

Supplementary Information: Demonstration of a Josephson vortex-based memory cell with microwave energy-efficient readout

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Supplementary Note 1: Numerical Model

The numerical simulations were performed within the resistively shunted junction (RSJ) model. Within the model, the current through a lumped Josephson junction can be represented as the sum of supercurrent, capacitive and resistive currents. For our SNS junctions, the capacitance is only about 1 pF and can be neglected. The total current through the junction is therefore

$$I = I_c \sin(\varphi) + V/R_N, \quad (1)$$

$$V = \frac{\hbar}{2e} \frac{d\varphi}{dt}, \quad (2)$$

where I_c is critical current, φ is Josephson phase and R_N is normal state resistance of the junction. By normalising the current to the critical current

and the time t to the Josephson frequency $w_c = I_c R_N \frac{2e}{\hbar}$, the equation takes the following form

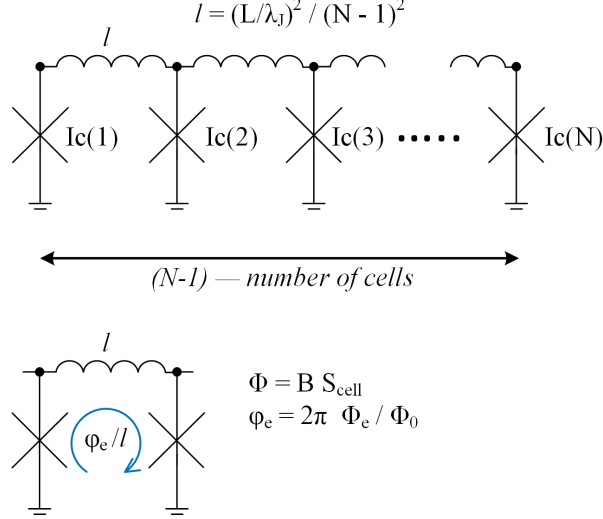
$$i = (I/I_c) = \dot{\varphi} + \sin(\varphi). \quad (3)$$

The long Josephson junction is described by the sine-gordon equation:

$$i = \lambda_J^2 \frac{d^2 \varphi(x, t)}{dx^2} + \sin(\varphi(x, t)) + \dot{\varphi}(x, t). \quad (4)$$

The spatial derivative can be represented as a difference scheme as follows

$$\lambda_J^2 \frac{d^2 \varphi(x)}{dx^2} = \frac{2\varphi(x) - \varphi(x + dx) - \varphi(x - dx)}{(dx/\lambda_J)^2}. \quad (5)$$



Supplementary Figure 1 Distributed resistively shunted junction (RSJ) model of a long SNS junction used in this work.

So the long Josephson junction can be represented as a chain of N junctions connected in parallel as shown in Suppl. Fig. 1. We have assumed that the inductances $l = (x/\lambda_J)^2/(N-1)^2$ are equal to each other and that they are normalised to the Josephson inductance, $L_J = \frac{\Phi_0}{2eI_{c1}}$, where I_{c1} is some normalization critical current of one junction. The equation (3) for k^{th} - junction can be written as

$$\dot{\varphi}_k + A_k(k) \sin(\varphi_k) + (\varphi_{k-1} - 2\varphi_k + \varphi_{k+1})/(2l) + \varphi_{k-1}^{ext}/l - \varphi_k^{ext}/l, \quad (6)$$

where φ_{k-1}^{ext} and φ_k^{ext} are normalized external fluxes through the neighboring cells. In the first and last equations, a term $0.5i_{ext}$ of the same sign is added for the current through the junction, and of opposite signs for the current along the junction. The equation (6) is a discrete representation of the sine-Gordon equation.

The exponential decay of the critical current at the edges is taken into account in our model:

$$A_k = I_c(k)/I_{c1} \sim \begin{cases} \exp(\alpha(k-6)), & k \leq 5 \\ 1, & 5 \leq k \leq N-6 \\ \exp(\alpha(N-5-k)), & k \geq 5 \end{cases}$$

Fitting procedure

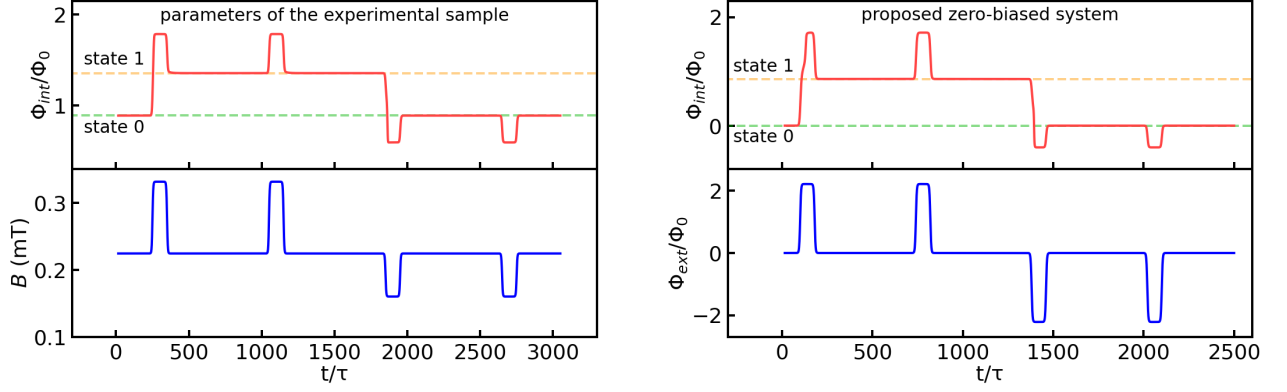
For the fitting procedure, we used the experimental values of the magnetic field at the entry and exit points of the vortex.

Within our model, the coupling inductance is represented as $l = (x/\lambda_J)^2/(N-1)^2$, where (x/λ_J) is the lateral coordinate x normalized to the Josephson penetration depth λ_J . This is *a priori* unknown quantity, and is the first parameter of the model. The critical current I_{c1} is also unknown, but it can be obtained by normalizing both the model and the experimental data to their maximum values.

The normalized flux φ_k^{ext} is evaluated from the experimental values H_{ext} taking into account the effective area of one cell, which is the second parameter of the model. Note that this parameter is not really free as it is constrained by the device geometry and λ_J .

Supplementary Note 2: Simulation of pulses sequences

We analysed the performance of our memory cell for a sequence of identical pulses. It was produced as part of our model. Suppl. Fig. 2 (left) shows the simulation result for the model parameters corresponding to the experiment. It can be seen that the first pulse moves the cell from state “0” to state “1”. The subsequent pulse leaves the cell in the same state. The reset pulse then changes the state from “1” to “0”. The second reset pulse



Supplementary Figure 2 Time dependence of the magnetic flux inside the Josephson contact for a sequence of external pulses. On the left for the experimental sample. On the right for the proposed memory cell for the zero-biased system

does not change the state of the memory cell. It is important to note that in the modelling the pulse duration was chosen much shorter than in the experiment. Pulses of such duration, for example, can be obtained using current pulses along the Josephson junction.

Suppl. Fig. 2 (right) demonstrates the similar behaviour of the memory cell proposed in the text of the paper in Figure 4. The modelling was carried out for the zero operating point (no constant magnetic flux). In this case, in state “0”, there is no magnetic flux in the Josephson transition, and in the case of state “1”, the magnetic flux corresponds to the Josephson vortex. Figure 4 shows that the required magnitude of the magnetic flux pulse amplitude is about $2\Phi_0$. The duration of the operation is determined by the relaxation time and the time of entry of the Josephson vortex. The pulse puts the system into some auxiliary state in which the vortex enters the Josephson contact. At a sufficiently sharp pulse with a large amplitude, the system relaxes to the zero state because the kinetic energy exceeds the value of the potential barrier holding the vortex. If one chooses a working point near the entry point of the Josephson vortex, a pulse of small amplitude will be required. In this case, the operation time will be determined only by the vortex entry time ~ 10 ps. In case of zero operating point, the recording time will be as follows ~ 100 ps.