nature portfolio

Peer Review File

Spin operator, Bell nonlocality and Tsirelson bound in quantum-gravity induced minimal-length quantum mechanics

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

The manuscript aims to explore phenomenological consequences of a minimal length scale, as predicted by various quantum gravity models. The authors do this by deriving several consequences of a deformed canonical commutation relation, which is given in eq. (1). One of the main results is the derivation of a modified Tsirelson bound. The authors then discuss an experimental setting, in which such deviations can be measured. Finally, they give a concrete example of how such experiments can be used to constrain deformation parameters in quantum gravity models.

The paper is well written and understandable for a broad audience of physicists. The derivation presented in the paper seems solid and can be reproduced by other researchers in the field. Furthermore, I find the results interesting, and they are, to my knowledge, novel. In my opinion, the paper is a very valuable and timely addition to the growing field of quantum gravity phenomenology, since it concisely guides the reader from basic theoretical notions to a possible experimental test of quantum gravity models.

I have just one minor remark about the current presentation of the paper. In their analysis, the authors assume coordinate commutativity, isotropy and rotational invariance. There exists, however, an important class of quantum gravity models, in which the notion of minimal length is induced by a non-commutativity of the coordinates. The existence of other classes of models, which are not necessarily covered by the analysis, does not make the results less relevant. However, to readers who are less familiar with deformed commutation relations, the introduction suggests that the analysis is applicable to all models that incorporate a minimal length scale. Therefore, I believe that it would be appropriate, if the authors briefly comment on such subtleties in the introduction and/or conclusion.

In addition, I believe there to be a typo in eqs. (9) and (17):
- in eq. (9), it seems that a factor 1/2 is missing;
- in eq. (17), I believe the right hand side should be multiplied by the field phi.
I would urge the authors to check these equations, and make modifications, if they agree.

Finally, I have two minor suggestions that could improve the readability of the paper:
- In eq. (1), two analytic functions f and \bar{f} are introduced. In the literature, the complex conjugate of a function is often denoted by either \bar{f} or f^\ast. In this work, \bar{f} does not seem to denote the conjugate of f. Therefore, in order to avoid any confusion, the authors might want to reconsider their notation.
- In eq. (7), the authors introduce a deformation parameter beta. In the further analysis, the authors seem to assume that beta \geq 0. The authors might want to specify this assumption in the text between eqs. (7) and (8).

In conclusion, I do have some minor remarks that should be considered by the authors, but, if this is done, I would recommend the paper for publication in Communications Physics.

Reviewer #2 (Remarks to the Author):

The paper is written very well and has some valuable results that can use in the future studying. In my opinion after some minor revision, this paper will get enough potential to publish in this journal. My comments are as follows:

1. It will make the paper more complete if briefly other famous models of GUP by their references including (Phys. Lett. B 646, 63 (2007), Phys. Lett. B 714, 317 (2012), Phys. Lett. B 718, 638 (2012),

Phys. Lett. B 770, 445 (2017)). Also, as an example of the application of GUP in other areas of physics, authors can mention some references such as Phys. Lett. B 452, 39 (1999), int. J. Mod. Phys. D 24, 1550016 (2015), J. Phys. A 32, 7691 (1999), Grav. Cosmol. 18, 211 (2012), Int. J. Theor. Phys. 61, 205 (2022). Eur. Phys. J. plus 137, 376 (2022) and so on.

2. Before sec. IV, the authors used the model of GUP (and noncommutative relation) that admits the minimal length. But in sec. IV they used the model of GUP that admits both minimal length and maximal momentum (since Eq. (25) is deduced from the models of Doubly Special Relativity type). What is the reason? It is better they follow a single model or at least in the abstract section, it is mentioned that they used 2 models of GUP.

Best regards,
The referee

COMMSPHYS-22-1181-T: "Spin operator, Bell nonlocality and Tsirelson bound in quantum-gravity induced minimal-length quantum mechanics"
Response to Referees' reports

March 17, 2023

In compliance with the Referees' comments, in what follows we present our point-by-point response. The reference numbers appearing below correspond to those in the revised version.

Referee 1

Referee's comment #1

In their analysis, the authors assume coordinate commutativity, isotropy and rotational invariance. There exists, however, an important class of quantum gravity models, in which the notion of minimal length is induced by a non-commutativity of the coordinates. The existence of other classes of models, which are not necessarily covered by the analysis, does not make the results less relevant. However, to readers who are less familiar with deformed commutation relations, the introduction suggests that the analysis is applicable to all models that incorporate a minimal length scale. Therefore, I believe that it would be appropriate, if the authors briefly comment on such subtleties in the introduction and/or conclusion.

Response

We appreciate the comment of the Referee, which allowed for a refinement of the general reasoning contained in our work. Indeed, in the revised version of the manuscript, we have specified the distinction between classes of models encompassing a minimal length scale and/or spatial non-commutativity. For each of these classes, we have also provided an explicit example (see item #1 in the List of Changes).

Referee's comment #2

In addition, I believe there to be a typo in eqs. (9) and (17):
- in eq. (9), it seems that a factor 1/2 is missing;
- in eq. (17), I believe the right hand side should be multiplied by the field phi.
I would urge the authors to check these equations, and make modifications, if they agree.

Response

We are grateful to the Reviewer for having addressed these typos; we have now corrected the above misprints in accordance with the suggestions (see item #2 in the List of Changes).

Referee's comment #3

Finally, I have two minor suggestions that could improve the readability of the paper:
- In eq. (1), two analytic functions f and $\bar{f}$ are introduced. In the literature, the complex conjugate of a function is often denoted by either $\bar{f}$ or f^*. In this work, $\bar{f}$ does not seem to denote the conjugate of f. Therefore, in order to avoid any confusion, the authors might want to reconsider their notation.
- In eq. (7), the authors introduce a deformation parameter beta. In the further analysis, the authors seem to assume that $\beta \geq 0$. The authors might want to specify this assumption in the text between eqs. (7) and (8).

Response

We thank the Reviewer for the above suggestions. In compliance with the recommendation, we have made the following changes/additions to the text:

- We have changed the notation from $\bar{f}$ to g (see item #3 in the List of Changes).
- We have indeed assumed $\beta > 0$. We have now specified this in the text (see item #4 in the List of Changes).

Referee 2

Referee's comment #1

1. It will make the paper more complete if briefly other famous models of GUP by their references including (Phys. Lett. B 646, 63 (2007), Phys. Lett. B 714, 317 (2012), Phys. Lett. B 718, 638 (2012), Phys. Lett. B 770, 445 (2017)). Also, as an example of the application of GUP in other areas of physics, authors can mention some references such as Phys. Lett. B 452, 39 (1999), int. J. Mod. Phys. D 24, 1550016 (2015), J. Phys. A 32, 7691 (1999), Grav. Cosmol. 18, 211 (2012), Int. J. Theor. Phys. 61, 205 (2022). Eur. Phys. J. plus 137, 376 (2022) and so on.

Response

We thank the Referee for the suggested references. In particular, we have included the majority of the ones that were not already present in the previous version of the manuscript (see item #5 in the List of Changes).

Referee's comment #2

2. Before sec. IV, the authors used the model of GUP (and noncommutative relation) that admits the minimal length. But in sec. IV they used the model of GUP that admits both minimal length and maximal momentum (since Eq. (25) is deduced from the models of Doubly Special Relativity type). What is the reason? It is better they follow a single model or at least in the abstract section, it is mentioned that they used 2 models of GUP.

Response

We are grateful to the Reviewer for the observation, which helped us to better comment on this point. As a matter of fact, at the end of Section IV, our sole interest is estimating the modifying factor in Eq. (24). This is done by separately considering a correction of the order $\sqrt{\hat{\pi}^2}$ and one that is of the order $\hat{\pi}^2$, simply because these choices represent two archetypical models that are commonly accounted for in the literature. Hence, our intent does not consist in the analysis of implications associated with specific models, but rather in the evaluation of possible corrections related to explicit examples. We clarify this concept in the text (see item #6 in the List of Changes).

List of changes made in the paper

The equation and reference numbers correspond to those appearing in the revised version of the paper.

1. Right after Eq. (3), we have added the following paragraph to briefly discuss non-commutative scenarios:

> A comment is in order here: coordinate non-commutativity and minimal length are in principle two independent aspects. Distinct models may feature minimal length and no coordinate non-commutativity, no minimal length and coordinate non-commutativity, or both minimal length and coordinate non-commutativity at the same time. The first and the third type of models can be directly implemented by either fulfilling or violating the identity (2), respectively. To mention a simple example of the second kind, one can consider the set of commutators $[\hat{x}^i, \hat{x}^j] = i\theta^{ij}$ with θ^{ij} constants and $[\hat{x}^i, \hat{p}_j] = i\delta^i_j$. Such a scheme evidently does not predict any minimal length, but does encompass spatial non-commutativity.

2. We have corrected the typos in Eqs. (9) and (17) as follows:

$$\Delta x^i \geq \frac{1}{2\Delta\pi_i}\left[1 + 3\beta\left(\frac{\Delta\pi_i}{m_p}\right)^2\right]. \tag{9}$$

$$i\partial_t\varphi = \left[\frac{f^2}{2m}\hat{\Pi}^2 - ef\left(\hat{L}_i + 2\hat{S}_i\right)B^i\right]\varphi = \left[\frac{\hat{\pi}^2}{2m} - e\left(\hat{l}_i + 2\hat{s}_i\right)B^i\right]\varphi\,, \tag{17}$$

3. We have changed the notation from $\bar{f}$ to g wherever the former occurred. Specifically:

   - In Eq. (1b) and right after

$$\left[\hat{x}^i, \hat{\pi}_j\right] = i\left[f\left(\hat{\pi}^2\right)\delta^i_j + g\left(\hat{\pi}^2\right)\frac{\hat{\pi}^i\hat{\pi}_j}{\hat{\pi}^2}\right], \tag{1b}$$

> with f and g being two arbitrary, analytic, dimensionless functions [...]

   - In Eq. (2)

$$g = \frac{2\left(\log f\right)'\hat{\pi}^2}{1 - 2\left(\log f\right)'\hat{\pi}^2}f\,, \tag{2}$$

   - Around Eq. (4)

> Since the functions f and g must be dimensionless [...]
> Ordinary QM is recovered in the low-energy limit, yielding $\lim_{\hat{\pi}^2\to 0} f = 1$ and $\lim_{\hat{\pi}^2\to 0} g = 0$.

   - Equations (6)–(8)

$$\Delta x^i \geq \frac{|\langle[\hat{x}^i, \hat{\pi}_i]\rangle|}{2(\Delta\pi_i)} = \frac{1}{2\Delta\pi_i}\left|\langle f(\hat{\pi}^2)\rangle + \left\langle g(\hat{\pi}^2)\frac{(\hat{\pi}^i)^2}{\hat{\pi}^2}\right\rangle\right|, \tag{6}$$

[...] We obtain an immediate visualization of this occurrence by expanding f and g to leading order in $\hat{\pi}^2/m_\mathrm{p}^2$ in a generic scheme of MLQM

$$f(\hat{\pi}^2) \simeq 1 + \beta \frac{\hat{\pi}^2}{m_\mathrm{p}^2} \,, \qquad g(\hat{\pi}^2) \simeq 2\beta \frac{\hat{\pi}^2}{m_\mathrm{p}^2} \,, \tag{7}$$

[...]

$$\left| \langle f(\hat{\pi}^2) \rangle + \left\langle g(\hat{\pi}^2) \frac{(\hat{\pi}^i)^2}{\hat{\pi}^2} \right\rangle \right| \gtrsim \left| 1 + \beta \frac{\sum_j \Delta\pi^j \Delta\pi_j}{m_\mathrm{p}^2} + 2\beta \frac{(\Delta\pi^i)^2}{m_\mathrm{p}^2} \right| \,, \tag{8}$$

4. We have rewritten the sentence before Eq. (8) as

Then, for the relevant case $\beta > 0$, to leading order:

5. We have included the following references:

[35] K. Nouicer, "Quantum-corrected black hole thermodynamics to all orders in the Planck length," Phys. Lett. B **646** (2007), 63-71.

[36] P. Pedram, "A Higher Order GUP with Minimal Length Uncertainty and Maximal Momentum," *Phys. Lett. B* **714** (2012) 317–323.

[39] H. Shababi and W. S. Chung, "On the two new types of the higher order GUP with minimal length uncertainty and maximal momentum," *Phys. Lett. B* **770** (2017) 445–450.

[44] E. Al-Nasrallah, S. Das, F. Illuminati, L. Petruzziello and E. C. Vagenas, "Discriminating quantum gravity models by gravitational decoherence," arXiv:2110.10288 [gr-qc] (2021).

[45] P. Pedram, M. Amirfakhrian and H. Shababi, "On the (2+1)-dimensional Dirac equation in a constant magnetic field with a minimal length uncertainty," Int. J. Mod. Phys. D **24** (2014) no.02, 1550016.

[46] H. Shababi, "On the Thomas–Fermi model at the noncommutative framework," Eur. Phys. J. Plus **137** (2022) no.3, 376.

[47] M. Fadel and M. Maggiore, "Revisiting the algebraic structure of the generalized uncertainty principle," Phys. Rev. D **105** (2022) no.10, 106017.

[55] P. Bosso, F. Illuminati, L. Petruzziello, and F. Wagner. In preparation.

6. We have added a sentence after Eq. (25) to discuss the linear model used for the estimations, that is

It is worth stressing that the above scheme predicts not only the existence of a minimal length, but also the presence of a maximal momentum. However, one can prove that the framework introduced so far can be extended to arbitrary deformations of standard quantum mechanics that account for minimal/maximal length and/or minimal/maximal momentum [55].