where $f(WeRe^{-2/5})$ takes the form of a single power law, instead of interpolating between two regimes via a Padé approximant. Thus, although the Scheller et al. model was an empirical fit, it is consistent with the functional dependence of Equation 3 reported more recently.

Figure S3b shows both the Padé approximation (black dashed line) and a power law fit (red dashed line) to our spreading data of $\beta/Re_{eff}^{1/5}$ vs. $WeRe_{eff}^{-2/5}$.The Padé approximation fit gives



Comparison of maximum spread scalings for impacting colloidal drops. (a) β vs. $Re_{eff}We^{1/2}$ for spreading colloidal drops. The dashed black line is the power law fit, $\beta = (0.79 \pm 0.02)(Re_{eff}We^{1/2})^{0.165\pm0.002}$. The power law exponent shows a very good agreement with the exponent reported by Scheller et al. for Newtonian fluids. (b) $\beta/Re_{eff}^{1/5}$ vs. $WeRe_{eff}^{-2/5}$ for colloidal suspensions of $\phi \leq 0.47$. The data shows a good collapse. The black dashed line indicates the first-order Padé approximation reported by Laan et al. fit to our data, Equation 4. The red line indicates a single power law fit. In both panels, error bars indicate one standard deviation over multiple trials. For the parameter range concerned, both equations describe our data well.

$A = 0.09 \pm 0.01$, and the power-law fit gives the exponent of $n = 0.050 \pm 0.02$. This exponent deviates from the value 0.083 $(1/12)$ in order for Equation 5 to be satisfied. However, we note that the exponent value is very small, and the dynamic range of our data is limited. Moreover, both curves seem to describe our data well. As discussed by Josserand and Thoroddsen[3] in detail, the number of different scaling models reported in the literature are hard to differentiate, as they all describe experimental data well. This also holds true for our experimental data of colloidal suspension drops in the spreading regime, as evident from Figure S3b.

Lee et al.[4] have recently reported a data scaling with a wettability correction, where they calculate the spreading at zero impact velocity, $\beta_{v\rightarrow 0}$ using another Padé approximant with four fitting parameters. Corrected this way, they plot $\sqrt{\beta^2 - \beta^2_{v\rightarrow 0}}/Re^{1/5}$ against We, instead of the parameter $WeRe^{-2/5}$, and show a good collapse for several fluids. Correcting for wettability is important when comparing several fluids with varying surface tensions. However, in our case, all the suspensions we use are water-based, and the contact angle stays constant over ϕ [Figure S1]. Therefore, we did not apply these wetting corrections.

Supplementary References

1. Scheller, B. L. & Bousfield, D. W. Newtonian drop impact with a solid surface. *AIChE Journal* **41**, 1357–1367 (1995).

2. Laan, N., de Bruin, K. G., Bartolo, D., Josserand, C. & Bonn, D. Maximum diameter of impacting liquid droplets. *Physical Review Applied* **2**, 044018 (2014).

3. Josserand, C. & Thoroddsen, S. T. Drop impact on a solid surface. *Annual review of fluid mechanics* **48**, 365–391 (2016).

4. Lee, J. B. *et al.* Universal rescaling of drop impact on smooth and rough surfaces. *Journal of Fluid Mechanics* **786**, R4 (2016).