Phonon transmission through a nonlocal metamaterial slab

– Supplementary Materials

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Supplementary Note 1. Conditions of bound states in the continuum (BIC)

In the main paper, the BIC condition and frequency are provided for the simplest non-trivial case with $N = 3$ and $L/a = 4$. The BIC condition and frequency can be derived by analyzing eigenmodes of the small mass-and-spring system. The slab (cf. Fig. 1) contains in total five masses coupled by springs with spring constant K_1 and K_3. Newton's law of motion for the five masses write as

$$m\begin{pmatrix}\ddot{u}_1\\ \ddot{u}_2\\ \ddot{u}_3\\ \ddot{u}_4\\ \ddot{u}_5\end{pmatrix} = \begin{pmatrix}-K_1-K_3 & K_1 & 0 & K_3 & 0\\ K_1 & -2K_1-K_3 & K_1 & 0 & K_3\\ 0 & K_1 & -2K_1 & K_1 & 0\\ K_3 & 0 & K_1 & -2K_1-K_3 & K_1\\ 0 & K_3 & 0 & K_1 & -K_1-K_3\end{pmatrix}\begin{pmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\end{pmatrix}, \tag{S1}$$

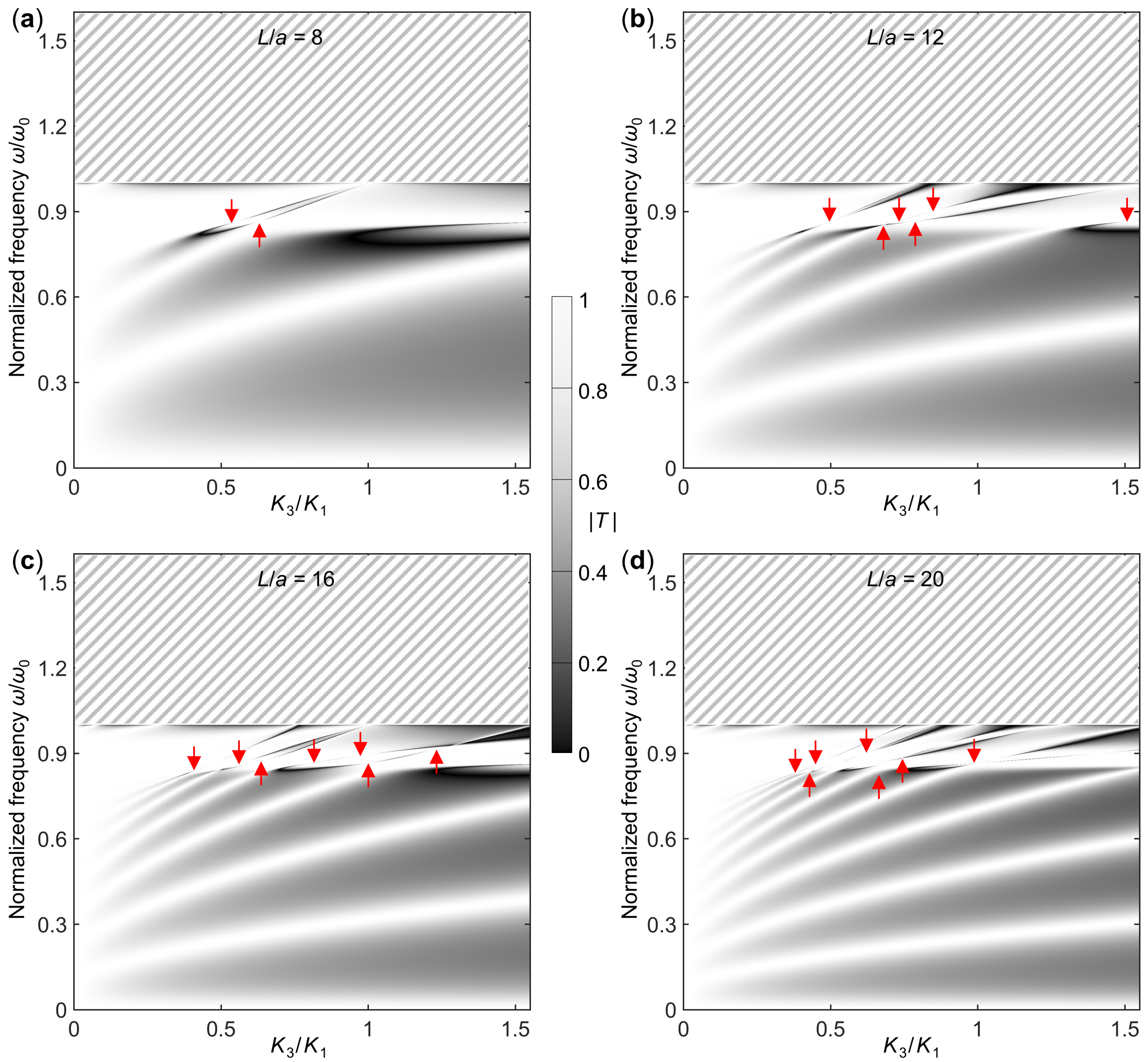
where, u_i $(i = 1,2 \dots 5)$ corresponds to the displacement of the five masses from left to right, and $\ddot{u}_i$ stands for secod-order derivative with respect to time. By assuming a time harmonic solution, $u_i = \tilde{u}_i \exp(-i\omega t)$ with ω being the angular frequency, we derive the following eigenfrequency problem

$$\begin{pmatrix}m\omega^2-K_1-K_3 & K_1 & 0 & K_3 & 0\\ K_1 & m\omega^2-2K_1-K_3 & K_1 & 0 & K_3\\ 0 & K_1 & m\omega^2-2K_1 & K_1 & 0\\ K_3 & 0 & K_1 & m\omega^2-2K_1-K_3 & K_1\\ 0 & K_3 & 0 & K_1 & m\omega^2-K_1-K_3\end{pmatrix}\mathbf{u} = \mathbf{0}, \tag{S2}$$

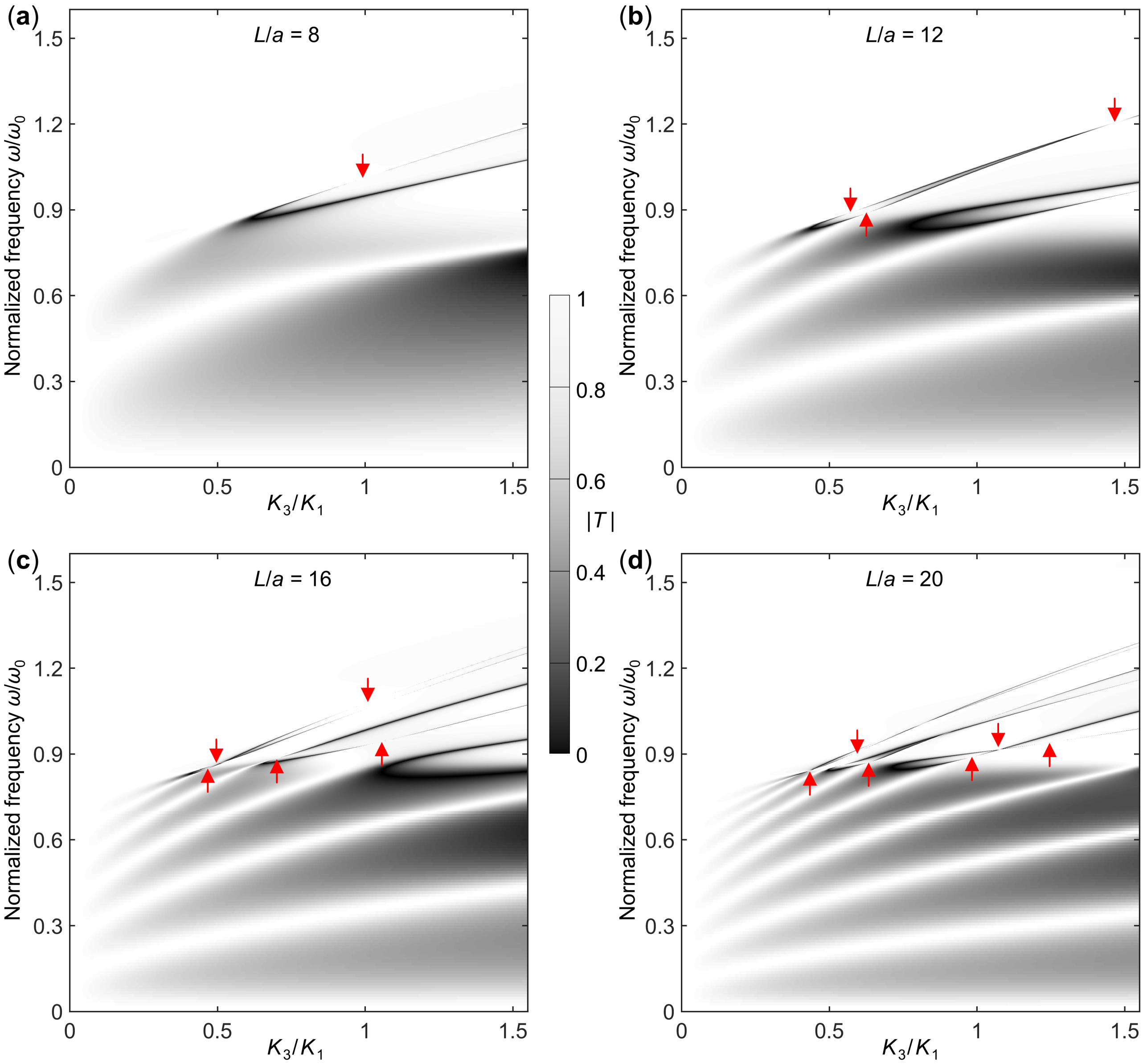
where $\mathbf{u} = (\tilde{u}_1 \quad \tilde{u}_2 \quad \tilde{u}_3 \quad \tilde{u}_4 \quad \tilde{u}_5)^{\mathrm{T}}$ for simplicity. By demanding the first and the last masses have zero-displacements, i.e. $\tilde{u}_1 = \tilde{u}_5 = 0$, we have the conditions of BIC $K_3/K_1 = 1$. In total, we obtain five eigenfrequencies and eigenmodes

$$\begin{aligned}
\omega_1 &= 0, & \mathbf{u} &= (1 \quad 1 \quad 1 \quad 1 \quad 1)^{\mathrm{T}}\\
\omega_2 &= \sqrt{\frac{2K_3}{m}}, & \mathbf{u} &= (-1 \quad 0 \quad 0 \quad 0 \quad 1)^{\mathrm{T}}\\
\omega_3 &= \sqrt{\frac{2K_3}{m}}, & \mathbf{u} &= (-1 \quad 0 \quad 1 \quad 0 \quad 0)^{\mathrm{T}}\\
\omega_4 &= \sqrt{\frac{3K_3}{m}}, & \mathbf{u} &= (0 \quad -1 \quad 0 \quad 1 \quad 0)^{\mathrm{T}}\\
\omega_5 &= \sqrt{\frac{5K_3}{m}}, & \mathbf{u} &= (2 \quad -3 \quad 2 \quad -3 \quad 2)^{\mathrm{T}}
\end{aligned}. \tag{S3}$$

The fourth eigenfrequency ω_4 is the same as Eq. (12) in the main paper. For the corresponding eigenmode, the two masses on both ends have zero displacements as wanted.



Supplementary Figure 1. As Fig. 3(a), but for different relative slab thicknesses L/a. (a) - (d) $L/a = 8$, $L/a = 12$, $L/a = 16$, and $L/a = 20$, respectively. BIC points are marked by arrows. In the hatched region above the cut-off frequency $\omega/\omega_0 = 1.0$, waves cannot propagate in the surrounding medium.



Supplementary Figure 2: As Fig. 6, but for different relative thicknesses of the slab. (a) - (d) $L/a = 8$, $L/a = 12$, $L/a = 16$, and $L/a = 20$, respectively. It can be seen that the number of BIC points marked by arrows increases with increasing relative slab thickness L/a.