nature portfolio

Peer Review File

Probe for bound states of SU(3) fermions and colour deconfinement.

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Reviewers' comments:

Reviewer #1 (Remarks to the Author):

In this manuscript, the authors have theoretically investigated the persistent current in attractively interacting SU(3) fermions in optical lattices. The persistent current shows a signal for the bound-state formation in the periodicity with respect to the effective magnetic flux. The results are new and interesting so that I can recommend the publication of the manuscript if the authors can address the following points.

1. The authors mentioned that the persistent current can be probed via the TOF measurement and also presented the numerical result of TOF showing some characteristics of the bound-state formation in the latter part of the manuscript.
In this sense, naively, I am wondering about the advantages to study the persistent current rather than the TOF itself.

2. It would be helpful for readers if the authors could provide the explicit definition of the Fourier weight C_3 and its intuitive physical meaning.

3. The three-body loss would be an obstacle for observing trions and the associated currents in cold atom experiments. Can the authors make a comment on this point?

4. The authors mentioned the analogy with finite-temperature crossover from the hadronic phase to the QGP phase, which has already been confirmed by the lattice QCD. Along this direction, is it possible to see some features of the crossover with increasing the filling (or density) at zero temperature?
It would be interesting because recently the crossover behavior from hadronic phase to the quark phase at zero temperature has been discussed in connection with the observation of neutron stars (e.g.T. Kojo, et al 2022 ApJ 934 46).

Reviewer #2 (Remarks to the Author):

The most significant findings of this manuscript with regard to attractive SU(2) fermions in a 1D lattice are that (1) A change in the current response of the system to an applied flux provides information about the formation of bound states, (2) This would be a much more experimentally accessible way of obtaining information that attempting to directly measure e.g. two and three-body correlations(3) There is enough information in the current response to distinguish between a color superfluid of pairs and trions (4) the interplay of interactions and temperature on deconfinement in this system is non-trivial and interesting.

The specific results in this manuscript combine several well-established elements in a way that is novel and has not been addressed elsewhere. There is a long history of using persistent currents as an indicator of the formation of bound states, dating all the way back to the Deaver-Fairbank experiments providing early evidence for BCS pairing. The essential features of attractive SU(2) fermion systems have been identified previously, including the existence of BCS color superfluid pairs and trions. This paper's main novelty is in determining how a ring of attractive SU(2) fermions would respond to an applied rotational flux. The results in this paper are also very clearly distinct from the authors' previous work involving bosons and repulsive fermions, both in terms of the types of phenomena observed and the theoretical and computational tools used to study the system. The authors also go considerably beyond previous work in their treatment of the effects of finite temperature.

This paper will likely be of substantial interest to the various different groups that have realized different parts of the capabilities that would be required to realize an experimental implementation of the system described in the paper, especially those working with ytterbium or other atoms that allow for creation of an SU(N) system. The authors have highlighted that there has been a recent increase in experimental work connected to this work: Persistent currents have recently been observed in much larger systems of fermions in the continuum limit, and another very recent publication (which would be good to include as a supporting citation, 10.1103/PhysRevLett.129.123201) has shown progress toward realization of an experimentally useful ring lattices, although this is for a light atomic species (lithium) rather than ytterbium. Combining these capabilities in a single experiment would be very challenging, but the publication of this result will add significant motivation to efforts to develop these capabilities. There are many interesting aspects of the physics raised in this analysis and in contrast with earlier works (e.g. different fractionalization mechanism than repulsive fermions) that warrant attention. While the obvious analogies with quark matter are interesting, trying to draw inferences from this geometrically simplified 1D ring model that would translate usefully to more realistic models of quark matter seems unlikely. There is enough intrinsically interesting physics without relying on that analogy to drive interest in this work, and I think this work will attract significant attention.

The work presented in the paper and in the supplemental material is very thorough in establishing the approach used in various limits, necessary cross-checks of results obtained from DMRG or exact diagonalization against Bethe-ansatz etc. The Supplemental material answers essentially every question I might have thought to ask. There is so much information in fact that it is challenging to sort out exactly what is being said or shown in many places, but with some effort it was clear that the authors thought carefully about what should be included and went to some lengths to do so. Overall, I do think the authors have been very thorough in justifying their approach and presenting numerous sensible checks on its validity, and I am adequately convinced that the work is sound.

One point of concern with regard to clarity is that Figure 3 in the main manuscript is rather difficult to understand at a glance. Someone not already very familiar with certain elements of the authors' approach will not understand what is shown without going through the main body of the manuscript in detail. It would be good to state in simple text that what is being plotted in the vertical axis is a Fourier weight, and what the significance of that quantity is. The normalization is also not entirely trivial and could be better explained. This and several other key parts of the manuscript would be more accessible with clearer textual descriptions of the physical meanings of otherwise obscure symbols and expressions.

In conclusion, this manuscript is novel, technically sound, thorough in its execution, and it is sufficiently important within the sub-field that I recommend that this manuscript be published. It could be revised for clarity in a few places but is in need of no major corrections or additions.

Reviewer #3 (Remarks to the Author):

In their manuscript, Chetcuti et al. analyze persistent currents of 3-component fermions in periodic 1D systems. They point out the period of the response to a magnetic field as a useful measure for distinguishing a color superfluid (CSF) from a trion liquid. Their study is interesting and timely and the paper is largely well written and accessible. As such the work should be published. Before I can fully recommend publication in Communications Physics however, the following issues should be addressed (in order of relevance):

1) In Fig. 3, there appear to be much fewer data points along the T-axes in the small lower-left insets than in the main figures. Am I missing something / can the authors explain?

2) The author's description on page 3 of how persistent current can be measured from time of flight (TOF) images is too short to follow. The variance of TOF images appears to be the main quantity of interest, but notations are not defined in this context and it's not immediately obvious what needs to

be done. This makes it harder to appreciate the results in Fig. 4. Moreover, on page 3 the authors cite [53], but in that reference I also could not find a crisp explanation how TOF can directly measure the persistent current (Ref. [53] is surely relevant here, but given that the authors propose their method as a way for cold atom experiments to measure the persistent currents, a more pedagogical brief explanation of the key physics and of what needs to be measured precisely would be appropriate I believe).

3) The authors talk about 'deconfinement' and make connections to QCD, but their model does not feature SU(3) gauge fields, and SU(3) is broken by hand in the CSF case. They mention this clearly in the end, but I would wish for a brief statement a bit earlier on when they make a connection to quark matter that the model studied here lacks some key features. Overall I find the analogies drawn by the authors justified, so I'm just asking to avoid any potential confusion.

4) The authors should be more careful with how they define units. In all figures, they just state U = 0.01, 5 etc without mentioning that they set t=1 in the figures. Likewise, figure axes should always indicate the unit (e.g. use T/t, U/t instead of just T or U). The authors state in the text that t=1, which is fine, but often enough as a field moves on other units become more important (here e.g. the super-exchange J ~ t^4/U) and there is simply no reason to work with axes without units.

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1. R: "The authors mentioned that the persistent current can be probed via the TOF measurement and also presented the numerical result of TOF showing some characteristics of the bound-state formation in the latter part of the manuscript. In this sense, naively, I am wondering about the advantages to study the persistent current rather than the TOF itself.

   A: For the persistent current to be experimentally observed, one utilizes TOF measurement as a probe. For this reason we focused our attention in the aforementioned experimental osbervable. Through the images obtained from TOF expansion, we are able to gather information about the features of the persistent current flowing through the system. In the case considered by us, the discrete steps of the variance of the momentum distribution is a direct reflection of the tri-partite periodicity. As such, the study of persistent currents and TOF is intrinsically linked and one cannot study one without the other.

2. R: "It would be helpful for readers if the authors could provide the explicit definition of the Fourier weight C_3 and its intuitive physical meaning."

   A: We agree with the Referee that the paper would benefit by better explaining the significance of the Fourier weight C_3. The explicit definition of C_3 is provided in the Appendix and we have added a small sentence to notify the readers of this fact. In the main text, we have added a small explanation with regards to the physical meaning of using C_3 in our analysis. Changes to manuscript 1.

3. R: "The three-body loss would be an obstacle for observing trions and the associated currents in cold atom experiments. Can the authors make a comment on this point?"

   A: We thank the Referee for bringing up this point. Three-body losses prove to be the main obstacle in observing trions when considering Efimov states, which are very highly excited states. Our trions/CSF ground-states have nothing to do with Efimov states. In our work, we emphasize that we refer to ground-states and not to excitations, where such a decay appears. Following discussions with experimentalists, we were informed that the system under consideration would be robust to three-body losses since we consider the ground-state. Furthermore, we highlight that for SU(N) symmetric systems, there are no spin changing collisions such that the number of particles per colour is conserved.

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   A: We thank the Referee for putting forward this interesting question. We have updated the Conclusions section in the manuscript to briefly discuss this connection. Changes to manuscript 2.

REFEREE 2

R:"The most significant findings of this manuscript with regard to attractive SU(2) fermions in a 1D lattice are that (1) A change in the current response of the system to an applied flux provides information about the formation of bound states, (2) This would be a much more experimentally accessible way of obtaining information that attempting to directly measure e.g. two and three-body correlations(3) There is enough information in the current response to distinguish between a color superfluid of pairs and trions (4) the interplay of interactions and temperature on deconfinement in this system is non-trivial and interesting.

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This paper will likely be of substantial interest to the various different groups that have realized different parts of the capabilities that would be required to realize an experimental implementation of the system described in the paper, especially those working with ytterbium or other atoms that allow for creation of an SU(N) system. The authors have highlighted that there has been a recent increase in experimental work connected to this work: Persistent currents have recently been observed in much larger systems of fermions in the continuum limit, and another very recent publication (which would be good to include as a supporting citation, 10.1103/PhysRevLett.129.123201) has shown progress toward realization of an experimentally useful ring lattices, although this is for a light atomic species (lithium) rather than ytterbium. Combining these capabilities in a single experiment would be very challenging, but the publication of this result will add significant motivation to efforts to develop these capabilities. There are many interesting aspects of the physics raised in this analysis and in contrast with earlier works (e.g. different fractionalization mechanism than repulsive fermions) that warrant attention. While the obvious analogies with quark matter are interesting, trying to draw inferences from this geometrically simplified 1D ring model that would translate usefully to more realistic models of quark matter seems unlikely. There is enough intrinsically interesting physics without relying on that analogy to drive interest in this work, and I think this work will attract significant attention.

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   A: We agree with the Referee that Figure 3 has a lot of information to digest. As such, we have updated the caption to explain better what is being plotted in the figure. Changes to manuscript 3.

   Additionally, we have added the reference pointed out by the Referee. Changes to manuscript 4.

R: “In conclusion, this manuscript is novel, technically sound, thorough in its execution, and it is sufficiently important within the sub-field that I recommend that this manuscript be published. It could be revised for clarity in a few places but is in need of no major corrections or additions. ”

REFEREE 3

R:“ In their manuscript, Chetcuti et al. analyze persistent currents of 3-component fermions in periodic 1D systems. They point out the period of the response to a magnetic field as a useful measure for distinguishing a color superfluid (CSF) from a trion liquid. Their study is interesting and timely and the paper is largely well written and accessible. As such the work should be published. Before I can fully recommend publication in Communications Physics however, the following issues should be addressed (in order of relevance): ”

1. R:“ In Fig. 3, there appear to be much fewer data points along the T-axes in the small lower-left insets than in the main figures. Am I missing something / can the authors explain? ”

   A: The lower left-insets in Fig. 3 contain the same number of data points as in the main panel. In order to preserve the readability of the figure, we opted to display less data points in the inset as otherwise it would be too crowded due to its smaller size. We have updated the caption to make this clearer to the reader. Changes to manuscript 5.

2. R:“ The author’s description on page 3 of how persistent current can be measured from time of flight (TOF) images is too short to follow. The variance of TOF images appears to be the main quantity of interest, but notations are not defined in this context and it’s not immediately obvious what needs to be done. This makes it harder to appreciate the results in Fig. 4. Moreover, on page 3 the authors cite [53], but in that reference I also could not find a crisp explanation how TOF can directly measure the persistent current (Ref. [53] is surely relevant here, but given that the authors propose their method as a way for cold atom experiments to measure the persistent currents, a more pedagogical brief explanation of the key physics and of what needs to be measured precisely would be appropriate I believe). ”

   A: We agree with the Referee that our description on how to measure the persistent current through TOF is a bit lacking. The reason being that we expanded on this further in a follow-up work [1]. Nonetheless, it would be better to make the manuscript more self-consistent. To this end, we changed the paragraph of TOF in the methods section of the manuscript to give the readers a better picture of how TOF images enable us to measure persistent currents. Additionally, we have added a reference of our follow-up work that explains in great detail on how to measure the persistent current through TOF. Changes in manuscript 6.

3. R:“The authors talk about ’deconfinement’ and make connections to QCD, but their model does not feature SU(3) gauge fields, and SU(3) is broken by hand in the CSF case. They mention this clearly in the end, but I would wish for a brief statement a bit earlier on when they make a connection to quark matter that the model studied here lacks some key features. Overall I find the analogies drawn by the authors justified, so I’m just asking to avoid any potential confusion. ”

   A: We agree with the Referee that it would be much better to make such a statement clear early on. In the methods section of the manuscript, where we first make the analogy, we have updated the manuscript to stress that our cold atoms system is only an analogue to QCD and does not possess all the features of the latter. Changes to manuscript 7.

4. R:“ The authors should be more careful with how they define units. In all figures, they just state U = 0.01, 5 etc without mentioning that they set t=1 in the figures. Likewise, figure axes should always indicate the unit (e.g. use T/t, U/t instead of just T or U). The authors state in the text that t=1, which is fine, but often enough as a field moves on other units become more important (here e.g. the super-exchange $J\ t^4/U$) and there is simply no reason to work with axes without units. ”

   A: We thank the Referee for pointing this out. Amending all the figure axes and labels to say that U/t would overcrowd the figures and decrease the readability of the figures. We agree with the Referee that it is better to make the figures self-consistent. To this end, in all the captions we have explicitly given the value for all the units that we utilized such that the figures can be easily reproduced and understood. Changes to manuscript 8.

LIST OF CHANGES

All changes carried out in the manuscript are highlighted red.

1. On page 4 paragraph 2 line 5, we added "(see Appendix for the explicit definition). Such a weight corresponds to the reduced tri-partite periodicity corresponding to the formation of trions."

2. In the penultimate paragraph of the conclusion on page 5, we added "Moreover, the introduction of a chemical potential, μ, could permit us to study of the "critical line" in the T-μ plane, in analogy with QCD at finite temperature and density, related with relativistic, but lower energy, heavy ion collisions and with the equation of state in the neutron stars core [61,62]. However this aspect requires a dedicated forthcoming analysis."

3. In the caption of Figure 3 on page 3, we added "by monitoring the Fourier weight C_3 of the current that reflects its tri-partite periodicity. This coefficient decreases upon increasing temperature showing the breakdown of trions and in turn deconfinement." and "where I_{max} is the maximum persistent current used to re-scale the Fourier weight across different interactions" to make the figure self-consistent.

4. In the last paragraph of the conclusion on page 5, we added "Additionally, we point out that recent advancements in the platform of programmable tweezers have paved the way to experimentally realize fermionic ring lattices".

5. In the caption of Figure 3 on page 3, we have added "Note that in the lower left insets only a few data points are displayed to enhance the readability of the figure."

6. In paragraph 2 line 7 on page 3, we added "From the experimental side, $n(\mathbf{k})$ can be accessed through contrast image measurement of the density distribution after a TOF expansion of the condensate is carried out by switching off the confinement potential". Furthermore, we added " Nevertheless, current states and the corresponding angular momentum quantization emerge in the variance σ of the width of the momentum distribution as discrete steps. Here, trions display three steps in σ reflecting the reduced tri-partite periodicity of the current." on page 4 paragraph 3 line 10.

7. On page 2 paragraph 6 line 2 we added "due to their relevance in emulating quark matter [15,23,26]. It is important to stress such systems are only analogues as they lack key features of quark matter such as string breaking and colour charge screening." .

8. We have updated all the captions of the figures to say that $t = k_B = 1$.

[1] W. J. Chetcuti, A. Osterloh, L. Amico, and J. Polo, "Interference dynamics of matter-waves of su(n) fermions," (2022).

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

Basically, I am satisfied with the author's reply and revision except for the discussion about the three-body loss. As far as I know, three-body recombination, which is the main origin of the three-body loss, can occur even in the ground-state trions. While the three-body loss of excited-state trimers has been well studied to confirm the Efimov discrete scaling of the binding energies [Phys. Rev. Lett. 103, 130404(2009)], it does not mean that the three-body loss can be avoidable in the ground state as there are no experiments of stable degenerate gases of ground-state trions. In this regard, I am not convinced by the argument of the suppressed three-body loss in the ground state. The three-body loss effect has also been theoretically studied even in the case with optical lattice [Phys. Rev. Lett. 103, 240401 (2009)]. Another possible reduction of the three-body loss might be the 1D confinement as observed in 1D Bose gases [Phys. Rev. Lett. 92, 190401 (2004)] and in 1D Fermi gases [arXiv:2007.15783].
Please elaborate on this point.

Reviewer #2 (Remarks to the Author):

After examining changes made to the manuscript, I am satisfied that the authors have suitably addressed my comments/questions and those of the other reviewers. The readability of the manuscript has also been significantly improved, and it is suitable for publication in Communications Physics.

Reviewer #3 (Remarks to the Author):

As I concluded in my first report, the paper deserves publication in Communications Physics. But let me add two comments:

I was surprised that the authors essentially ignored my comment regarding units, and chose not to indicate units properly on the figures or in the figure captions. I'm not saying what they're doing is factually wrong, but I firmly believe that lots of confusion can be avoided in the future by simply *always* indicating proper units throughout. After all, comparing to previous work one doesn't always read the entire paper to search for a tiny comment somewhere which units are used. I also find the author's argument that figures get 'overcrowded' invalid: writing U/t instead of U does not increase the size of the axes labels significantly.

I have another brief remark about the three-body losses brought up by Reviewer #1: I think this is an interesting point, but I'm not entirely sure the authors response is completely valid. After all, the atoms may react and form lower-lying atomic bound states. Cold atom experiments usually do *not* operate in the actual overall ground state (e.g. ultracold Rb in its ground state doesn't form a BEC, but a solid! The BEC is just a very long-lived state of matter, from which it is hard to reach the solid ground state.) Hence, I'm not sure why three-body losses would be completely absent in their case. That said, however, I don't think the presence of three-body losses (an experimental challenge) invalidates the overall claims and results of the paper.

REFEREE 1

1. R: “Basically, I am satisfied with the author’s reply and revision except for the discussion about the three-body loss. As far as I know, three-body recombination, which is the main origin of the three-body loss, can occur even in the ground-state trions. While the three-body loss of excited-state trimers has been well studied to confirm the Efimov discrete scaling of the binding energies [Phys. Rev. Lett. 103, 130404(2009)], it does not mean that the three-body loss can be avoidable in the ground state as there are no experiments of stable degenerate gases of ground-state trions. In this regard, I am not convinced by the argument of the suppressed three-body loss in the ground state. The three-body loss effect has also been theoretically studied even in the case with optical lattice [Phys. Rev. Lett. 103, 240401 (2009)]. Another possible reduction of the three-body loss might be the 1D confinement as observed in 1D Bose gases [Phys. Rev. Lett. 92, 190401 (2004)] and in 1D Fermi gases [arXiv:2007.15783]. Please elaborate on this point. ”

   A: We thank the Referee for sharing these references with us. After a careful reading of these references and literature on three-body losses, a paragraph was added in the conclusions addressing the expected impact of Efimov states and three-body losses in our system. Changes to the manuscript 1.

REFEREE 2

1. R:“After examining changes made to the manuscript, I am satisfied that the authors have suitably addressed my comments/questions and those of the other reviewers. The readability of the manuscript has also been significantly improved, and it is suitable for publication in Communications Physics. ”

   A: We thank the Referee for recommending our manuscript for publication.

REFEREE 3

1. R:“I was surprised that the authors essentially ignored my comment regarding units, and chose not to indicate units properly on the figures or in the figure captions. I’m not saying what they’re doing is factually wrong, but I firmly believe that lots of confusion can be avoided in the future by simply *always* indicating proper units throughout. After all, comparing to previous work one doesn’t always read the entire paper to search for a tiny comment somewhere which units are used. I also find the author’s argument that figures get ’overcrowded’ invalid: writing U/t instead of U does not increase the size of the axes labels significantly. ”

   A: We want to highlight that we have not simply ignored those comments: indeed we have in all relevant figures modified the captions and the corresponding text. But in the last version of the submitted manuscript, we followed the suggestion of the Referee and updated all of the figure captions to clearly state all of the units (see Changes to manuscript 8 of the last report). In the new re-submitted version of the manuscript, we have updated all of the figure axes to clearly display the units. Changes to manuscript 2.

2. R:“I have another brief remark about the three-body losses brought up by Reviewer 1: I think this is an interesting point, but I’m not entirely sure the authors response is completely valid. After all, the atoms may react and form lower-lying atomic bound states. Cold atom experiments usually do *not* operate in the actual overall ground state (e.g. ultracold Rb in its ground state doesn’t form a BEC, but a solid! The BEC is just a very long-lived state of matter, from which it is hard to reach the solid ground state.) Hence, I’m not sure why three-body losses would be completely absent in their case. That said, however, I don’t think the presence of three-body losses (an experimental challenge) invalidates the overall claims and results of the paper. ”

3. We thank the Referee for the question, which was also raised by the first Referee. Thus, we refer to our answer to the comment of Referee 1.

LIST OF CHANGES

All changes carried out in the manuscript are highlighted red.

1. We have added a paragraph on three-body losses in the Conclusions.
2. We have updated all the axes of the figures to include all the units, as requested by Referee 3.