nature portfolio

Peer Review File

Resource Theory of Imaginarity: Distributed Scenarios

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

This manuscript extends the recently developed "resource theory of imaginarity" to include exact analytical conditions for imaginarity distillation, state conversion and channel discrimination. I am convinced that the theorems presented in section IV and the supplementary material of the manuscript are valid and proved rigorously. Additionally, the authors implement the distillation and channel discrimination protocols using entangled photons, which shows agreement with the theoretical statements presented in section IV. A few minor concerns/questions I have are listed below:

The authors do not provide a description of the error bars for the experimental results presented in Figures 2 and 3.

What do the solid black lines in Fig. 3 represent?

The resource theory of coherence (mentioned in the introduction) for example, is another basis dependent resource theory characterizing the resourcefulness of off-diagonal amplitudes of a state. Is there any comment that can be made to compare/contrast the experimental relevance of imaginarity vs. coherence or other basis-dependent resource theories?

I believe that this work is novel, advances the subject of quantum resource theories and is of broader interest to the quantum information community. After addressing the aforementioned questions and concerns, I recommend this manuscript for publication in Communications Physics.

Reviewer #2 (Remarks to the Author):

In the manuscript, the authors study the resource theory of imaginarity in distributed scenarios. Specifically, a set of operations, Local Quantum-Real Operations and Classical Communication (LQRCC), is introduced to investigate the role of imaginarity in a quantum state to perform distillation and ancilla-free channel discrimination tasks. Furthermore, the authors report the experimental demonstration of their formalism using a quantum optical setup.

Overall, I find this work to be a valuable contribution to the field of quantum resource theory. Formulation of the authors' main idea is clearly described in the manuscript with a strong mathematical background. However, I have a few concerns regarding the connection between the theory and the experiment, as well as some aspects of the experimental procedure.

Firstly, it is important to establish a stronger connection between the theory and the experiment. The quantum states [Eqs. (31) and (32)] adopted in the experiment appear to be different from the example states studied in the theory part, and the manuscript lacks a clear description of the experimental procedure for these states. I recommend the authors explain why these states are chosen for the experiment and provide a more detailed experimental procedure, such as explicit forms of the optimal measurement operators adopted for imaginarity distillation and channel discrimination.

Furthermore, in Figure 2, several data points deviate from the theoretical expectation beyond the error bars. Identifying and discussing the major source of error responsible for these deviations will help strengthen the experimental results and interpretations. For instance, it should be clarified whether the dominant error arises from the state preparation, tomography, discrimination process, or any other potential sources.

Once these issues are fully addressed, I believe the manuscript will be suitable for publication in Communications Physics.

Reviewer #3 (Remarks to the Author):

The manuscript by Wu et al. examines imaginarity as a resource in quantum information theory. This uses a recently developed theoretical framework in which the imaginary elements of states and operations (in a given basis) are quantified as resources. The free (resourceless) states are those with only real terms, and the free operations are those with a set of real Kraus operators (equivalently, having a dilation involving all real elements). This is in part motivated by earlier and more recent work showing that “real” quantum theory differs from the full complex quantum theory in a variety of quantum information processing tasks. The main idea of this manuscript is to describe tasks in which the free operations are further constrained to local operations and classical communication between parties (giving the LRCC and LQRCC classes). The authors study a bipartite assisted distillation protocol in which Alice helps Bob to distil a maximally resourceful state via LQRCC operations, as well as the discrimination of two real channels using general/real probe states and measurements, and with/without an entangled ancilla. The results of a two-photon optical experiment are shown to demonstrate these tasks.

The claims are novel and technically correct, as far as I can tell. A general criticism of this topic (not specific to this paper) would be that the resource theory does not seem to correspond to any realistic experimental restriction that one may encounter in practice. Aside from the fixed basis, even the relative phases have to be fixed, while usually these are easily changed in most systems. (In fact, I suppose that in the optics experiment, some effort is required to fix these phases.) It is fine to use resource theory to study an abstract structure of quantum theory, however, the authors often refer to practical usefulness of imaginarity, which I do not see as well justified.

The discussed protocols are interesting, and helpful examples are given to aid intuition. One notably unusual feature is the dimension-independent maximally resourceful state; I would like to have seen a discussion of why this is the correct definition, as it is at odds with many other resource theories. It also leads to quite an unusual type of distillation protocol.

I am not sure how much scientific value is added by the experiment – it looks to be a quite routine set-up, and does not provide any additional insights. There are data points in Fig. 2(b) that are off the theoretical prediction by more than the error bars, and I would expect to see a discussion of possible error sources, other than saying there is agreement "within reasonable experimental imperfections".

The writing is generally very clear and is structured well – except that maybe the "Imaginarity in channel discrimination" part should not appear as a sub-section of the assisted distillation section.

Overall, while interesting from a theoretical viewpoint and certainly publishable, I am not convinced that the paper represents a significant enough step to influence thinking in the field, and therefore does not quite meet the requirements for this journal.

Minor comments:

- It would be helpful for non-expert readers to fully define "completely non-imaginarity-creating operations" and their physical interpretation via a real dilation

- Line 131: an "i" index should be "j"

- Line 133: are the terms with j=k being double-counted? (I don't think this affects the conclusions anyhow)

- Perhaps define the Choi matrix in the main text

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We first thank the Reviewer for his/her time in reviewing our manuscript and providing constructive comments to further improve this paper.

The authors do not provide a description of the error bars for the experimental results presented in Figures 2 and 3.
Thanks for pointing this out, we have added some text to make the clarification of the errorbars.

What do the solid black lines in Fig. 3 represent?
Thanks for pointing this out, we have added some text to explain the meaning of solid lines.

The resource theory of coherence (mentioned in the introduction) for example, is another basis dependent resource theory characterizing the resourcefulness of off-diagonal amplitudes of a state. Is there any comment that can be made to compare/contrast the experimental relevance of imaginarity vs. coherence or other basis-dependent resource theories?
Thanks for these comments. Imaginarity is a resource which is different from coherence, since the structures are different. Nevertheless, there is some relation between imaginarity and coherence since there are both basis-dependent, eg., in certain scenario they can be converted from one to another. The experimental relevance between coherence and imginarity is unclear now since each resource theory assume a different scenario where different resource is neended.

I believe that this work is novel, advances the subject of quantum resource theories and is of broader interest to the quantum information community. After addressing the aforementioned questions and concerns, I recommend this manuscript for publication in Communications Physics.
Thanks for the recommendation. We believe the current revised manuscript will fulfill the high standard of the journal.

Reviewer #2 (Remarks to the Author):

In the manuscript, the authors study the resource theory of imaginarity in distributed scenarios. Specifically, a set of operations, Local Quantum-Real Operations and Classical Communication (LQRCC), is introduced to investigate the role of imaginarity in a quantum state to perform distillation and ancilla-free channel discrimination tasks. Furthermore, the authors report the experimental demonstration of their formalism using a quantum optical setup.

Overall, I find this work to be a valuable contribution to the field of quantum resource theory. Formulation of the authors' main idea is clearly described in the manuscript with a strong mathematical background.
First we do appreciate your time for reviewing our manuscript. And we are also delighted for your encouraging comments.. The response to address all the referee comments are listed below.

However, I have a few concerns regarding the connection between the theory and the experiment, as well as some aspects of the experimental procedure.

Firstly, it is important to establish a stronger connection between the theory and the experiment. The quantum states [Eqs. (31) and (32)] adopted in the experiment appear to be different from the example states studied in the theory part, and the manuscript lacks a clear description of the experimental procedure for these states. I recommend the authors explain why these states are chosen for the experiment and provide a more detailed experimental procedure, such as explicit forms of the optimal measurement operators adopted for imaginarity distillation and channel discrimination.
Thanks for pointing this out. In the revised version of the manuscript, we have detailed the experimental procedures. Regarding the comment "quantum states [Eqs. (31) and (32), now Eqs (33,34)] adopted in the experiment appear to be different from the example states studied in the theory part", we have not experimentally realized these states due their greater difficulty in preparation, while Eqs (33,34) can be straightly prepared. Also we think these states [Eqs (33,34)] are enough for us to clarify the main theoretical conclusion to the authors.

Furthermore, in Figure 2, several data points deviate from the theoretical expectation beyond the error bars. Identifying and discussing the major source of error responsible for these deviations will help strengthen the experimental results and interpretations. For instance, it should be clarified whether the dominant error arises from the state preparation, tomography, discrimination process, or any other potential sources.
Thanks for these comments, we have added some text to clarify this issue.

Once these issues are fully addressed, I believe the manuscript will be suitable for

publication in Communications Physics.
Thanks for the recommendation, we believe the current version is suitable for publishing in Communications Physics.

Reviewer #3 (Remarks to the Author):

The manuscript by Wu et al. examines imaginarity as a resource in quantum information theory. This uses a recently developed theoretical framework in which the imaginary elements of states and operations (in a given basis) are quantified as resources. The free (resourceless) states are those with only real terms, and the free operations are those with a set of real Kraus operators (equivalently, having a dilation involving all real elements). This is in part motivated by earlier and more recent work showing that “real” quantum theory differs from the full complex quantum theory in a variety of quantum information processing tasks. The main idea of this manuscript is to describe tasks in which the free operations are further constrained to local operations and classical communication between parties (giving the LRCC and LQRCC classes). The authors study a bipartite assisted distillation protocol in which Alice helps Bob to distil a maximally resourceful state via LQRCC operations, as well as the discrimination of two real channels using general/real probe states and measurements, and with/without an entangled ancilla. The results of a two-photon optical experiment are shown to demonstrate these tasks.

Thank you for your time in reviewing our paper and making a proper summary of our research work.

The claims are novel and technically correct, as far as I can tell. A general criticism of this topic (not specific to this paper) would be that the resource theory does not seem to correspond to any realistic experimental restriction that one may encounter in practice. Aside from the fixed basis, even the relative phases have to be fixed, while usually these are easily changed in most systems. (In fact, I suppose that in the optics experiment, some effort is required to fix these phases.) It is fine to use resource theory to study an abstract structure of quantum theory, however, the authors often refer to practical usefulness of imaginarity, which I do not see as well justified.
Thanks for the comments. Indeed, the comment “resource theory does not seem to correspond to any realistic experimental restriction that one may encounter in practice” is correct, and we agree that resource theory mainly solves the problem of quantifying an abstract quantum phenomena. However, it has been shown that resource theory does have practical relevance [Rev. Mod. Phys. 91, 025001 (2019)] that resource theory can be desirable from a practical perspective as they often restrict attention to quantum operations that reflect current experimental capabilities, eg., quantum entanglement and LOCC. In our framework of imaginarity theory, we main solve the operational meaning of imginarity in quantum states or channels. In optics experiments, we regard imaginarity as a precious resource , it will cost more resource to control optical phase than manipulating real quantum states. Thus we suppose imaginarity as an proper “resource” in quantum

mechanics. In the revised paper, we added a statement regarding this point.

The discussed protocols are interesting, and helpful examples are given to aid intuition. One notably unusual feature is the dimension-independent maximally resourceful state; I would like to have seen a discussion of why this is the correct definition, as it is at odds with many other resource theories. It also leads to quite an unusual type of distillation protocol.

The reason behind this choice of the maximally resourceful state follows from the fact that one "imbit" is necessary and sufficient to implement any general quantum operation (independent of the dimension) by performing real operations. This fact was also mentioned in our article (see Eq. (9) of the main text).

I am not sure how much scientific value is added by the experiment – it looks to be a quite routine set-up, and does not provide any additional insights.

The experiments are indeed a proof of principle test of the theory presented in the main text, and it is based on standard description of quantum theory and routine optical setup. We need to stress that the main innovation lies in the theoretical part, where we study the resource of imaginarity in distributed scenario.

There are data points in Fig. 2(b) that are off the theoretical prediction by more than the error bars, and I would expect to see a discussion of possible error sources, other than saying there is agreement "within reasonable experimental imperfections".

Thanks for pointing this out, we have added some to text to make the clarification on the possible experimental imperfections.

The writing is generally very clear and is structured well – except that maybe the "Imaginarity in channel discrimination" part should not appear as a sub-section of the assisted distillation section.

Thanks for pointing this out, we have divided the text into two sections.

Overall, while interesting from a theoretical viewpoint and certainly publishable, I am not convinced that the paper represents a significant enough step to influence thinking in the field, and therefore does not quite meet the requirements for this journal.

We do appreciate your time in reviewing our manuscript and bring up several useful comments. Following your constructive comments, we have revised the paper and solve the comments you have raised to match the high standard for publishing in *Communication Physics*.

Minor comments:
- It would be helpful for non-expert readers to fully define "completely non-imaginarity-creating operations" and their physical interpretation via a real dilation
- Line 131: an "i" index should be "j"
- Line 133: are the terms with j=k being double-counted? (I don't think this affects the

conclusions anyhow)

- Perhaps define the Choi matrix in the main text

We have added these definitions to the updated version of our manuscript, and corrected all these issues.

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

The authors have adequately addressed each of my comments and I am satisfied with this version of the manuscript. I recommend the publication of this article in Communications Physics.

Reviewer #2 (Remarks to the Author):

The authors have carefully addressed all the comments from the previous report. In particular, the experimental details added in the current version have improved the readability of the manuscript. I recommend the revised manuscript for publication in Communications Physics.

Reviewer #3 (Remarks to the Author):

The authors have improved the manuscript according to the questions and criticisms of all reviewers. Details about connection to the experiment are now much clearer. The only point which I do not see as convincingly argued is the practical relevance of this resource theory setting. My question regarded this particular theory involving Real Operations – I agree that resource theories in general can have practical relevance, but this depends on the specific chosen set of operations. I still do not see a reason why controlling a complex phase is harder than a real phase (and would suggest the authors go over the text added to the start of section VI, as it is hard to comprehend). I agree that there is theoretical interest to this topic, but see the motivation in terms of practical resource value of imaginarity as weak. My overall evaluation is that this paper is an advance within the specific topic of imaginarity but may not have large influence outside this sub-field.

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Thank you for your time in reviewing our paper and providing several useful comments. Regarding the comment "I still do not see a reason why controlling a complex phase is harder than a real phase", we think in some certain scenarios (polarization or path-encoded linear optical system), optical phases are harder to manipulating, though generally it appears naturally in many optical process. It is also noted that complex measurement can help us get larger non-locality [Nature 600, 625–629 (2021)] or higher success probability in local state discrimination[Phys. Rev. Lett. 126, 090401(2021)].

Besides, we have added an explanation in the text (highlighted), “It is noted that complex numbers in describing wave mechanics are universal, and optical phase is naturally generated in many processes. In our linear-optical settings, where imaginarity is regarded as a precious resource, manipulating (rather than) complex phase between different optical modes (path or polarization) always need more optical elements. For example, generating an arbitrary real polarization states (starting from a free state, \ket{H} or \ket{V}) requires one half-wave plate while for states of complex-valued amplitudes, we need additional quarter-wave plates to control the exact value of phase.”

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Thank you for your time in reviewing our paper and providing several useful comments. In our previous manuscript we agree that we did not provide a more detailed discussion on the experimental usefulness of imaginarity. Now we will extend the discussion over the experimental relevance of complex numbers in quantum physics in the framework of resource theory in two aspects:.

1. First we would like to remark that complex numbers are indispensable in quantum physics, and have operational meanings. More specifically, complex-valued measurement can help us experimentally get larger non-locality [Nature 600, 625–629 (2021)] or higher success probability in local state discrimination[Phys. Rev. Lett. 126, 090401(2021)].
2. Regarding the comment "I still do not see a reason why controlling a complex phase is harder than a real phase", we think in some certain scenarios (polarization or path-encoded linear optical system), optical phases are harder to manipulating, though generally it appears naturally in many optical processes. Besides, we have added an explanation in the text (highlighted), "It is noted that complex numbers in describing wave mechanics are universal, and optical phase is naturally generated in many processes. In our linear-optical settings, where imaginarity is regarded as precious resource, manipulating (rather than) complex phase between different optical modes (path or polarization) always need more optical elements. For example, generating an arbitrary real polarization states (starting from a free state, \ket{H} or \ket{V}) requires one half-wave plate while for states of complex-valued amplitudes, we need additional quarter-wave plates to control the exact value of phase."

REVIEWERS' COMMENTS:

Reviewer #3 (Remarks to the Author):

I appreciate the authors clarifying the motivation. Although I do not agree with the bigger picture and do not find the reasoning especially strong, the manuscript is suitable for publication in this journal.