

Dynamics of inertialess sedimentation of a rigid U-shaped disk



Open Access This file is licensed under a Creative Commons Attribution 4.0

International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to

the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. In the cases where the authors are anonymous, such as is the case for the reports of anonymous peer reviewers, author attribution should be to 'Anonymous Referee' followed by a clear attribution to the source work. The images or other third party material in this file are included in the article's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this license, visit <http://creativecommons.org/licenses/by/4.0/>.

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

The paper by Miara et al investigates in a very elegant way the complex sedimentation dynamics of a U-shaped disc. The dynamics of this seemingly simple system turn out to be extremely rich and the authors have combined very well controlled experiments using sophisticated image analysis tools together with modeling to elucidate and explain the underlying mechanisms.

The reorientation dynamics of the discs prove to be very slow compared to the vertical sedimentation velocity of the disc, leading to motions with a periodicity that can be of thousands of disc radii along the z axis. Obviously in experiments only a small fraction of such a trajectory can be visualized making it very difficult to rationalize and understand the disc motion.

The authors performed very careful and well controlled experiments with the aim of uncovering the sedimentation dynamics. A very sophisticated and inventive image treatment method allowed them to extract the full 3D orientation of the discs only from the image projection into one plane and recorded with a single camera. From the reconstructed disc trajectories and orientations, they obtained first hints about the periodicity of the trajectories. Finally full modeling of the problem revealed the full spectra of trajectory types and their characteristics.

Using the combination of experiments and results from modeling the authors then investigate the role of the initial orientation of a disc of given radius and curvature in the framework of a dynamic system, with saddle points and fixed points. The localization of the initial conditions in this phase space determines the observed type of trajectory.

The main findings concern the fact that first, all trajectories are periodic, and a continuous sideways drift is never observed, as is for example the case for a rigid fiber. And second, chiral trajectories are observed for the sedimentation of a non-chiral particle with the handedness given by the initial disc orientation, a very intriguing observation.

This is a very nicely written paper elucidating and explaining a very complicated dynamical system in a comprehensive and relatively easy to follow way. It describes the complexity obtained during sedimentation of a simple object, a rigid U-shaped disc. These findings constitute by themselves a complete and very interesting study but can also serve as a starting point for the understanding of the sedimentation of more complex particles, as rigid particles with different symmetries or deformable particles. In addition, I feel that not only the results themselves, but also the methods used and described in this paper are valuable for future studies. In my opinion the paper deserves to be published in Communications Physics with minor modifications.

I ask the authors to address the following points before final acceptance of the paper.

- 1) I am very intrigued by the fact that chiral trajectories occur without chirality in the sedimenting object (chiral trajectories under sedimentations have been predicted before but for chiral objects, like helices). So, I wonder if there is chirality in some projected plane or cut through the disc that could explain this finding and also explain how and why the initial orientation determines the handedness.
- 2) Different types of trajectories are observed and predicted, is there a certain type of trajectories observed most frequently in the experiments as others occur only for a very narrow parameter range? May be the authors could point out what dynamics are nearly impossible to observe in the experiments?
- 3) I understand that in this work a given disc geometry has been investigated. At one point in the paper the authors comment on the role of the disc size, is there anything that can be said on the role of the curvature of the bent disc? How would that influence the periodicity?
- 4) If I understand correctly, the mobility matrix is corrected for the influence of the container

boundaries by optimizing the agreement between the experimental and numerical trajectories. How important are these corrections? Also, what does "These values are in good agreement with those obtained from direct numerical simulations based on the Stokes equations." mean? If there are exact solutions, why use the optimized results?

5) Is there any easy explanation why the fixed point is found at $(90^\circ, 8.6^\circ)$? What sets the values of this fixed point?

6) And a very minor comment. Can you define what you mean by helix and what you mean by spiral? It seems you use both interchangeable?

Reviewer #2 (Remarks to the Author):

The authors report the sedimentation dynamics of a rigid U-shaped disk in the limit of negligible inertia by the combination of experimental and numerical approaches. In comparison with previous works on sedimentation behavior of a single particle, the authors test a U-shaped disk as an achiral object. The major finding of this work is the demonstration of chiral trajectories exhibited by the achiral object as determined by the initial state. The presence of chiral trajectory created by rigid bodies (including achiral objects) has been theoretically discussed in Ref. [14]. In this work, the experimentally observed trajectories are classified, and they are modeled in the frame of the linear Stokes equations. I have some questions on the completeness of experimental data and the depth of the analysis on the experimental results, which may impair the reliability of the conclusion (see below for detailed questions). Furthermore, the authors shall discuss the impact of this work on relevant fields.

1. The sedimentation dynamics of the disk is described in the frame of the Eqs.(3) and (4). Is the linearity of the motion validated by experiment? Will any nonlinear effect arise in the long-time evolution of the disk?

2. In Fig.6, the presented experimental data are incomplete. Are the numerically predicted periodic patterns formed in the x-y plane observable in experiment?

3. Mathematically, due to the first-order nature of the governing dynamical equations, the initial position and orientation of the disk determine the subsequent dynamical evolution of the system. In general, the dynamical state is a linear superposition of multiple eigenstates of distinct frequencies. For the combination of incommensurate frequencies, the motion is aperiodic. However, according to Fig. 5, the observed motion is periodic. The authors may want to discuss on this issue. Furthermore, how shall one understand the dependence of the period on the initial orientation?

4. In Fig.5, a slight change of β_0 leads a drastic fall of the period in the region of small β_0 , showing the sensitivity of the dynamics on disk orientation. What is happening here in physics? Is this result reproducible?

5. It is of interest to examine the variation of key dynamical behaviors as the disk is gradually changed from a planar shape to a U-shaped one. This investigation may shed light on the development of chiral trajectories.

6. A technical question: the authors employ the tool of neural network to fit the disk shape. Considering that the key parameters of the disk geometry could be directly measured, what are the necessity and advantage of using the tool of auto-encoder?

Reviewer #3 (Remarks to the Author):

The authors studied the sedimentation of our U-shaped disk in a tank filled with silicone oil by experiments and theory. Experiments were performed for different placed orientation. Then, the dynamics of the U-shaped disk is well described using a ballistic model by considering the leading-order interactions of the disk with the boundaries of the fluid domain. The works are carefully done. I am therefore recommending publication of the manuscript in Commsphys, but I have a few questions that would help the readability and clarity of the manuscript.

The released way of the U-shaped disk is not very clear in the text. It is uncertain whether there will be any changes in the initial state and velocity during the release process. Besides, I noticed that U-shaped disk is made up of polyamide nylon, how stiff is it? Is it possible to deform during sedimentation.

2, Will vigorously stirring the tank cause bubbles to form in the liquid, thereby affecting the movement of objects?

3, The significance of the modification to the theoretical model appears to be questionable. The modification is centered around correcting the forces exerted by the boundaries on the U-shaped disk, whereas the subsequent predictions and experiments are specifically aimed at scenarios involving fluid with unbounded fluid.

4, Furthermore, the referenced literature regarding the important content of data processing in the article has not been published. Are there any arXiv preprints available for reference?

Response to Reviewers' comments:

Reviewer #1:

We thank the reviewer for their careful reading of our paper and their insightful comments. We are delighted that they appreciated the study as careful and elegant and found that it had a dual value of both elucidating complex sedimentation dynamics and providing novel methods of measurement and interpretation valuable to future studies. Please find below the itemized response to their comments and suggestions.

1) I am very intrigued by the fact that chiral trajectories occur without chirality in the sedimenting object (chiral trajectories under sedimentations have been predicted before but for chiral objects, like helices). So, I wonder if there is chirality in some projected plane or cut through the disc that could explain this finding and also explain how and why the initial orientation determines the handedness.

A chiral particle sediments preferentially along a trajectory of particular handedness set by the shape of the particle and not by the initial conditions. Our particle is a cylindrically-deformed disk, with two planes of reflectional symmetry; it is identical to its mirror image, and therefore is non-chiral. Hence, there is no preferred handedness to the trajectory it follows; this is set by the initial orientation alone. As stated on p.3 (column 1, second paragraph), positive and negative values of α (or β) result in the same orientation relative to the direction of gravity, but with the direction of pitching (or rolling) reversed. So the handedness of the trajectories differs depending on whether the initial angles α and β have the same or opposite signs.

We previously only stated this on p.6 (first column, second paragraph) of the paper but we have now included the following sentence in the conclusion: "In spite of our particle's two planes of symmetry (which make it identical to its mirror image and therefore non-chiral), our particle sediments along chiral trajectories. The handedness of these trajectories is set by the initial orientation alone, which is determined by whether the initial angles (α , β) have the same or opposite signs."

2) Different types of trajectories are observed and predicted, is there a certain type of trajectories observed most frequently in the experiments as others occur only for a very narrow parameter range? Maybe the authors could point out what dynamics are nearly impossible to observe in the experiments?

The trajectories shown in the α - β phase plane in Fig. 3 are closed orbits, and therefore, the particular trajectory observed is uniquely determined by its initial conditions. As such, all trajectories are equally likely to be experimentally observed if the initial conditions for α and β are random. However, because pitching occurs on a much longer timescale than rolling, the disk's β angle rapidly decays towards zero far from the dynamic centre. The disk then undergoes a long period of near-pure pitching, during which very small differences in the β angle lead to very different orientations later in the trajectory. The sensitivity in this region of phase-space means that the outermost trajectories will not be experimentally observable in full. This is now shown in the new Fig. 3b (see item 3) where the disk can be seen to be perturbed during the period of near pure-pitching from outermost trajectories which start very close to the line of pure pitching onto trajectories with a smaller β span due to unavoidable experimental fluctuations. We have added this discussion in the last paragraph of column 2 on p.5 of the revised paper.

3) I understand that in this work a given disc geometry has been investigated. At one point in the

paper the authors comment on the role of the disc size, is there anything that can be said on the role of the curvature of the bent disc? How would that influence the periodicity?

Thank you for nudging us to elaborate on the role of the curvature of the U-shaped disk. We have added a second panel to Fig. 3 (see Fig. 3b below), which shows the phase-plane trajectories of the smaller and more curved disk shown in Fig. 2b of the paper, whose non-dimensional radius of curvature $R_c/R=0.21$ is less than half of that of the larger disk ($R_c/R=0.52$). As can be seen in Fig 3b, the orbits have a centre at $\beta_c \approx 18^\circ$, which is more than twice that of the less curved disk. Additionally, we recently conducted some experiments on a flatter disk ($R_c/R=1.19$) and found the centre to be $\beta_c=6.1^\circ$, suggesting that β_c increases monotonically with increasing curvature. We have added these additional details in the last paragraph of column 1 on p.3 of the revised paper.

We found that for the flatter disk, the coupling terms in the fitted mobility matrix which govern the rate of reorientation are smaller and the period is longer. This is consistent with the intuition that in the limit of zero curvature, the coupling terms must go to zero and the period tends to infinity (so that a flat disk sediments steadily). We have added this in the discussion of the period on p.6 (column 1, paragraph 2).

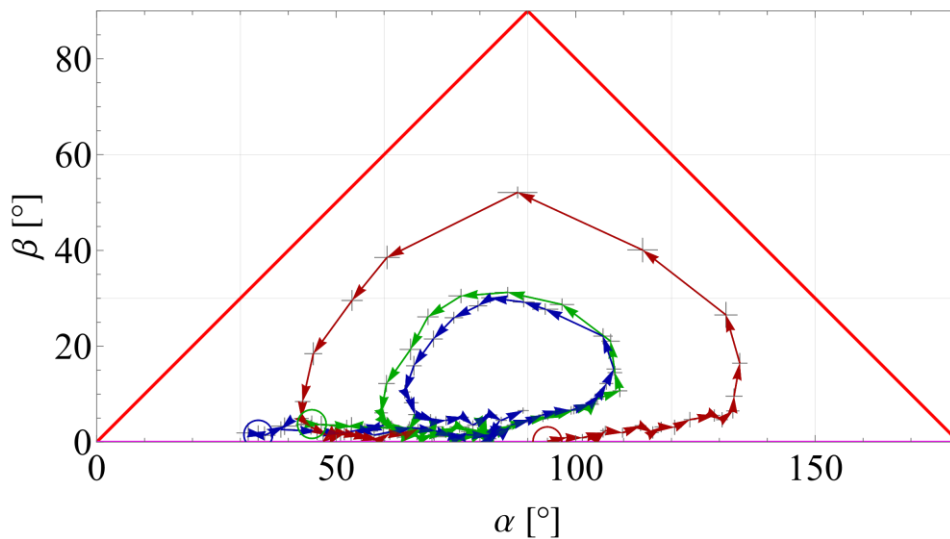


Figure 3b: Phase plane trajectories of the small curved disk ($R = 5$ mm and $R_c = 1.05$ mm) shown in Fig. 2b showing concentric orbits around the dynamical centre at $\beta \approx 18^\circ$, which is twice the value for the less curved disk.

4) If I understand correctly, the mobility matrix is corrected for the influence of the container boundaries by optimizing the agreement between the experimental and numerical trajectories. How important are these corrections? Also, what does “These values are in good agreement with those obtained from direct numerical simulations based on the Stokes equations.” mean? If there are exact solutions, why use the optimized results?

We applied the leading-order corrections derived by Caswell [25] for the interactions of a particle with the boundaries of the fluid domain. Without this correction, the different components of the fitted mobility matrix differ by between 3% and 14%, which is reassuringly modest and justifies our choice of container size. We apologize for the confusion introduced by the “These values are in good agreement with those obtained from direct numerical simulations based on the Stokes equations.” Here we refer to finite element simulations of the sedimentation of a similar disk. However, the comparison with numerical simulations is not straight-forward and a detailed account would detract from the focus of this paper, which is the characterisation of the dynamics of a U-shaped disk

sedimenting in a large tank. Hence, we have decided to remove the sentence referring to the direct numerical simulations from the paper to avoid confusion.

The issue with the comparison to the computational model which is described in detail in Ref. [28] is that numerical results are not from time-stepped simulations in an equivalently-sized container which would replicate the experimental setup. Instead, for reasons of computational cost, they are performed as six steady-state solutions of the Stokes equations in which the three translations and three rotations about the body-fitted axes are imposed sequentially, and the resulting forces and torques on the body are computed after each solve. This allows us to assemble the resistance matrix (the inverse of the mobility matrix), which can be subsequently used in a post-processing step to time-integrate the equations of motion (3) and (4) to determine the trajectory for given initial conditions (see Ref. [28]). Thus, this resistance matrix captures the instantaneous resistance of the disk when it is located at the centre of the finite computational domain. It is therefore also affected by the presence of the container walls, but not quite in the same way as the matrix that we obtained by fitting its coefficients to the experimentally observed trajectories.

5) Is there any easy explanation why the fixed point is found at $(90^\circ, 8.6^\circ)$? What sets the values of this fixed point?

For a perfectly symmetrically bent sheet, we expect $\alpha_c = 90^\circ$ for reasons of symmetry, and the experimental deviation indicates imperfections in the sheet geometry. As discussed in item 3, the value of β_c is set by the geometry of the disk and we find that β_c increases monotonically with increasing curvature. We now discuss this on p.3 (last paragraph of column 1). To the best of our knowledge, there is no analytical expression which would allow us to derive *a priori* the position of the fixed point from a given geometry.

6) And a very minor comment. Can you define what you mean by helix and what you mean by spiral? It seems you use both interchangeably?

We did not mean to use the terms “helix” and “spiral” interchangeably, and in response to this comment we have now defined the former by “a spiral with constant pitch”. We have also updated the caption of Fig. 4 where helical motion is first illustrated. A perfectly helical trajectory in our system is present only at the dynamical centre (see Fig. 4e), while all other closed orbits manifest in the laboratory frame as complex spirals.

Reviewer #2:

We thank the reviewer for their careful reading of the paper and thought-provoking comments which have helped us improve the paper. We provide an itemized response below.

General comments:

a. I have some questions on the completeness of experimental data and the depth of the analysis on the experimental results, which may impair the reliability of the conclusion (see below for detailed questions).

Please see our detailed response to these concerns in the itemized response to specific comments.

b. Furthermore, the authors shall discuss the impact of this work on relevant fields.

As stated in the conclusion, we find that a dilute suspension of our U-shaped disks does not exhibit horizontal dispersion and we demonstrate that the sedimentation of rigid particles of complex shape is not chaotic. We have now added that our analysis also provides a novel framework in which to interpret physically the motion of sedimenting particles of arbitrary shape.

Specific comments:

1. The sedimentation dynamics of the disk is described in the frame of the Eqs.(3) and (4). Is the linearity of the motion validated by experiment? Will any nonlinear effect arise in the long-time evolution of the disk?

The system is non-inertial (we have verified the validity of the Stokes approximation in the experiment), but also nonlinear since the equations of motion (3) and (4) are written in a body-fitted coordinate system which rotates with respect to the lab frame throughout the motion. It is this nonlinearity which gives rise to the rich dynamics presented in the paper.

2. In Fig.6, the presented experimental data are incomplete. Are the numerically predicted periodic patterns formed in the x-y plane observable in experiment?

We appreciate the reviewer's concern that experimental trajectories are relatively short compared to the typical lengthscale of the spiral trajectories shown in Fig. 6. This is unavoidable because experimentation on our larger U-shaped disk over a depth of ~ 3000 radii shown in Fig. 6 would require a ~ 30 metre tall tank, which is clearly incompatible with controlled experiments in the Stokes limit. However, Fig. 3 indicates that multiple experiments can be used just as effectively as a long experiment to reconstruct the phase plane dynamics of our disk, because of the absence of measurable inertia. We now highlight that the trajectories cover 3000 disk radii in the caption of Fig. 6.

However, we also appreciate that the fact that our 90 cm tall tank does not allow our larger U-shaped disk ($R=12\text{mm}$, $R_c/R=0.52$) to complete a full cycle in the phase plane (Fig. 3) is a concern. In response, we have added a second panel to Fig. 3 (see Fig. 3b below), which shows the phase-plane trajectories of the smaller and more curved disk shown in Fig. 2b of the paper, whose non-dimensional radius of curvature $R_c/R=0.21$ is less than half of that of the larger disk ($R_c/R=0.52$). We believe that Fig 2b together with the supplied Fig 3b make a convincing case that the U-shaped disk has completed a cycle and therefore that the U-shaped disk would continue to reorient in the same fashion, producing a quasi-periodic spiralling trajectory.

3. Mathematically, due to the first-order nature of the governing dynamical equations, the initial position and orientation of the disk determine the subsequent dynamical evolution of the system. In general, the dynamical state is a linear superposition of multiple eigenstates of distinct frequencies. For the combination of incommensurate frequencies, the motion is aperiodic. However, according to Fig. 5, the observed motion is periodic. The authors may want to discuss on this issue. Furthermore, how shall one understand the dependence of the period on the initial orientation?

Fig. 5 shows the period of the orbits in the two-dimensional phase plane, i.e. the time it takes for the sheet to return to its initial inclination described by the angles α and β which are defined only relative to the axis of gravity. It is only these two orientation angles which affect the kinematics, because the fluid is isotropic and therefore rotations about the gravitational axis (yaw) merely add a rigid-body rotation to the trajectories. In the laboratory frame, however, the motion is indeed aperiodic due to the fact that the period in (α, β) plane is generally incommensurate with the period of rotation of the particle about the gravitational axis. This leads to the precession of the orbit about

this axis, as observed in the horizontal projections shown in Fig. 6. Specific periods are commensurate with the yawing frequency, for example at the dynamic centre; this trajectory traces a true periodic path in phase-space, corresponding to a perfect helical path followed by the centre of mass (Fig. 4(e)). This is now stressed more clearly in the discussion.

4. In Fig.5, a slight change of β_0 leads a drastic fall of the period in the region of small β_0 , showing the sensitivity of the dynamics on disk orientation. What is happening here in physics? Is this result reproducible?

The steep curve at small β_0 in Fig. 5 is associated with the difference in timescales between pitching and rolling reorientations. Pitching occurs on a much longer timescale than rolling, and so for small β_0 , perturbations in the particle's β angle rapidly decay towards zero, and the particle then undergoes a long period of near-pure pitching motion, during which time very small differences in β will lead to very different orientations later in the trajectory. This is reflected in Fig. 3a, where the outer trajectories away from the dynamic centre all pass extremely close to the horizontal axis. The high sensitivity in this region of phase-space means that the outermost trajectories will not be experimentally observable/reproducible in full; the parts of the trajectories away from the line of pure pitching will be readily reproducible, but unavoidable experimental fluctuations during the period of near pure-pitching will tend to perturb the particle onto a different trajectory. This is now shown in Fig. 3b (see item 5) where the disk can be seen to be perturbed due to unavoidable experimental fluctuations.

5. It is of interest to examine the variation of key dynamical behaviors as the disk is gradually changed from a planar shape to a U-shaped one. This investigation may shed light on the development of chiral trajectories.

Thank you for asking us to elaborate on the role of the curvature of the U-shaped disk. We have added a second panel to Fig. 3 (see Fig. 3b below). It shows the phase-plane trajectories of the smaller and more curved disk shown in Fig. 2b of the paper, whose non-dimensional radius of curvature $R_c/R=0.21$ is less than half of that of the larger disk ($R_c/R=0.52$). As can be seen in Fig 3b, the orbits have a centre at $\beta_c \approx 18^\circ$, which is more than twice that of the less curved disk. Additionally, we recently conducted some experiments on a flatter disk ($R_c/R=1.19$) and found the centre to be $\beta_c=6.1^\circ$, suggesting that β_c increases monotonically with increasing curvature. We have added these additional details in the last paragraph of column 1 on p.3 of the revised paper.

We found that for the flatter disk, the coupling terms in the fitted mobility matrix which govern the rate of reorientation are smaller and the period is longer, consistent with the intuition that in the limit of zero curvature, the coupling terms must go to zero and the period tends to infinity (so that a flat disk sediments steadily). We have added this in the discussion of the period on p.6 (column 1, paragraph 2).

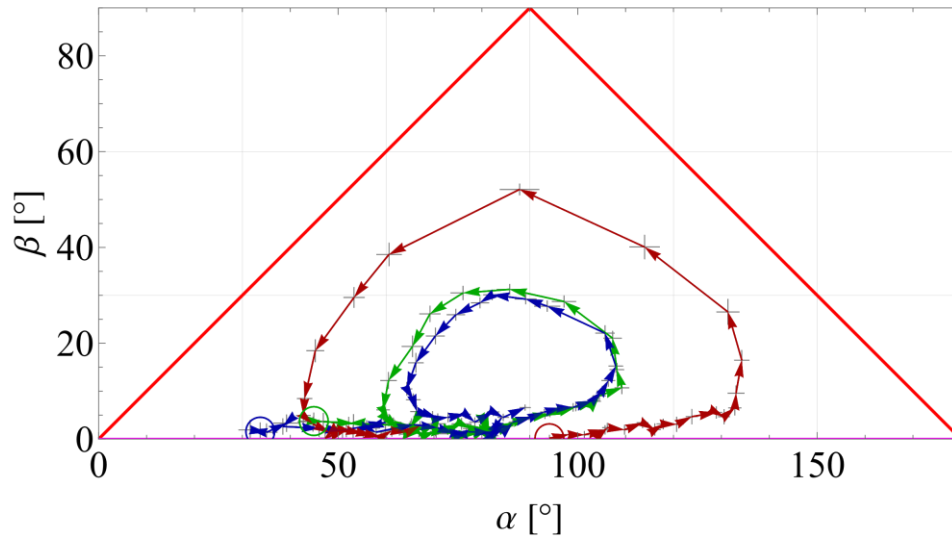


Figure 3b: Phase plane trajectories of the small curved disk ($R = 5 \text{ mm}$ and $R_c = 1.05 \text{ mm}$) shown in Fig. 2b showing concentric orbits around the dynamical centre at $\beta \approx 18^\circ$, which is twice the value for the less curved disk.

6. A technical question: the authors employ the tool of neural network to fit the disk shape. Considering that the key parameters of the disk geometry could be directly measured, what are the necessity and advantage of using the tool of auto-encoder?

Thank you for prompting this clarification. The auto-encoder method we used was developed for the purpose of reconstructing the time-dependent evolution of sedimenting elastic sheets. Fig S1 in the supplement shows an example of an uncrumpling sheet reconstructed at various timesteps. We adapted this already existing method to the case of our rigid particle in that we only needed to reconstruct the shape once and used the result in the analysis of all experiments. We have now uploaded a preprint on ArXiv, which describes the method in detail, and updated the corresponding Ref. [23] in the paper.

Reviewer #3:

We thank the reviewer for their careful reading of our manuscript and insightful comments which have helped us improve the paper. We are pleased by the reviewer's positive assessment of our work and provide an itemised response to their comments and suggestions below.

1. The released way of the U-shaped disk is not very clear in the text. It is uncertain whether there will be any changes in the initial state and velocity during the release process. Besides, I noticed that U-shaped disk is made up of polyamide nylon, how stiff is it? Is it possible to deform during sedimentation.

Since our U-shaped disk moved at a speed of less than 1.2 mm/s (taking approximately 7-10 minutes to fall through the tank), it was sufficient to manually manipulate the particle using a thin rod several centimetres above region of interest where the sheet was visualised, remove the rod and wait 20-60 seconds for the sheet to enter the region of interest. We carefully tested that by that time, the residual motion of the fluid in the tank had dissipated. We use the first reconstructed orientation in the region of interest as an initial condition in our analysis.

We now provide evidence of the stiffness of our polyamide nylon disks in the supplementary material. We use Kirchhoff-Love plate theory to estimate by how much the viscous loading changes the curvature of the U-shaped sheet. We find that the viscous forces change the curvature of the disk by $O(0.0005 \text{ m}^{-1})$ which is vanishingly small compared to the curvature of the U-shaped disk ($1/R_c = 159 \text{ m}^{-1}$).

We have also calculated the relevant fluid-structure interaction parameter for this system, the elasto-gravitational number describing the ratio of elastic to viscous forces which is around 1800, thus confirming that the U-shaped disk is rigid.

2. Will vigorously stirring the tank cause bubbles to form in the liquid, thereby affecting the movement of objects?

We took great care to avoid generating bubbles and if any were formed, we waited for them to leave the system before performing experiments. We have added “taking care to avoid the entrainment of bubbles” on p.2 (column 1, paragraph 2) of the paper.

3. The significance of the modification to the theoretical model appears to be questionable. The modification is centered around correcting the forces exerted by the boundaries on the U-shaped disk, whereas the subsequent predictions and experiments are specifically aimed at scenarios involving fluid with unbounded fluid.

The height of the experimental container allowed us only to observe a small part of the trajectory in any given experiment. Therefore, we used our theoretical framework to extend the trajectories to facilitate interpretation of the complex dynamics of our U-shaped disk. Since we are in a Stokes flow regime, boundaries introduce long-range effects which we addressed with theoretical corrections derived by Caswell [25]. As the particle moved through the tank, it approaches the side walls and the bottom, so different portions of the trajectory were influenced by the boundaries to a different extent. Our method allows to subtract this effect and infer the particle motion independently of the container. The correction turns out to have a modest impact on the mobility matrix. Without it, the different components of the fitted mobility matrix differed by between 3% and 14%. Specifically, the walls only contributed a modest 11%-14% to the total drag experienced by the particle which did not affect the behaviour qualitatively, thus justifying our choice of container size. We have now added a sentence to quantify the corrections on p.5 (column 1, first paragraph).

4. Furthermore, the referenced literature regarding the important content of data processing in the article has not been published. Are there any arXiv preprints available for reference?

We have uploaded the article on ArXiv and updated Ref. [23] in the paper.

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

I thank the authors for their answers to my questions and the modifications made to the manuscript.

From my side that paper can now be published as is.

Reviewer #2 (Remarks to the Author):

The authors have adequately addressed the questions in the previous report. Especially, the authors have clarified questions related to the reliability of the conclusion. I recommend the publication of the manuscript in the current form.

Reviewer #3 (Remarks to the Author):

The authors have addressed most of the concerns that I raised in my previous report and the response to my previous recommendations is satisfactory. I recommend acceptance in COMMSPHYS.