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Reviewers' comments:

Reviewer #1 (Remarks to the Author):

Rivy et.al. shows the first realization of a single-atom trapped in a superoscillatory light field. That is, a modified optical profile that can beat the diffraction limit by the interference of multiple coherent modes. They achieve this by using two spatial light modulators in series to adiabatically transform their optical potential from a close to diffraction limited normal TEM00 mode to a superoscillatory mode with a central "hotspot" that beats the diffraction limit by a factor of $\sim 70\%$ at the expense of most of the power being in an outer ring. They perform a thorough characterization of the superoscillatory trap in comparison to the normal diffraction limited trap, finding that the transformation effectively cools down their atoms to very cold temperatures.

However, while the writing of the manuscript is mostly clear, I find the presentation confusing to the reader. As an atomic physicist, I found the explanation of how a magneto optical trap works redundant. While more relevant discussions such as technical aspects of superoscillatory optical potentials are left for the reader to figure out by piecing together statements across the manuscript. Therefore, I cannot recommend publication of this manuscript in its current form. In the following, I will mention my concerns in order of most important to least:

1.- There is an absence of any discussion about the shape of superoscillatory optical traps and their implications. By carefully reading the Methods section and the Supplement, you can find out that for constant power, the effective power of the hotspot is reduced by a factor of ~ 21 . This means that most of the power in the trap is actually on the outer ring. Not only does this seem like a clear limitation due to an increase by an order of magnitude for power overhead, but there is no discussion on what this outer ring can mean for a trap itself, or how it could limit experiments with multiple traps. This seems like a clear limitation to generate any sort of sub-wavelength ordered arrays like mentioned in the abstract. I request that the authors add a clear discussion about the shape of superoscillatory optical traps.

2.- It is unclear to me how their technique can be scaled towards many traps. As it stands, it seems that the double SLM technique can only work for a single trap? If this is correct, do the authors have any information as to whether superoscillatory modes are compatible with techniques such as Acousto Optic Deflectors? I would request that mentions of many atom arrays be removed or that a clear small description of a pathway towards this be added in the conclusion.

3.- You mention that abrupt changes from Airy to superoscillatory lead to atom loss, but there is no data to compare. By reading Figure 2's caption we find that it takes 390 ms for the adiabatic transformation, while in the supplement this is 300ms. I would request a clearer explanation and presentation of the timescales of the transformation as well as a section in the supplement for the survivability vs transformation time ideally with a figure.

4.- Figure 2. Please add errorbars to panels c and d. In the case of d, you quote a fitted temperature of 1.5(5) μK in the text, but in the figure, your data seems to agree the most with the 2.5 μK curve. Please explain this. And consider just putting the fitted temperature with the fitted standard deviation as the extra curves on the figure.

5.- End of paragraph 4 of Results section. You claim that the superoscillatory field gives you an enhancement of 6.2 on the required power for the same trapping frequencies. This seems like an easy thing to measure directly, can you add a comparison to a measured value? Also, is this for the power only on the hotspot or on the whole profile including ring? please make it clear.

6.- There is no discussion on the axial trapping frequencies, can these be measured as well? At least for the normal Airy potential, measuring both axial and radial would give you an estimate of the trap waist that you can compare to your imaging system one.

6.- Middle of paragraph 4 of Results section. It should be "... dip of the survival probability..." not "deep".

7.- Adiabatic cooling paragraph in Method section. "OT" is not defined as optical potential in the text, only in the supplement. Spell out "optical potential".

Reviewer #2 (Remarks to the Author):

In the manuscript "Single atom in a superoscillatory optical trap" the authors present a new approach to trapping by using a pair of SLMS to perform phase and amplitude encoding of a dipole trap beam to permit continuous tuning of the beam profile from an Airy spot to a sub-wavelength hotspot, offering tighter confinement for a given beam power and suitable for a wide range of potential experiments.

The authors carefully motivate the work and provide experimental evidence for the observed trapping, but in its current form the paper lacks important details that are required to really show the novelty and impact of the work. Below I list the main points that are not discussed, or not included with sufficient detail:

(1) The method requires a pair of SLMS - what is the efficiency? How much power is lost in this process, and can it scale?

(2) Figure 1 shows a cartoon of the setup, but it is not entirely clear - I actually think the Fig 1 from the supplementary material with an inset of Fig1 from the paper would be far more instructive (albeit with fewer distances and perhaps a simplified optical setup).

(3) The main result of the paper is characterising the trapping of a single atom as the trap is transformed, however I would have expected more experimental results to show this characterisation. For example, the trap frequency of the sub-wavelength trap is measured, but what was the frequency of the initial Airy spot? This is not quoted. As the size is tuned, how does the trap frequency scale given as the spot is reduced more power is placed into the diffraction ring? Whilst changed confinement is quoted, and numbers of the power within the spot is mentioned, is there not a continuous tuning between the two beam profiles? How does this trade off in trap frequency and spot power vary between the two limits - is there an optimum trap frequency between the two chosen operating points? For example if the power drops by a factor of 20, but 6x more power was required to get the same trap frequency then the net result is lower confinement in a subwavelength trap than an Airy trap. Am I perhaps missing then the benefit of less power per trap for the same trap frequency when accounting for power lost to the rings?

(4) An adiabatic ramp is described but no data showing transfer efficiency vs ramp time etc to show how this was optimised, how sensitive it is.

(5) Many of the proposed applications relate to arrays - how does this method work scaling to more than a single spot? The group in Paris have pioneered methods for generating arrays of traps with SLMs, but if this method requires a third SLM or other techniques it is unclear how useful the result really is.

If the authors can address these questions and provide a more careful discussion then I believe it may be possible for this result to be sufficiently novel for consideration in this journal, but at present for the reasons outlined above I am not convinced.

A final minor point - in the introduction optical traps are described as useful for atoms, but also many

groups are now making optical traps for molecules and a suitable reference or two should be added to reflect this.

Reviewer #1 (Remarks to the Author):

We thank the reviewer for their constructive comments and criticism. In the following, we remind the reviewer's comments, and we give our answers in red. The changes are summarized in blue. Finally, changes are highlighted in blue in the manuscript.

Rivy et.al. shows the first realization of a single-atom trapped in a superoscillatory light field. That is, a modified optical profile that can beat the diffraction limit by the interference of multiple coherent modes. They achieve this by using two spatial light modulators in series to adiabatically transform their optical potential from a close to diffraction limited normal TEM00 mode to a superoscillatory mode with a central "hotspot" that beats the diffraction limit by a factor of $\sim 70\%$ at the expense of most of the power being in an outer ring. They perform a thorough characterization of the superoscillatory trap in comparison to the normal diffraction limited trap, finding that the transformation effectively cools down their atoms to very cold temperatures.

However, while the writing of the manuscript is mostly clear, I find the presentation confusing to the reader. As an atomic physicist, I found the explanation of how a magneto optical trap works redundant. While more relevant discussions such as technical aspects of superoscillatory optical potentials are left for the reader to figure out by piecing together statements across the manuscript. Therefore, I cannot recommend publication of this manuscript in its current form. In the following, I will mention my concerns in order of most important to least:

Authors' reply: We thanks the reviewer for pointing out that our manuscript is mostly clear with thorough characterization. We briefly discuss the magneto-optical trap having the photonics community in mind, who is not necessarily familiar with laser cooling techniques. We now add more technical aspects on superoscillatory optical traps as requested. Those changes are detailed in our replies below.

1.- There is an absence of any discussion about the shape of superoscillatory optical traps and their implications. By carefully reading the Methods section and the Supplement, you can find out that for constant power, the effective power of the hotspot is reduced by a factor of ~ 21 . This means that most of the power in the trap is actually on the outer ring. Not only does this seem like a clear limitation due to an increase by an order of magnitude for power overhead, but there is no discussion on what this outer ring can mean for a trap itself, or how it could limit experiments with multiple traps. This seems like a clear limitation to generate any sort of sub-wavelength ordered arrays like mentioned in the abstract. I request that the authors add a clear discussion about the shape of superoscillatory optical traps.

Authors' reply: It is correct that the Airy-trap power is 23 mW and the superoscillatory-trap power is 1.1 mW, thus a reduction of ~ 21 as the reviewer pointed out. However, it does not mean that the remaining power, namely $23 - 1.1 \sim 22$ mW are located in the outer ring. Indeed, the superoscillatory mask contains an amplitude part that reduces the total power available in the superoscillatory configuration to ~ 8 mW. Hence the hotspot counts for 14% of the total power and ~ 7 mW are within the ring. We overlooked this important point, which is now discussed in the manuscript.

By transferring from an airy hotspot to a superoscillatory hotspot, we prevent atoms to be trapped in the outer ring, since the overlap of the two trap occurs only within the superoscillatory hotspot location. Without transfer, we did not observe atom trapping which rule out trapping in the outer ring within our parameter range. We further clarify this point in the manuscript.

We agree that the presence of strong outer rings could be a limitation for a traps array. Nevertheless, there is at least one interesting approach to address this potential limitation. So far, superoscillation was used for super-resolution where the purpose is to push away the outer ring as far as possible from the hotspot. We used similar approach for our work. For a hotspot array, one might need a new paradigm where the optical field out of the hotspot shall be spatially spread to reduce the peak energy density in outer rings. This point is currently under investigation in our group, but beyond the scope of the present publication. We extend the discussion of hotspot array in the manuscript.

Changes in the manuscript:

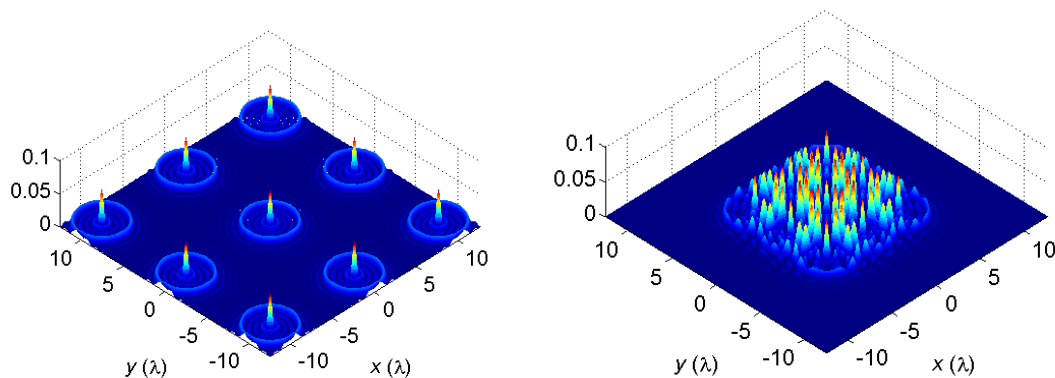
- In the result section 2nd paragraph, we discuss the power distribution: “The Air-trap power is 23 mW. ... is spread into the outer ring.”

- In the result section 4th paragraph, we clarify why possible loading of the outer ring can be ruled out: “We did not observe direct loading of the superoscillatory pattern ... the weak trapping depth of the outer ring.” And “Since the Airy profile does not overlap with the outer ring, we can safely rule out transfer and trapping outside the hotspot”

-In the conclusion section last paragraph, we discuss possible implementation of the hotspot array: “Creation of standard optical trap arrays ... outside the hotspot.”

2.- It is unclear to me how their technique can be scaled towards many traps. As it stands, it seems that the double SLM technique can only work for a single trap? If this is correct, do the authors have any information as to whether superoscillatory modes are compatible with techniques such as Acousto Optic Deflectors? I would request that mentions of many atom arrays be removed or that a clear small description of a pathway towards this be added in the conclusion.

Authors’ reply: The double SLM technique could also be used to generate an array of super oscillatory traps. We show examples below that were numerically generated.



If the traps are far apart, each trap behaves independently (left figure). If the period is set such that trap (and outer rings) start to overlap, interference take place and destroy the periodicity (right figure). The superoscillatory pattern can also be sent into an acousto-optic modulator (AOM), as long as the angular spread of the pattern is compatible with the Bragg diffraction condition of the device. Eventually, possible distortions in the AOM can also be corrected with the SLMs.

To realize a sub-wavelength pattern, one can deflect with an AOM the left pattern alternatively at different angles, leading to displaced array at the atoms position. If the change is fast enough (with respect to the trapping frequency), we can realized the same array than in the left figure but with shorter periodicity.

We gave a possible approach, but maybe other alternatives are also possible, and we hope that our work will trigger fruitful ideas for implementations. However, one has to keep in mind that implementation of superoscillatory arrays is not a central point of the manuscript but a potential future application still speculative at this stage.

Changes in the manuscript:

- We rephrase the first sentence of the abstract to avoid potential confusions about the objectives of the manuscript.

- In the conclusion last paragraph, we discuss a possible implementation of a sub-wavelength array: "For practical realisation of a hotspot array ... alternatively turn on neighbouring sites, in the same spirit than Ref. [16]."

3.- You mention that abrupt changes from Airy to superoscillatory lead to atom loss, but there is no data to compare. By reading Figure 2's caption we find that it takes 390 ms for the adiabatic transformation, while in the supplement this is 300ms. I would request a clearer explanantion and presentation of the timescales of the transformation as well as a section in the supplement for the survivability vs transformation time ideally with a figure.

Authors' reply: For an abrupt change from Airy to superoscillatory, we barely see transfer, and the survivable probability below our measurement capability.

The adiabatic transformation from Airy to Superoscillatory trap took 300 ms. The transformation from superoscillatory back to Airy trap took 90 ms. Hence, the minimal total time of a sequence is 390 ms. This time is taken into account when we compute the trap lifetime shown on Fig. 3b (previously Fig. 2b). The survival probability after a sequence of minimal duration (i.e. 390 ms) is 0.8 in agreement with the trap lifetime, meaning that no extra atom losses are detected when we switch from Airy to superoscillatory trap. This point is now clarified in the manuscript.

Changes in the manuscript: in the result section 4th paragraph, we clarify the time scale involves in the Airy to superoscillatory trap sequence: "We note that the adiabatic transfer from Airy to superoscillatory profile takes 300 ms ... no extra significant losses occur during the transfer from Airy to superoscillatory trap."

4.- Figure 2. Please add errorbars to panels c and d. In the case of d, you quote a fitted temperature of 1.5(5) uK in the text, but in the figure, your data seems to agree the most with the 2.5 uK curve. Please explain this. And consider just putting the fitted temperature with the fitted standard deviation as the extra curves on the figure.

Authors' reply: The typical fluctuations in Fig. 3c (Previously Fig. 2c) are visible on the graph for modulation frequencies outside the resonance peak at 100 kHz. We evaluate the standard deviation of the survival probability to be 0.07.

We thank the reviewer for pointing out possible inconsistency in the discussion about temperature. In fact, the example shown on Fig. 3d (previously Fig. 2d) does not correspond to the observed

minimal temperature, thus there is not inconsistency here. Both points are now clarified in the manuscript.

Changes in the manuscript:

- in figure 3 caption, we give the mean and standard deviation of data points in panel c: "Outside the resonance at 100 kHz, the survival probability reads 0.76(7), where the error correspond to one standard deviation"

- in figure 3 caption, we clarify the temperature error and give the MOT power to identify the corresponding point in the inset of panel d: "We then deduce an effective temperature of 2.5(10) μ K here, for an optical power per MOT beam of 130 μ W"

5.- End of paragraph 4 of Results section. You claim that the superoscillatory field gives you an enhancement of 6.2 on the required power for the same trapping frequencies. This seems like an easy thing to measure directly, can you add a comparison to a measured value? Also, is this for the power only on the hotspot or on the whole profile including ring? please make it clear.

Authors' reply: We agree that this discussion might be confusing and not supported by a direct measurement of the trapping frequency in that regime. We simply remove the incriminate discussion and replace it by a discussion on the measurement of the trapping frequency of the Airy trap at 23 mW.

The power is given for the hotspot only and not for the whole profile. We clarify this point in the manuscript.

Changes in the manuscript: in section results 5th paragraph, we remove "As the trap characteristic transverse cross-section scales ... larger than the superoscillatory hotspot", and add a discussion on the trapping frequency of the Airy trap "A measurement on the Airy hotspot ... agreement with the measurements"

6.- There is no discussion on the axial trapping frequencies, can these be measured as well? At least for the normal Airy potential, measuring both axial and radial would give you an estimate of the trap waist that you can compare to your imaging system one.

Authors' reply: Approximating the Airy profile with a Gaussian profile, we expect a longitudinal trapping frequency 3 times smaller than the radial frequency. For the superoscillatory trap, the longitudinal frequency reduction factor is expected to be larger, since the trap has a needle shape. This point is discussed in the method section. In both case, we were not able to obtain a measurement of the longitudinal frequency.

6.- Middle of paragraph 4 of Results section. It should be "... dip of the survival probability..." not "deep".

Authors' reply: corrected.

7.- Adiabatic cooling paragraph in Method section. "OT" is not defined as optical potential in the text, only in the supplement. Spell out "optical potential".

Authors' reply: corrected.

Reviewer #2 (Remarks to the Author):

We thank the reviewer for their constructive comments and criticism. In the following, we remind the reviewer's comments, and we give our answers in red. The changes are summarized in blue. Finally, changes are highlighted in blue in the manuscript.

The authors carefully motivate the work and provide experimental evidence for the observed trapping, but in its current form the paper lacks important details that are required to really show the novelty and impact of the work. Below I list the main points that are not discussed, or not included with sufficient detail:

Authors' reply: We thanks the reviewer for pointing out that we provided experimental evidence of the observed trapping. We now add technical details as requested. Those changes are detailed in our replies below.

(1) The method requires a pair of SLMS - what is the efficiency? How much power is lost in this process, and can it scale?

Authors' reply: The efficiency of the SLMs is about 50%, mainly due to the efficiency of the phase grating to diffract the pattern in the proper diffracting order. Another 70% of the power is lost once we switch from Airy to superoscillatory trap mainly because of the presence of the amplitude mask. The latter point is added in the manuscript. The former point as well as a detailed description of the setup is given in ref. [1] of the supplemental material.

Changes in the manuscript: In the result section 2nd paragraph, we discuss the power distribution: "The Airy-trap power is 23 mW. ... is spread into the outer rings."

(2) Figure 1 shows a cartoon of the setup, but it is not entirely clear - I actually think the Fig 1 from the supplementary material with an inset of Fig1 from the paper would be far more instructive (albeit with fewer distances and perhaps a simplified optical setup).

Authors' reply: We thanks the reviewer for his/her comment. We now add a new figure (Fig 2) where panel a gives a simplified schematic of the superoscillatory optical system.

-Changes in the manuscript: We add a new figure to present the superoscillatory optical setup in the main text. A more detailed scheme is still given in the supplemental material.

(3) The main result of the paper is characterising the trapping of a single atom as the trap is transformed, however I would have expected more experimental results to show this characterisation. For example, the trap frequency of the sub-wavelength trap is measured, but what was the frequency of the initial Airy spot? This is not quoted. As the size is tuned, how does the trap frequency scale given as the spot is reduced more power is placed into the diffraction ring? Whilst changed confinement is quoted, and numbers of the power within the spot is mentioned, is there not a continuous tuning between the two beam profiles? How does this trade of in trap frequency and spot power vary between the two limits - is there an optimum trap frequency between the two chosen operating points? For example if the power drops by a factor of 20, but 6x more power was required to get the same trap frequency then the net result is lower confinement in a subwavelength trap than an Airy trap. Am I perhaps missing then the benefit of less power per trap for the same trap frequency when accounting for power lost to the rings?

Authors' reply: The trapping frequency of the Airy trap is measured to be 75 kHz, thus a factor ~ 1.5 larger than for the superoscillatory trap. This value is given in the method section, but now discuss also in the main text. There is a continuous changes between the Airy and superoscillatory profile which explain the adiabatic cooling discuss in the manuscript. As we go from Airy to superoscillatory trap, the power in the hotspot reduces from 23mW to 1.1mW. At the same time the size reduce as well but not enough so the trapping frequency decreases. As a result, for an experiment limited by the laser power, with a single trap, the best option would be to use an Airy (or Gaussian) profile. In contrast, if the experiment is limited by the peak intensity (to limit multi-photons absorption, for example), the superoscillatory trap, with smaller trap size, is the best option. We discussed this point in the conclusion and now elaborate more on that important point.

Changes in the manuscript:

- in section results 5th paragraph, we add a discussion on the trapping frequency of the Airy trap "A measurement on the Airy hotspot ... agreement with the measurements"

-in conclusion section 3rd paragraph, we add a critical comparison between the Airy and the superoscillatory trap "We measured a transverse trapping frequency ... similar trapping frequency at lower laser intensity".

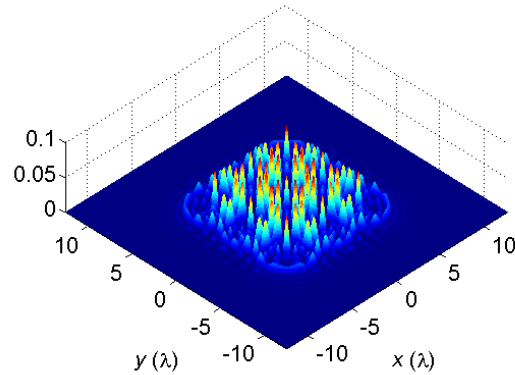
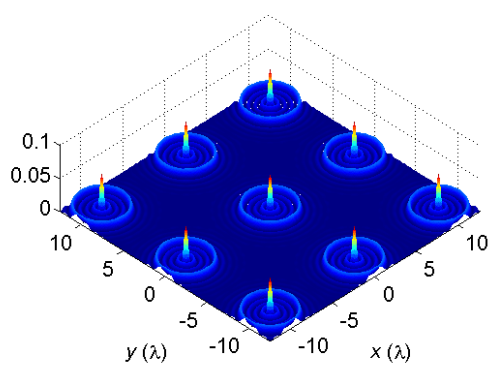
(4) An adiabatic ramp is described but no data showing transfer efficiency vs ramp time etc to show how this was optimised, how sensitive it is.

Authors' reply: The transfer is performed by inserting two intermediate SLM patterns from the Airy to the final superoscillatory profile. This point is now clearly stated in the manuscript. The patterns were chosen to minimize the difference between two consecutive patterns. The total time to realize the transfer is 300 ms, limited by the response time of the SLMs. Here, we are already deeply into the adiabatic limit, so longer transfer time will only reduce the survival probability because of the finite trap lifetime. We show on Fig. 3b (previously Fig. 2b) that the transfer efficiency is close to unity, and the reduction of the survival probability is dominated by the trap lifetime. We note that a direct transfer from Airy to superoscillatory trap leads to an almost null transfer efficiency. By inserting one intermediate image, the transfer efficiency is improved but still not satisfactory. In the view of the experimental constrains (number of SLM patterns, slow transfer rate with respect to the adiabatic conditions) and the excellent transfer with two intermediate patterns, we did not perform a systematic study of the transfer rate.

Changes in the manuscript: in section results 4th paragraph, we add more details on the transfer "In practical, using two intermediate profiles is enough to obtain a transfer from Airy to superoscillatory trap with a probability close to unity."

(5) Many of the proposed applications relate to arrays - how does this method work scaling to more than a single spot? The group in Paris have pioneered methods for generating arrays of traps with SLMs, but if this method requires a third SLM or other techniques it is unclear how useful the result really is.

Authors' reply: Reviewer 1 addressed similar comment. We copy here the reply we give to reviewer 1. The double SLM technique could also be used to generate an array of super oscillatory traps. We show examples below that were numerically generated



If the traps are far apart, each trap behaves independently (left figure). If the period is set such that trap (and outer rings) start to overlap, interference take place and destroy the periodicity (right figure). The superoscillatory pattern can also be sent into an acousto-optic modulator (AOM), as long as the angular spread of the pattern is compatible with the Bragg diffraction condition of the device. Eventually, possible distortions in the AOM can also be corrected with the SLMs.

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A final minor point - in the introduction optical traps are described as useful for atoms, but also many groups are now making optical traps for molecules and a suitable reference or two should be added to reflect this.

Authors' reply: We add the work of Anderegg and co-authors (ref. [3])

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

The authors have satisfactorily replied to my comments and amended their manuscript accordingly. I can now recommend publication.

Reviewer #2 (Remarks to the Author):

In the revised manuscript the authors have addressed the major concerns raised by the referees and have significantly improved both the clarity and significance of the research. Specifically, they have addressed questions around relative trap power and efficiencies as well as prospects for making arrays.

With the changes, this paper now meets the threshold for novelty and quality listed under the journal scope.

A minor change is required to the first sentence of the abstract to pluralise "atoms" and "molecules"