

Continuous heralding control of vortex beams using quantum metasurface

Hong Liang¹⁺, Hammad Ahmed²⁺, Wing Yim Tam¹, Xianzhong Chen^{2,*}, and Jensen Li^{1,*}

¹Department of Physics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, 999077, P. R. China

²School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH144AS, UK

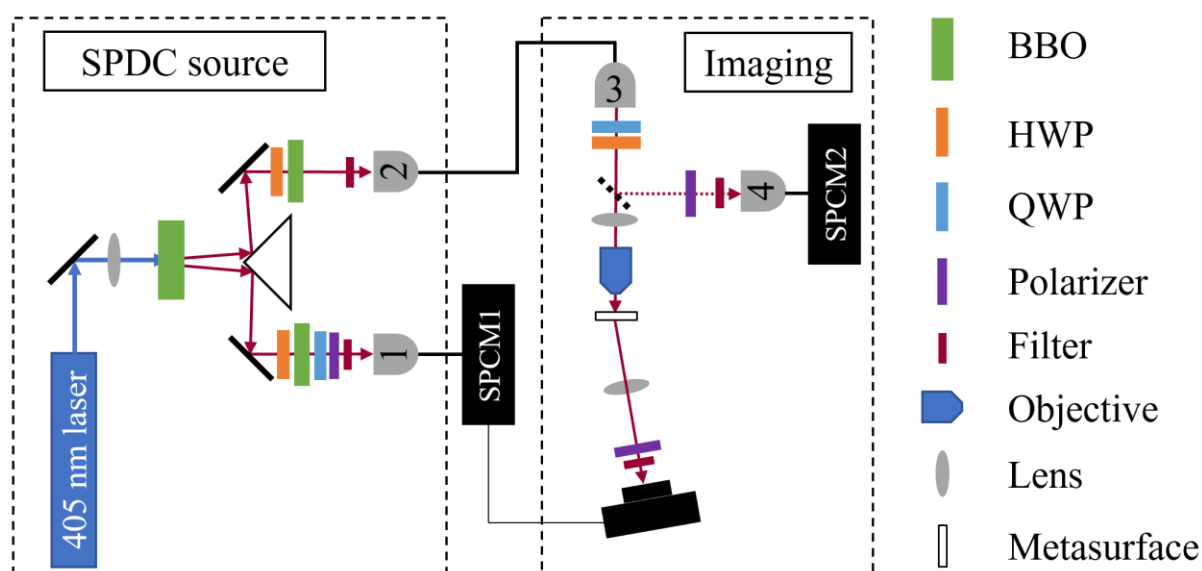
Supplementary information

Supplementary Note 1: Fabrication Details

The geometric-phase metasurface consists of planar gold nanorods with spatially variant orientations sitting on an ITO-coated glass substrate. First, the substrate is cleaned with acetone for 10 min in an ultrasonic bath, followed by isopropyl alcohol (IPA) for 10 min. Then, a photoresist poly(methyl methacrylate) (PMMA) 950 A2 resist is spin-coated on the substrate at 1000 rpm for 60 s, producing a 100 nm thick film. After that, the sample is baked on a hotplate at 180°C for 5 min. Electron beam lithography (Raith PIONEER, 30 kV) is used to direct-write the nanorod pattern, which is then developed in MIBK:IPA (1:3) for 45 s and rinsed IPA for 45 s. Then an electron beam evaporator is used to deposit a 40 nm-thick gold film on the sample. Finally, the metasurface is ready for characterization after a lift-off process.

Supplementary Note 2: Experimental details

In the experimental setup (Supplementary Fig. S1), we use a 2-mm-thick type-II BBO and a 200mW 405 nm laser (CrystaLaser DL-405-400) to generate the polarization-entangled photon pairs in a state of $1/\sqrt{2}(|HV\rangle + |VH\rangle)$, with a fidelity of 91.2% and a Bell parameter of $2.54 > 2$. The half-opening angle of the generated photon pairs is designed to be 3° . The photons are split into the signal arm and heralding arm using a prism. To compensate for the translational and longitudinal walk-off effects of the photon pairs, a half-wave plate (HWP), with an optical axis at 45° with the horizontal axis, and a BBO with half of the thickness of the main BBO are installed in both arms.



Supplementary Figure S1 The full experimental setup. The polarization-entangled photon pair are generated in the BBO type-II β -barium borate (BBO) crystal pumped by a 405 nm laser. The pair are separated by a prism into the heralding arm and signal arm. There are a half-wave plate (HWP) and BBO in each arm to compensate for the walk-off effects. The heralding photon would be polarization-selectively detected with a quarter-wave plate (QWP) and a polarizer using a single-photon counting module (SPCM). The signal photon is sent to interact with the metasurface and imaged by a single-photon avalanche diode (SPAD) camera, which is gated by the heralding detection signal.

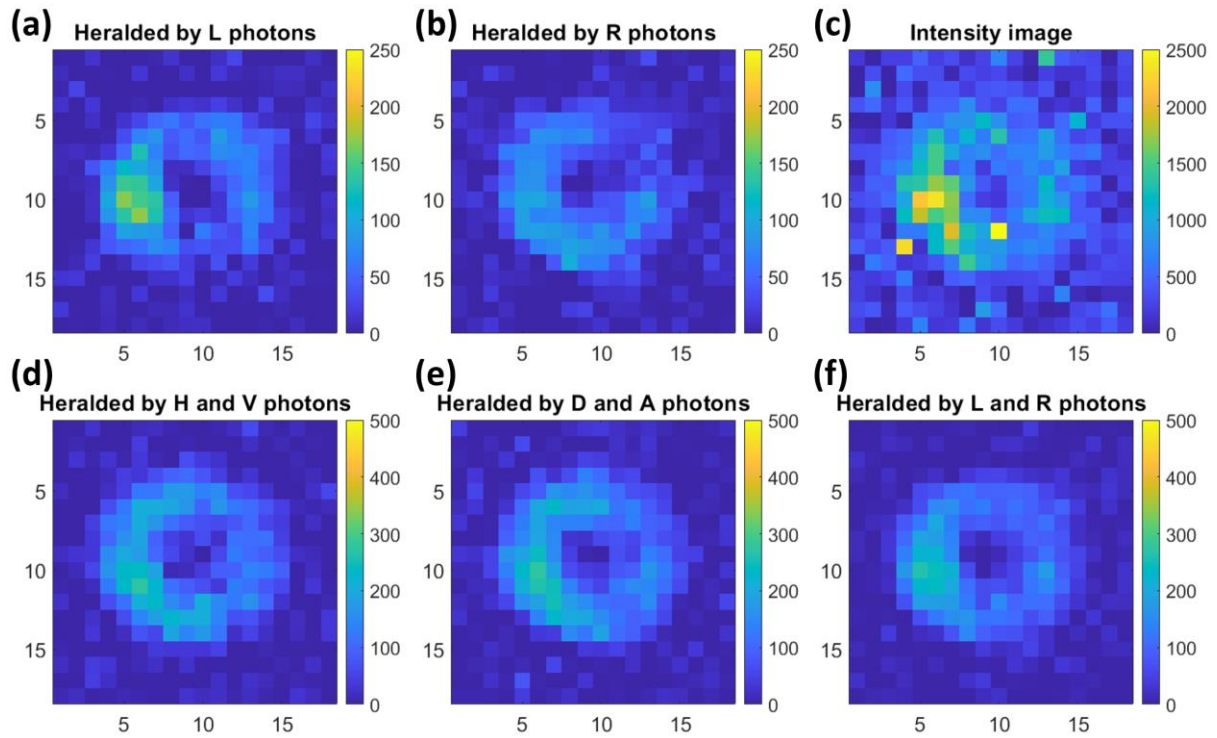
The photons in the heralding arm are detected with polarization selection using a single photon counting module (SPCM) (Excelitas-SPCM-800-14-FC), of which the detection signals are sent to herald signal photons' arrival on the SPAD camera. Meanwhile, the signal photons

are sent through a 10-meter-long single-mode fiber to the imaging setup. The HWP and QWP before the metasurface are used to correct the polarization rotation due to the long fiber so that the input quantum states on the metasurface are $1/\sqrt{2}(|HV\rangle + |VH\rangle)$. The polarization state is checked with a flip mirror to direct the light, through a polarizer, into fiber coupler 4, which is detected by SPCM2 (dashed light path). The SPCM2 is correlated with SPCM1 with a correlator (UQD-Logic-16). We then use a lens (focal length 100 mm) with a 10x objective to focus the signal beam onto the metasurface. The vortex beam generated from the metasurface is imaged by the SPAD camera (SPAD512S) using a lens with a focal length of 45 mm.

The single-photon intensity profile of the signal photons in Supplementary Fig. S2(c) is acquired using 6000 frames without heralding triggers and with background white noise subtracted. Each frame spans 100 ms with a maximum of 255 photon counts in each pixel. The background is taken with the signal photons blocked (i.e., only white noise is captured). On the other hand, the heralded image of the signal photons in Figs. 4(a-d) and Supplementary Figs. S2(a,b) are all retrieved using 6000 frames with external triggers from SPCM1 and with background subtracted. Each frame spans 100 ms with a maximum of 255 photon counts in each pixel. Each trigger from the SPCM1 would turn on the camera for a detection window of 10 ns. With heralding photon counting rate up to 300 kHz, there would be multiple triggers in one frame, and the detection events in each triggered window would accumulate into one frame. Again, the background is measured using the same triggering setting with blocked signal photons.

Supplementary Note 3: More experimental results

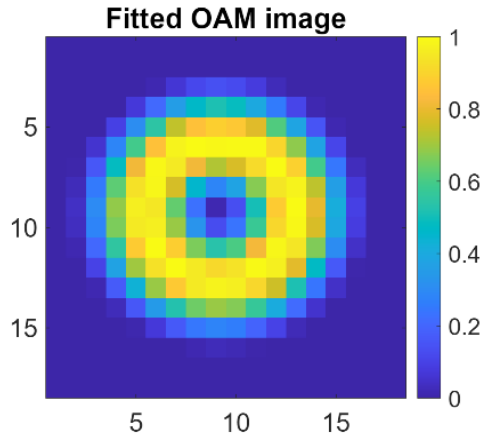
Here we show two more heralded results using left- and right-circular polarized heralding photons, as shown in Supplementary Figs. S2(a,b). These two results both show up a doughnut shape, corresponding to the $|+1\rangle$ and $| -1\rangle$ states. The single-photon intensity image is taken without any heralding trigger, shown in Supplementary Fig. S2(c). To consolidate the discussion about adding the heralded results for two orthogonal polarizations, Supplementary Figs. S2(d-f) show, respectively, the sum of heralded images for the horizontal and vertical, diagonal and antidiagonal, and left- and right-circular polarizations. All results display doughnut-shaped profiles, confirming the theoretical analysis.



Supplementary Figure S2 (a,b) The vortex beam profile of signal photons heralded by photons of left (L)- and right (R)-handed polarizations. (c) The intensity image of the signal photon without heralding mechanism. (d-f) The addition of the results using heralding photons of horizontal and vertical, diagonal and antidiagonal, and left- and right-handed circular polarizations.

Supplementary Note 4: Advantage of the heralded images regarding noise robustness

In evaluating the noise robustness of the heralded images, we use the signal-to-noise ratio (SNR) and the Pearson correlation coefficient to denote the quality of the images. We compare the doughnut shape image (Supplementary Figs. S2(c-f)), with and without the heralding trigger, to the expected result obtained by fitting the experimental heralded images with background subtracted. The fitting function is $A(J_1(\mathbf{r}))^2 + c$, where J_1 is the Bessel function of the first kind, $\mathbf{r} = \left(\frac{x-x_0}{s_x}, \frac{y-y_0}{s_y}\right)$ is the pixel position with scaling (s_x, s_y) and offset (x_0, y_0) , and $A(c)$ is signal amplitude (white noise background). With fitted noise background c subtracted, the normalized expected doughnut shape of the OAM is shown in Supplementary Fig. S3.



Supplementary Figure S3 The target OAM image for comparison with experimental results.

The Pearson correlation coefficient is then calculated as

$$r_{mt} = \frac{\sum_i^n (m_i - \bar{m})(t_i - \bar{t})}{\sqrt{\sum_i^n (m_i - \bar{m})^2} \sqrt{\sum_i^n (t_i - \bar{t})^2}}$$

where $m(t)$ denotes the measured experimental (target ideal) image and i denotes the pixel index ranging from 1 to n . To calculate SNR, we use

$$SNR = 20 \log_{10} \left(\frac{\sqrt{\sum t_i^2}}{\sqrt{\sum (m_i/s - t_i)^2}} \right),$$

where s is 95 percentiles of m_i counts being used to renormalize the experimental image. We avoid directly using the maximum count as a normalization factor in the image to alleviate the error due to outlier noisy pixels. The results for Supplementary Figs. S2(c-f) are summarized in Supplementary Table S1. We can see that the heralded results (Supplementary Figs. S2(d-f)) have an average SNR (Pearson correlation coefficient) of 8.5 dB (0.90), which is higher than that of the single-photon intensity image at 3.9 dB (0.71) (Supplementary Fig. S2(c)). This indicates that the heralding imaging mechanism has greater robustness to noise.

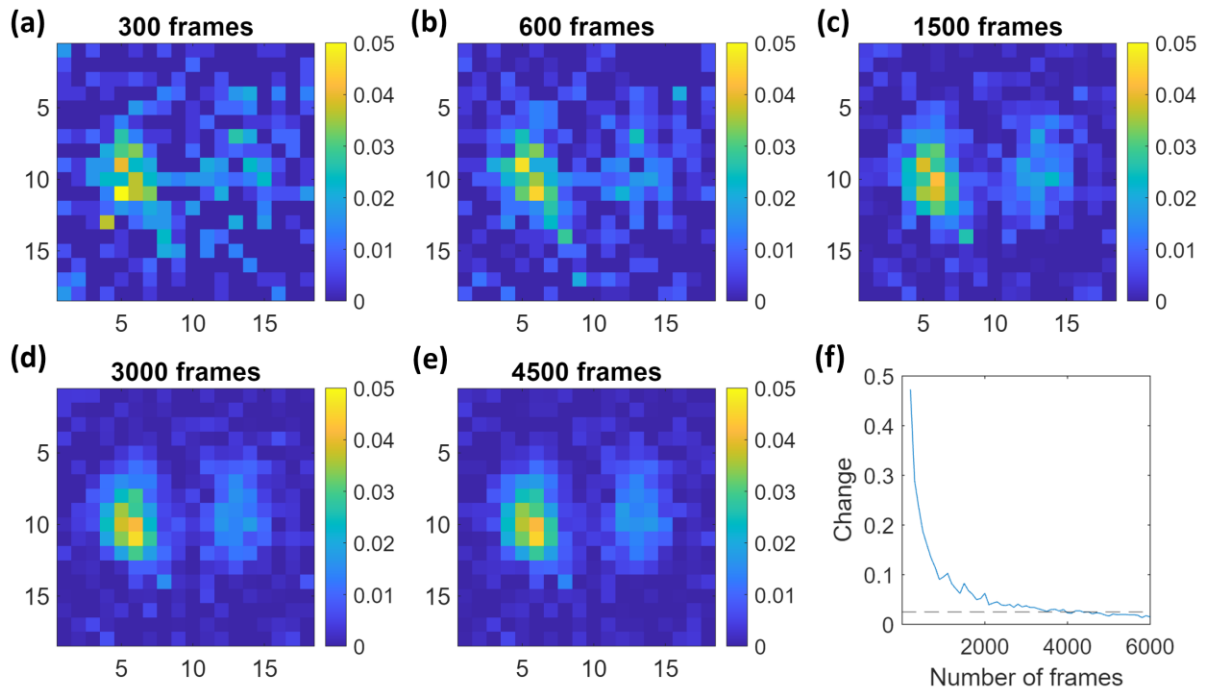
Supplementary Table S1 Pearson correlation coefficient and SNR in figures

Image of OAM	Pearson correlation coefficient	SNR
Supplementary Figure S2(c)	0.71	3.9
Supplementary Figure S2(d)	0.90	8.6
Supplementary Figure S2(e)	0.90	8.8
Supplementary Figure S2(f)	0.89	8.2

To show the effect of the number of frames on the quality of the heralded images, Supplementary Fig. S4 shows an example of heralded images taken with and normalized by the different number of frames. Using only 300 or 600 images, the images (Supplementary Figs. S4(a,b)) is noisy, and the vortex state is hardly identified. As the number of frames increases, the heralded image quality is gradually enhanced and vortex state becomes more distinct, as shown in Supplementary Figs. S4(c-e). Supplementary Fig. S4(f) shows the change of image quality as the number of frames increases, which is calculated by

$$D(N) = \frac{\sum_{i=1}^n |I_i(N) - I_i(N - 100)|^2}{\sum_{i=1}^n |I_i(N - 100)|^2},$$

where i is the pixel index ranging from 1 to n , $I_i(N)$ is the photon number at pixel i obtained with and normalized by a total the frame number of N . As shown in Supplementary Fig. S4(f), the change gradually decreases, indicating that the image quality gradually converges as the number of frames increases. The change of image quality is lower than 2.5% (dashed line) when the number of frames exceeds around 4000, indicating that the image quality becomes stable afterward. Therefore, within a practical acquisition time, we choose 6000 images in our experiment to achieve a converged image quality.

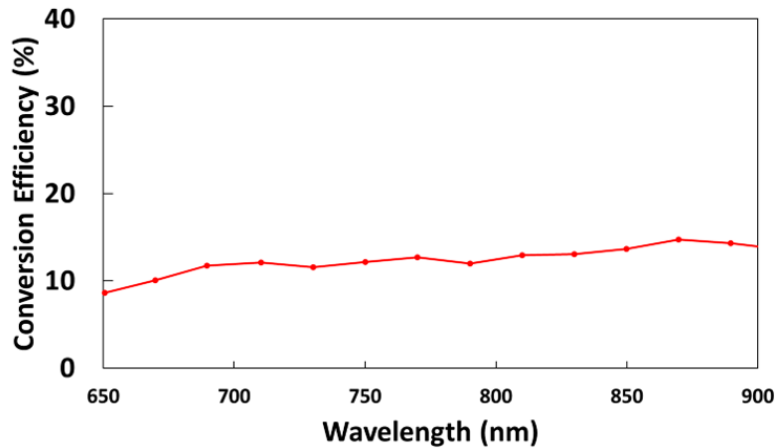


Supplementary Figure S4 The heralded images using vertical polarization taken with and normalized by the different number of frames, from (a) 300 frames to (e) 4500 frames. (f) The change of detected averaged photon number as the number of frames increases.

Supplementary Note 5: Efficiency of the experiment

In the experiment, the polarization conversion efficiency of the metasurface is critical and is defined as the ratio of the power of the modulated light with cross polarization and the input power. We measured the conversion efficiency of the metasurface at different wavelengths, as illustrated in Supplementary Fig. S5. At the working wavelength (810 nm), the efficiency is around 13%.

As for the heralding detection efficiency, we should also consider the camera efficiency (3%) and the detector efficiency (50%). Conditional to the photon pair arriving at the corresponding SPCM and camera, the heralding efficiency would be $3\% \times 50\% = 1.5\%$. We note that although the detection efficiency is quite low for the equipment, the results are obtained with high quality, as shown in the manuscript. And we believe that the efficiency of the metasurface can be further improved with a dielectric design.



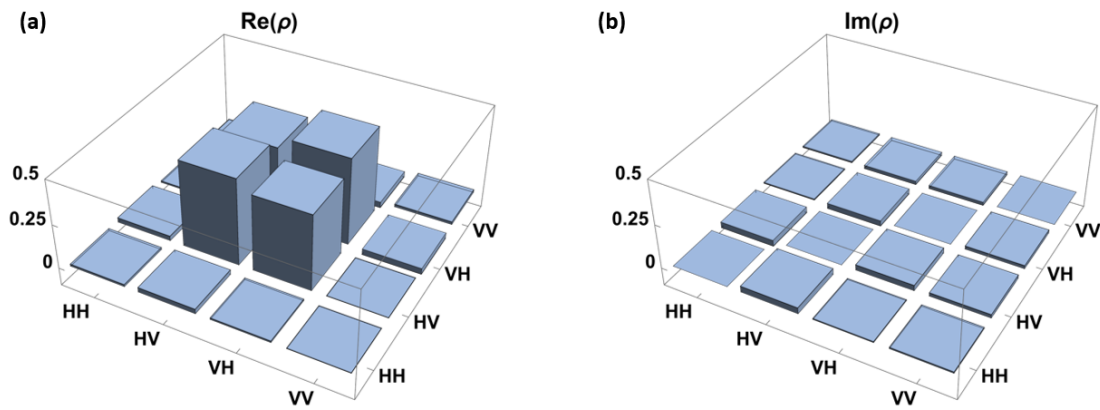
Supplementary Figure S5 The measured polarization conversion efficiency of the metasurface consisting of gold nanorods (200 nm long, 80 nm wide, and 40 nm thick).

Supplementary Note 6: Characterization of the quantum light source

To characterize the quantum light source, we measure the density matrix of the source using quarter-wave plates and polarizers to measure the coincidence of different analyzed states, as shown in Supplementary Table S2 [1]. With maximum likelihood estimation, the reconstructed density matrix is

$$\rho = \begin{pmatrix} 0.011 & 0.026 - 0.034i & 0.008 - 0.005i & 0.003 - 0.007i \\ 0.026 + 0.034i & 0.486 & 0.425 - 0.030i & 0.002 - 0.019i \\ 0.008 + 0.005i & 0.425 + 0.030i & 0.488 & -0.032 - 0.018i \\ 0.003 + 0.007i & 0.002 + 0.019i & -0.032 + 0.018i & 0.015 \end{pmatrix},$$

showing a state of $(|HV\rangle + |VH\rangle)/\sqrt{2}$ with a fidelity of 91.2%. We also plot the real part and imaginary part of the density matrix, as shown in Supplementary Fig. S6. With the coincidence data measured in Supplementary Table S3, the Bell parameter is estimated to be $2.54 > 2$, violating the Bell inequality and indicating the polarization entanglement of the quantum photon pair [2].



Supplementary Figure S6 The reconstructed density matrix of the quantum light source.

Supplementary Table S2 Coincidence of different analyzed states for density matrix reconstruction (unit: counts per second).

Signal Idler	H	D	V	R
H	919	14803	27726	14513
D	15769	26550	12425	15575
V	26859	14500	947	15439
R	16396	15097	15376	28406

Supplementary Table S3 Coincidence of different analyzed states for Bell test (unit: counts per second). The angle denotes the angle between the polarizer transmission axis respect to the horizontal axis in the signal (idler) arm.

Signal Idler	0°	90°	45°	135°
22.5°	5792	22735	23799	4026
112.5°	22550	6010	3409	23379
67.5°	22880	6012	22665	6134
157.5°	5585	22489	5252	22263

Supplementary References

- [1] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, *Measurement of Qubits*, Phys. Rev. A **64**, 052312 (2001).
- [2] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Proposed Experiment to Test Local Hidden-Variable Theories*, Phys. Rev. Lett. **23**, 880 (1969).