Supplementary information:

Single atom in a superoscillatory optical trap

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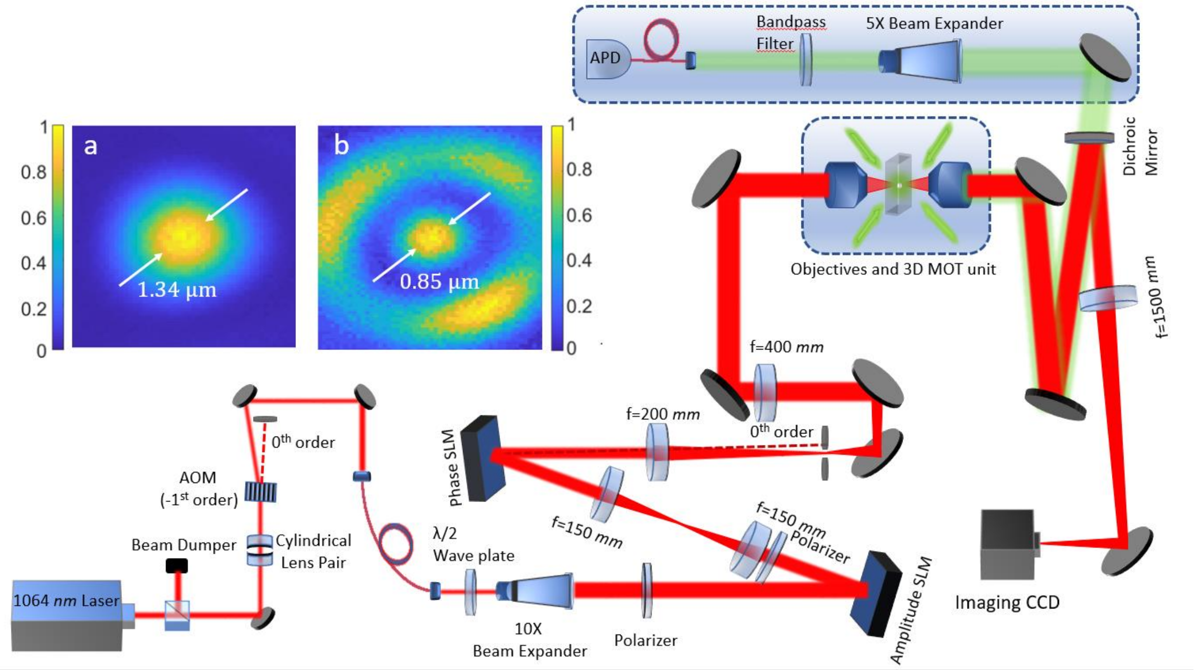
Supplementary Note 1: Superoscillatory fields optical set-up

The optical trap (OT) is created thanks to a $1064\ \mathrm{nm}$ linearly polarised $1.5\ \mathrm{W}$ laser. The OT beam is spatially filtered by a single mode fibre and expanded to a beam waist of $\sim 10\ \mathrm{mm}$ (see Supplementary Fig. 1). The core of the superoscillatory (SO) optical setup consists of two optically conjugated spatial light modulators (SLMs) for transversally resolved amplitude and phase reflection. The SLMs system is set to generate an SO optical field profile constructed from two circular prolate spheroidal functions $E(r/\lambda) = 3.123 S_2(r/\lambda) + S_3(r/\lambda)$, where r is radial distance from the hotspot centre, as described in [1].

The active aperture diameter of our SLM-based optical system is $9.65\ \mathrm{mm}$. The SLMs are imaged at the entrance pupil of a high numerical aperture microscope objective (working distance of $20.2\ \mathrm{mm}$, effective focal length $20.4\ \mathrm{mm}$), through a $2\times$ magnification achromatic lens doublet. The entrance pupil diameter of the microscope is $20.4\ \mathrm{mm}$, therefore slightly larger than the diameter $2 \times 9.65 = 19.3\ \mathrm{mm}$ of the SLMs image, leading to a numerical aperture of $\mathrm{NA} = 0.43$.

We use two microscope objectives, customized to compensate for the spherical aberration of the vacuum chamber windows, and provide a diffraction limited optical system at $852\ \mathrm{nm}$ and $1064\ \mathrm{nm}$. The first objective focuses the $1064\ \mathrm{nm}$ OT beam at the cold atoms cloud position. The second objective collects the fluorescence signal at 852 nm that is sent to an avalanche photodiode detector (APD), and serves to image the 1064 nm OT on a CCD camera. The latter imaging system is made of the second objective and a $f = 1500$ mm lens, and provide an image of OT beam with a 75× magnification. The images of the Airy profile used to load the OT and the SO profile are shown in Supplementary Fig. 1a and Fig. 1b, respectively.



Supplementary Figure 1. ***The optical setup, amplitude and phase masks****. Main picture: Sketch of the OT optical system. The phase and amplitude profiles of a 1064 nm beam (red optical path) are engineered thanks to two conjugated SLMs. The beam is focused at the centre of a cold atoms cloud in a Magneto-Optical Trap (MOT) represented here by laser beams (green arrows). The beam profile is imaged on a CCD camera with a 75x magnification optical system. The cold atoms fluorescence (green optical path), due to the MOT beams is collected on an APD. (a) and (b) Images of the Airy and the SO profiles, with an Abbe radii of 1.34(3) µm and 0.85(3) µm, respectively.*

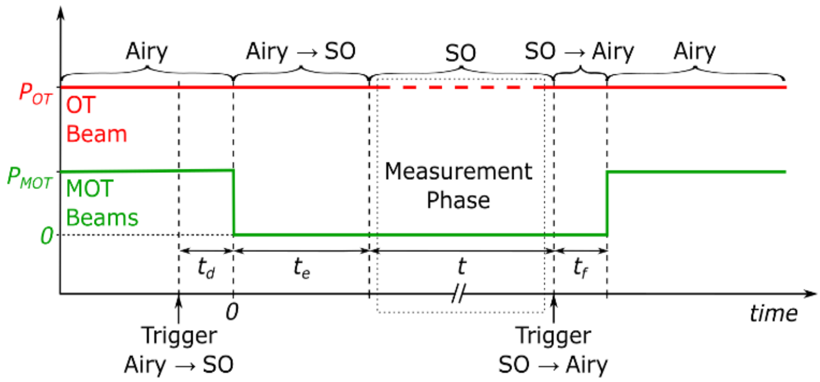
Supplementary Note 2: Temporal sequence for superoscillation trapping characterizations

The atom population in the Airy OT is determined from the APD photo-count integrated over 20 ms, see Fig. 2a in the main text. If the APD photo-count is above 100, the OT is considered to be populated with one atom, which triggers the temporal sequence sketched in Supplementary Fig. 2. During $t_d = 140$ ms, the MOT beams power is eventually either decreased or increase (not shown in Supplementary Fig. 2), to change the effective temperature of the atom in the OT. At $t = 0$, the MOT and repump beams are switched off and we adiabatically change the Airy profile into a SO profile during $t_e = 300$ ms. The SO OT is maintained for an extra variable time t to perform different type of measurements. Then, we send a trigger to go back to Airy profile within the next $t_f = 90$ ms. Finally, the MOT beams are turned on to measure the survival probability of the atom in the OT. Overall, the MOT beams are switched off during a time of $t + t_e + t_f = t + 390\,ms$.

During the measurement phases, we act on the OT power, according to the type of measurement, we are aiming to perform. As discussed in the main text, we did three measurements:

- The lifetime of the atom in the OT, which is done while maintaining the OT power at constant value.
- The trap frequency measurement, which is performed by modulating the OT beam during 300 ms with 4% of modulation depth. For each run, the modulation frequency is set within 10 kHz to 300 kHz.
- The effective temperature measurement, which is done by turning off the OT beam for a variable time $t = 0 - 400$ µs

We performed 80 runs for each data point shown in Supplementary Fig. 2b-d in the main text. To avoid possible systematic drifts, we randomized the order of the parameter-value occurrence.



*Supplementary Figure 2. **Temporal sequence for superoscillation trap characterizations**. The different sequences of the OT beam profiles are given on the top of the figure. The red and the green curve indicate the temporal evolution of the OT beam power and the MOT beams (cooling and repump beams) power, respectively. The measurement is performed during the SO phase (dotted rectangle). Here, the OT beam power might change (dashed red line) according to the type of measurement (see text for more details). At time -t_d, the Airy trap is loaded with a single atom and a trigger is sent to start the transfer from Airy to SO profile. $t_d = 140\,ms$ is the delay time associated with the software trigger, $t_e = 300\,ms$ is the time to change form Airy to SO profile, and $t_f = 90\,ms$ is the time to transfer back to Airy OT for measurement of the survival probability.*

References

[1] E. T. F. Rogers, S. Quraishe, K. S. Rogers, T. A. Newman, P. J. S. Smith, and N. I. Zheludev, “Far-field unlabeled super-resolution imaging with superoscillatory illumination,” *APL Photonics*, vol. 5, no. 6, p. 066107, Jun. 2020, doi: 10.1063/1.5144918.