nature portfolio

Peer Review File

Confirming the theoretical foundation of steady-state microbunching

Reviewers' comments:

Reviewer #1 (Remarks to the Author):

This manuscript presents a study of the steady-state microbunching (SSMB) mechanism in an electron storage ring, with the goal of comparing theoretical predictions of the impact of laser, beam and accelerator lattice parameters with experimental measurements of the coherent radiation produced by this technique. The article builds on previous results which showed the viability of preserving microbunching in a beam that has been modulated by a laser pulse over one turn, providing further evidence that could lead towards a full SSMB ring, which is currently under development.

The results appear to me to be novel, scientifically sound, and of potential interest to the light source community at least. The developments in this proof-of-principle experiment are impressive, and the comparison between theoretical predictions and measurements are promising. The significance of these comparisons for the future of SSMB, however, could perhaps be highlighted in more detail. How exactly will these results inform the facility being designed, or future experiments?

The point is made at the end of Sec. 4.3 that SSMB does not require the suppression of transverse-longitudinal coupling for the scheme to work, and only one potential application is cited as utilising the coupling effects presented in this paper for coherent power enhancement. How important, then, are these results? Are they crucial for the future of SSMB? This does not appear to the case. I would have expected the impact of the laser modulation amplitude to be a more significant parameter to check against theoretical predictions, but the authors seem to place more emphasis on the coupling terms.

Given the broad audience of this journal, some of the more technical details could be explained more clearly to readers without expertise in accelerator physics. Many terms are used which would make sense only to experts, and some of these ("optics", "resonance", etc.) could even cause confusion for people unfamiliar with how these words are used in accelerators. A number (but not all) of these jargon terms are given in my notes below, and I would recommend that the authors read carefully through the manuscript to ensure that they keep the broad readership in mind. (There are also various typographical errors and mistakes in the text which should be corrected.)

In summary, this paper represents a step forward in this interesting concept which could have important implications for some communities who make use of light sources. As such, the article is potentially worthy of publication in Communications Physics. However, the case for the significance of these results for the future of the SSMB project, and how they will impact the final generation of high average power coherent radiation, is not sufficiently strong, as mentioned above. If the authors could argue their case more strongly, and modify the technical accelerator jargon to appeal to a wider audience, then the paper could be accepted for publication.

Other notes
- L36: the introductory sentence is a bit hard to decipher, and no reference is given for potential spectroscopy applications for this wavelength range and power.
- L43: what average power of radiation is required for, e.g., the lithography applications mentioned?
- L45: perhaps mention a "typical" repetition rate of an electron storage ring such as the MLS.
- L47: the phrase "stored in a steady state" may not be clear to a non-expert (unless if they skip ahead to Section 2.1). Perhaps "[…] creating microbunches whose structure persists across many turns […]", or something to that effect. Additionally, is "longitudinal focusing" the right term to describe the laser modulation effect?
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- L60: what is meant by “unconfined microbunches”? It is mentioned further on but this term merits explanation here.
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- L75: what are the shortcomings of existing accelerator hardware? The high repetition rate laser? Is this referring only to what can be installed at the MLS, or does it imply the design and construction of entirely novel hardware?
- L86-87: “quasi-isochronous” and “momentum compaction” are technical terms that probably require some brief explanation. The authors could point to Section 3.1 for an explanation of the latter term.
- L90: “minimize the impact of” rather than “avoid” collective instabilities.
- L94: perhaps mention the value of the final radiated wavelength(s).
- L108: it should be mentioned that Eqs. (1) and (2) represent single-particle dynamics.
- L120: the bunching factor described here refers only to the **longitudinal** structure of the electron beam distribution.
- L131: it is not explained clearly what is meant by “coupling”, or what are the sources of this effect – this may not be clear to those without expertise in accelerator physics.
- Eq. (5): what does capital B signify?
- L140: it should perhaps be clarified that the alpha here is different to the momentum compaction mentioned above. A general description of, or reference to, the Courant-Snyder parameters given here could be useful for a more general audience. Perhaps even a section could be dedicated to these technical accelerator-based terms in the Methods.
- L147: what is meant by an “invariable pulse shape”? That it stays constant over each turn?
- Fig. 2A: while the background subtraction is discussed in the Methods section, it should also be mentioned in the main text or figure caption. A further question: has the background been subtracted from all of the subsequent figures that show the signal from the undulators? This point is not made clearly, and is important. A side point: the Methods section is referenced only once in the text, despite it being relevant to the main text in numerous places.
- Fig. 2: Can the results shown in Fig. 2 (especially 2b) be attributed entirely to the pulse shape of the laser? Presumably there is some minor variation in bunch charge and other beam properties along the train. If the laser was a perfect flat-top in terms of intensity should all pulses within a train have the same power? (A minor additional point: why does the x-axis in Fig. 2A start at 35 ns?)
- L177: Is the value for \beta_x the average throughout the ring? Should the units for \epsilon_x be nm-rad?
- Eqs. (10) and (11): the description of these parameters is not clear – it is stated that these are the expected values for the std. dev. on the radiation power, but then the values of sigma_Dx, etc. are given. More explanation would be helpful.
- Fig. 3a: The peak of the radiation power is skewed away from zero dispersion (assuming that zero on the horizontal axis means zero dispersion in the undulator). Can the change in momentum compaction induced by a non-zero dispersion be included in the calculation? Also, the units on the horizontal axis are a bit strange – why not just use the values of dispersion and dispersion angle and include horizontal error bars?
- L185: how exactly were the dispersion and angle measured? Point to the Methods section.
- L188: the values discussed here represent only the dispersion and angle at the undulator entrance, correct? How does the dispersion change through the undulator? Also, “acceptance widths” is a technical term.
- Fig. 4: Is the reduction in radiation power shown consistent with theoretical predictions? This could provide important support for the future SSMB proposal.
- L200: have the sources of this coupling been identified? Can the procedure described here be used to quantify the coupling, or the values of the vertical dispersion and angle?
- L205: the term “resonance” is jargon that will not mean much to a non-expert. How is the resonance changed?
- L209: does the enhancement of the signal at the 4th and 5th revolutions suggest that the x-y coupling can play a positive role in preserving microbunches?
- L210: how was it determined that the reduction of coupling was the sole reason for the change in the pattern of coherent emission? Again, a quantification of the effect could be helpful here.

- L220: the mechanism behind this technique, which beam properties they alter to increase the emittance, and how this relates to vertical-longitudinal coupling, should be explained. And if the effect on the emittance could be quantified, that would be even better. Presumably a fit could also be done to the analytical expression Eq. (7), even if the exact values of the emittance are not known.
- L225: Is there an explanation for the preservation of microbunching in the fourth and fifth revolutions as a function of vertical emittance increase?
- L246: are there any "smoking guns" to explain the large variability in coherent power?

Reviewer #2 (Remarks to the Author):

This paper experimentally verifies the transverse-longitudinal coupling motion effects the microbunching in a storage ring in a clear way, which is the basic beam dynamics for storage ring based microbunching scheme. It is worth publish on Communications Physics.
The radiation emitted by SSMB is not entirely embraced by the semiconductor industry; characteristics such as coherence and highly collimated radiation are not preferred. So both in the Abstract and in Introduction, I recommend to claim the EUV power can reaches the level required for industrial applications.

Equations (10-11) are challenging to comprehend; a clearer representation is provided in Fig.3. The figure illustrates the dependence of radiation power on Dx and D'x. To enhance clarity, consider adding a theoretical line in Fig.3 for comparison instead of Eq(10-11) . In this context, the authors concentrate on the accuracy of D, D'x, assuming emittance x is precise. However, if emittance x is not 30nm, it can also introduce discrepancies between theoretical and measured data as per Eq. (7). Therefore, I recommend reorganizing this section.

Reviewer #3 (Remarks to the Author):

The manuscript presents experimental findings from systematic studies in the ongoing proof-of-principle experiment of SSMB. The authors report on the confirmation of the theoretical expectations in multiple aspects, particularly focusing on the influence of transverse-longitudinal coupling dynamics on the microbunching process. The experimental findings are well-documented and supported by systematic studies. However, I have identified several areas that require further attention and clarification:

1. The manuscript focuses on the experimental investigation of transverse-longitudinal coupling, but the description of the experimental setup is insufficient to provide a clear understanding of the experimental process. I recommend including a schematic diagram of the magnetic lattice layout in the Methods section to illustrate how dispersion, dispersion angle, and the coupling between horizontal and vertical were adjusted during the experiment. This addition will enhance the clarity of the experimental setup for readers.

2. In Figure 3, the experimental results appear to be significantly larger than the theoretical predictions, particularly under the dispersion angle condition. The manuscript should provide a quantitative analysis of the potential effects of bunching tilt and the finite opening angle of the coherent radiation. Please explain which effect is the primary factor contributing to the observed discrepancy and clarify why the error impact under the dispersion angle condition is significantly larger than under the dispersion condition.

3. Figure 5 shows a trend in the radiation power with the white noise excitation amplitude, where the radiation power initially increases and then decreases, especially for turn 1-3. Please provide an explanation for this phenomenon.

Response to Referees' comments

Re: **Next steps towards steady-state microbunching: confirming the theoretical foundation**
by Arnold Kruschinski et al.

Dear referees,

Thank you very much for your time and efforts expended to review our manuscript. Your comments are very constructive and valuable to us. We have carefully read and studied the review report, and revised the manuscript accordingly, which we hope can meet with your approval. Before the point-to-point response, we want to show our sincere appreciation to you all for your appreciation of the potential significance of our work.

Major revisions in the manuscript are:

- Additional explanation and justification of the importance of the presented results in Introduction, Conclusion and final paragraph of section "Vertical-Longitudinal Coupling"
- Clarification on the experimental procedure in the section "Influence of modulation amplitude", with an additional figure (thereby splitting Fig. 2 into Fig. 2 and Fig. 3)
- Clarifications of the notations and procedure in the section "Horizontal-longitudinal coupling"
- Additional analysis and presentation of results in Fig. 5 (now Fig. 6) "Influence of vertical emittance" in section "Vertical-longitudinal coupling"
- Additional paragraph in the Methods section about the storage ring lattice
- Additional paragraph in the Methods section about Courant-Snyder parameters

Referee #1

General comment:

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We thank Referee #1 very much for this excellent summary of our work and the very careful, helpful review report. The support of the significance of our work is also much appreciated. The critical comments raised are also very constructive. We really appreciate the time and efforts devoted by Referee #1 in this process.

The major concern of Referee #1 is on the significance of our work for the future of SSMB. Indeed the manuscript should be strengthened in this respect. We have revised the manuscript correspondingly, adding a short section to better explain the implication of this work. Please allow us to explain a bit more here why we believe the result on coupling effect is of particular importance.

In the past several years, the SSMB concept is under active study, especially by the SSMB collaboration initiated by Tsinghua University. The main motivation of the group is to develop an SSMB ring that can generate high-power short-wavelength radiation, for example in the EUV regime, which could be important for both science and industry applications. The proof-of-principle experiment ongoing at the MLS is one of the key tasks of this collaboration. Another key task is to design such an EUV-SSMB storage ring.

To generate coherent EUV radiation, we need to compress the electron bunch length to nm level, which is a challenging task. To realize such a goal, we have developed and investigated three specific schemes in depth, namely, the longitudinal weak focusing, longitudinal strong focusing, and generalized longitudinal strong focusing. In a longitudinal weak focusing SSMB ring, we use a laser modulator to play the role of a conventional RF cavity in the ring to longitudinal focus the electron beam to form and constrain microbunches in the optical microbuckets in a steady state, just like conventional bunches in RF buckets. However, our study shows that this scheme can only generate a bunch whose equilibrium length is a couple of 10 nm, otherwise the required phase slippage factor of the ring will be too small from engineering viewpoint. To compress the bunch length further, we have invoked the longitudinal strong focusing scheme, not unlike the transverse strong focusing scheme, to compress the longitudinal beta function at the radiator by introducing a strong energy chirp strength before entering the radiator, similar to strong quadrupole focusing in the final focusing of a collider. This scheme can realize a bunch length of nm level, thus to generate coherent EUV radiation. But a strong energy chirp means a large modulation laser power is needed. This makes the optical cavity of such an SSMB ring can only work in a pulsed mode with low duty cycle, thus also limits the duty cycle of mcirobunches in the ring, and thus limits the average output radiation power.

This brings us to the generalized longitudinal strong focusing (GLSF) scheme [*Phys. Rev. Accel. Beams* 26, 110701 (2023)]. The basic idea is to take advantage of the fact that the vertical emittance in a planar ring is rather small, therefore, we can apply *y-z* coupling dynamics, similar to that in transverse-longitudinal emittance exchange, to compress the bunch length to nm level, and at the same time lower the required energy chirp strength or modulation laser power significantly compared to the longitudinal strong focusing scheme. Longitudinal strong focusing scheme plays with 2D phase space, while generalized longitudinal strong focusing play with 4D or 6D phase space. This GLSF scheme is at present the most promising one to generate high-power short-wavelength coherent radiation based on an SSMB ring. The SSMB collaboration is designing such a GLSF ring for the envisioned final SSMB. In this scheme, the precision transverse-longitudinal coupling dynamics is the key. This is the reason why we believe this experimental work on the coupling effect is of great importance for the future SSMB proposal.

But one could still question that we present a minimization of the coupling effect, while what we want in the final stage is an application or enhancement of coupling in a special way. Yes, this is true. But here the key point is that we are demonstrating that a real machine has this *ability* to manipulate the transverse-longitudinal coupling phase space dynamics with an ultra-high *precision* as that required by SSMB. Once we understand the physics and method of handling precision coupling dynamics in a real machine, this will give important support for our future proposal and design of an SSMB ring which is based on taking advantage of the coupling dynamics.

We believe this coupling effect result is also important outside of the SSMB project from a pure beam physics viewpoint. With the continuing development of accelerator photon science, the required precision of electron beam phase space manipulation becomes more and more demanding. This experimental result confirms that our theoretical understanding of beam dynamics in storage rings is still correct with the precision increased by three orders of magnitude. This provides confidence when we propose more bold ideas, which invoke precise electron beam phase space manipulation, to foster future accelerator light source development in general.

We have revised the other parts of the paper as suggested, trying to use less technical jargons or give clear definition, and to minimize typographical errors.

We hope the revised manuscript can meet with your approval.

Detailed comments:

- L36: the introductory sentence is a bit hard to decipher, and no reference is given for potential spectroscopy applications for this wavelength range and power.

Thank you for pointing out this issue. The introductory sentence has been revised and additional references have been added.

- L43: what average power of radiation is required for, e.g., the lithography applications mentioned?

The average power required by EUV lithography for high volume manufacturing and long-term development is kW level. This has been added.

- L45: perhaps mention a “typical” repetition rate of an electron storage ring such as the MLS.

Thanks for the suggestion. We now mention typical repetition rates at MHz level. The repetition rate of the MLS is 6.25 MHz.

- L47: the phrase "stored in a steady state" may not be clear to a non-expert (unless if they skip ahead to Section 2.1). Perhaps "[…] creating microbunches whose structure persists across many turns […]", or something to that effect. Additionally, is "longitudinal focusing" the right term to describe the laser modulation effect?

Thanks for the suggestion. We have revised the manuscript to read "...persistent microbunches…", meaning that the microbunches persist in the same as regular bunches in a conventional storage ring. In the final SSMB, as explained above, the electron beam is in a microbunched state and stays inside the optical microbuckets, just like the conventional bunches in RF buckets. The laser modulator in an SSMB ring plays the role of longitudinal focusing of the electrons, just like that provided by RF focusing. Or if we use Courant-Snyder analysis, the laser or RF focusing to longitudinal dimension in linear approximation is like a quadrupole focusing in the transverse dimension. For more discussions, refer to *Phys. Rev. Accel. Beams* 24, 094001 (2021). So we believe "longitudinal focusing" should be an adequate term.

- L49: with regard to the efficiency of this scheme, could the authors comment on how SSMB compares to energy recovery linac-based EUV generation?

Thanks for the question. It is not easy to give a comprehensive comparison since it is not easy to know the exact overall power consumption of an SSMB-EUV source and an ERL-FEL-EUV source. Here we can only give a rough estimation based on our understanding. An SSMB-EUV light source as whole may consume a few hundred kW of electricity to generate an EUV power 1 kW. This power consumption is similar to that of a small synchrotron radiation source like the MLS. For an ERL-FEL-EUV source, the number we find from literature is 7 MW electricity / 10 kW EUV (Nakamura 7th EUV-FEL Workshop, slide 19), which means 700 kW electricity / 1 kW EUV. Therefore, concerning power conversion efficiency, an SSMB-EUV source should be at about the same level as an ERL-FEL-EUV source.

- L53: "narrow bandwidth": how narrow can this be for an SSMB ring at EUV wavelengths?

The requirement from the EUV lithography industry is the bandwidth of EUV radiation should be less than 2%, which is due to the reflectivity curve of the EUV optical mirrors. For SSMB, if we gather the overall radiation (both on-axis and off-axis), the radiation bandwidth, depending on the transverse beam size, can reach 1% level. If we only collect the on-axis radiation, since the microbunches in different microbuckets are phase-locked defined by the modulation laser, the on-axis radiation is like a CW or quasi-CW laser, and the bandwidth can be even narrower than 1%. For more discussion on the SSMB radiation characteristics, especially the impact of transverse distribution of electron beam, refer to *J. Synchrotron Rad.* 30, 35–50

(2023) or Chapter 4 of Springer Thesis “Theoretical and Experimental Studies on Steady-State Microbunching”.

- Introduction: could the authors please provide an outline of the new results presented in the paper at the end of the introduction?

An outline of the results has been added as suggested.

- L60: what is meant by “unconfined microbunches”? It is mentioned further on but this term merits explanation here.

Thanks for the question. The “unconfined microbunches” here means the microbunches are not constrained in the optical microbuckets, since at present the laser is a single shot one. In the future experiment, we will replace the present laser with a high-repetition phase-locked one, such that it can interact with the electrons turn-by-turn and trap the microbunches in the microbuckets, like the conventional bunch trapped in RF buckets. We have added a clarification in the manuscript.

- L64: “optics” is a term that, in this context, only has significance for accelerator experts.

Thanks for pointing out this issue. We have changed the “optics” to “magnet settings”.

- L75: what are the shortcomings of existing accelerator hardware? The high repetition rate laser? Is this referring only to what can be installed at the MLS, or does it imply the design and construction of entirely novel hardware?

Thanks for raising this important question. High-repetition rate laser is one issue, but not the most fundamental one, since the phase-II experiment laser currently in preparation can already fire indefinitely in principle. The most fundamental reason from physics viewpoint is that the bending magnets of MLS have a too large bending angle (pi/4 for each dipole) and also the dispersion magnitude is large in these dipoles. Therefore, quantum excitation has a too large contribution to the longitudinal emittance. This quantum excitation and large local phase slippage factor of the ring will give a lower limit of bunch length even if we push the global momentum compaction of the ring to zero. This limit at the MLS is about 13.5 um at 250 MeV in the standard low-alpha mode, which is too large for the microbunches to be confined in the optical microbuckets, whose wavelength, i.e., the laser modulation laser wavelength, is only about 1 um. For the final SSMB, we need a dedicated new ring, optimized to minimize the longitudinal emittance and bunch length limit. Basically, we need a ring with small bending-angle dipoles, and to tailor the dispersion function in the dipole carefully. This requirement is similar to that in realizing low transverse emittance, but not exactly the same. For more details, refer to *Phys. Rev. Accel. Beams* 24, 094001 (2021).

- L86-87: “quasi-isochronous” and “momentum compaction” are technical terms that probably require some brief explanation. The authors could point to Section 3.1 for an explanation of the latter term.

Thanks for the suggestion. We have revised the manuscript accordingly.

- L90: “minimize the impact of” rather than “avoid” collective instabilities.

Thanks. We have revised the manuscript as suggested.

- L94: perhaps mention the value of the final radiated wavelength(s).

We have added the concrete wavelengths of 1064 nm and 532 nm.

- L108: it should be mentioned that Eqs. (1) and (2) represent single-particle dynamics.

Thanks for the suggestion. We have revised the manuscript as suggested.

- L120: the bunching factor described here refers only to the **longitudinal** structure of the electron beam distribution.

The word “longitudinal” has been added. Indeed, this bunching factor here refers only to the longitudinal structure. Actually, the electron beam transverse distribution will also affect the coherent radiation. For more discussion on this, refer to *J. Synchrotron Rad.* (2023). 30, 35-50.

- L131: it is not explained clearly what is meant by “coupling”, or what are the sources of this effect – this may not be clear to those without expertise in accelerator physics.

The coupling here means the longitudinal and transverse dynamics can affect each other. From a mathematical viewpoint, it means the one-turn map of the ring or transfer matrix of the lattice is not a block diagonal matrix and not all the R5[1,2,3,4] terms are zero. The physical reason of the coupling effect analyzed in the manuscript is: When particles with different transverse locations (betatron amplitudes) traverse curved trajectories in bending magnets, they will go through different path lengths. We have added an explanatory sentence to the manuscript.

- Eq. (5): what does capital B signify?

The intention of the subscript was to mean “betatron” as it is the bunch lengthening from coupled betatron motion. We have now chosen to remove the subscript to avoid confusion, as this is the only “Delta z” in the manuscript.

- L140: it should perhaps be clarified that the alpha here is different to the momentum compaction mentioned above. A general description of, or reference to, the Courant-Snyder parameters given here could be useful for a more general audience. Perhaps even a section could be dedicated to these technical accelerator-based terms in the Methods.

Thanks for the helpful suggestion. We have actually now changed over from using momentum compaction (alpha) to the more appropriate phase slippage (eta), so it is now a different symbol. We have added a reference for the Courant-Snyder parameters and a short paragraph in the methods section.

- L147: what is meant by an “invariable pulse shape”? That it stays constant over each turn?

It means the laser pulse length and envelope are assumed unchanged between individual laser shots. From laser pulse diagnostics we know this to be true to a reasonable standard (1% level). Then the electric field, thus the induced energy modulation depth on the electron beam can be assumed to be proportional to the square root of the total laser pulse energy.

- Fig. 2A: while the background subtraction is discussed in the Methods section, it should also be mentioned in the main text or figure caption. A further question: has the background been subtracted from all of the subsequent figures that show the signal from the undulators? This point is not made clearly, and is important. A side point: the Methods section is referenced only once in the text, despite it being relevant to the main text in numerous places.

Thanks for the question. Yes, the background subtraction is always employed as the first step of data analysis. We have added a comment in the main text and reference the Methods section more frequently.

- Fig. 2: Can the results shown in Fig. 2 (especially 2b) be attributed entirely to the pulse shape of the laser? Presumably there is some minor variation in bunch charge and other beam properties along the train. If the laser was a perfect flat-top in terms of intensity should all pulses within a train have the same power? (A minor additional point: why does the x-axis in Fig. 2A start at 35 ns?)

The result of Fig. 2b (now Fig. 3) is the data evaluation of one fixed bunch, i.e., the central bunch denoted by “C” in Fig. 2a, obtained by changing the overall laser pulse energy. Therefore, it really reflects the impact of laser power, not variation in bunch charge and other beam properties along the train. This has now been clarified again in the manuscript. Concerning Fig. 2a, the bunch train is setup with uniform charge and

we confirm this by observing that the incoherent radiation of the bunch train (without laser modulation) is uniform. In the experiment, when the laser power is not too high, we can observe the central bunch has a maximal coherent radiation power. Only when we increase the laser power to a large enough value, such that the microbunches in the central bunch will become over-bunched after one turn can we observe the signal shown in Fig. 2a.
The x-axis in Fig. 2a shows time relative to the master trigger of the experiment, which is positioned somewhat arbitrarily. The time has been offset now to reflect approximately the time since modulation.

- L177: Is the value for \beta_x the average throughout the ring? Should the units for \epsilon_x be nm-rad?

The \beta_x is the value in the center of the undulator, since the laser-electron interaction is in the undulator. We have clarified this in the manuscript. Yes, the unit for \epsilon_x is nm-rad, this is now changed.

- Eqs. (10) and (11): the description of these parameters is not clear – it is stated that these are the expected values for the std. dev. on the radiation power, but then the values of sigma_Dx, etc. are given. More explanation would be helpful.

Thank you for raising this issue. It should be more carefully worded. They are standard deviations of the coherent radiation power with respect to the variables Dx and Dx’. We have added more clarification and an additional equation explaining how one arrives at the values for sigma_Dx.

- Fig. 3a: The peak of the radiation power is skewed away from zero dispersion (assuming that zero on the horizontal axis means zero dispersion in the undulator). Can the change in momentum compaction induced by a non-zero dispersion be included in the calculation? Also, the units on the horizontal axis are a bit strange – why not just use the values of dispersion and dispersion angle and include horizontal error bars?

Yes, it can be included in the model and actually we had tried this in our analysis. As can be seen from the example calculation below, considering the impact of dispersion and dispersion angle knob on the phase slippage, indeed the peaks of radiation power of the first and second revolution appear at different values of Dx and Dx’. This is because the optimal phase slippage for microbunching in different revolutions are different.

In the fine tuning of D_x and D'_x, we assume there is a linear change of phase slippage η with respect to the change of D_x and D'_x, i.e., $\Delta\eta = c_1 \Delta D_x$, $\Delta\eta = c_2 \Delta D'_x$. Considering this, and ignoring the contribution of vertical dimension temporarily, Eq. (7) in the manuscript should be revised to be

$$b_{n,m} = J_n\left(nk_L m(\eta_0 + c_1 D_x + c_2 D'_x) C_0 A\right) \exp\left\{-\frac{(nk_L)^2}{2}\left[(m(\eta_0 + c_1 D_x + c_2 D'_x) C_0 \sigma_\delta)^2 + 4\epsilon_x\left(\frac{D_x^2}{\beta_x} + \beta_x D_x'^2\right)\sin^2(m\pi\nu_x)\right]\right\}. \quad (1)$$

Now, let us put in some numbers to get a feeling. $\epsilon_x = 30$ nm, $\beta_x = 11.5$ m, $\nu_x = 3.18$, $\sigma_\delta = 1.8 \times 10^{-4}$, $\eta_0 = 1 \times 10^{-5}$, $C_0 = 48$, $A = 3\sigma_\delta$, $c_1 = -2 \times 10^{-3}$, $c_2 = -2 \times 10^{-2}$ m. Since the first-turn and second-turn microbunching favors different optimal phase slippage η values, therefore the peaks are at different locations.

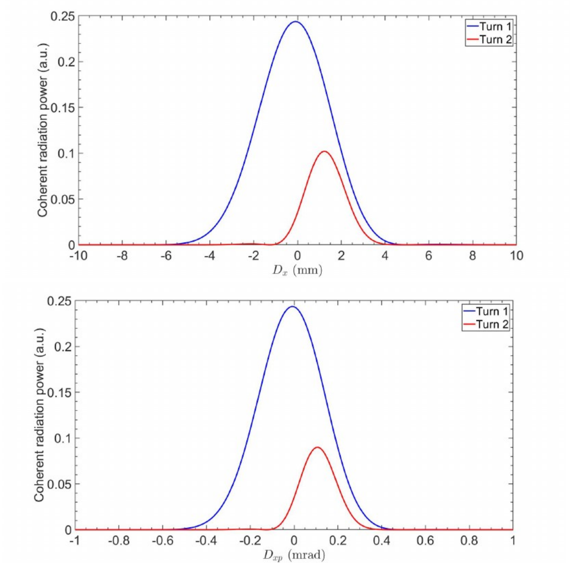


Figure 1: One example theoretical plot of dispersion and dispersion angle v.s. coherent radiation power, considering the change of η during the scan, based on Eq. (1).

Thanks for the suggestion, we have now used the values of dispersion and dispersion angle as the horizontal axis. The reasoning for using the original units was that the error from the calibration is systematic, impacting all data points equally, whereas error bars suggest individual statistical errors. For this reason, we still do not use error bars but add an explanation that there is an error on the scaling of the horizontal axis.

- L185: how exactly were the dispersion and angle measured? Point to the Methods section.

The section has been reorganized (see above comment on Fig. 3a), explaining a bit more about the measurement of the dispersion, also pointing to the Methods section.

- L188: the values discussed here represent only the dispersion and angle at the undulator entrance, correct? How does the dispersion change through the undulator? Also, “acceptance widths” is a technical term.

Thanks for this good question. The dispersion and dispersion angle here are that at the undulator center. Yes, the dispersion will wiggle in the undulator as shown in the figure below. The wiggling amplitude is actually quite large since the undulator here

has a long period length. During our evaluation of experiment results, we had also considered the possible impact of this. However, we realized that while the dispersion is wiggling in the undulator, its value at the entrance and exit should be the same, if the undulator as a whole can be viewed as an achromat. This wiggling is actually mainly due to the oscillating of the reference orbit direction in the undulator when the electron goes through a sinusoidal trajectory. If we keep observing the beam in the direction of the undulator axis, the microbunch length (thus bunching factor) actually does not change much inside the undulator. So we have only used the dispersion and dispersion angle, more accurately the chromatic H_x,y, at the undulator center to quantify the impact of transverse-longitudinal coupling on microbunching formation and coherent radiation generation. But we should admit whether or not such wiggling can really affect the coherent radiation and acceptance of the Dx and Dx' need further study, and this is one of the tasks on our to-do-list, and we appreciate your insightful question.
Yes, we have changed the "acceptance widths" to "tolerance".

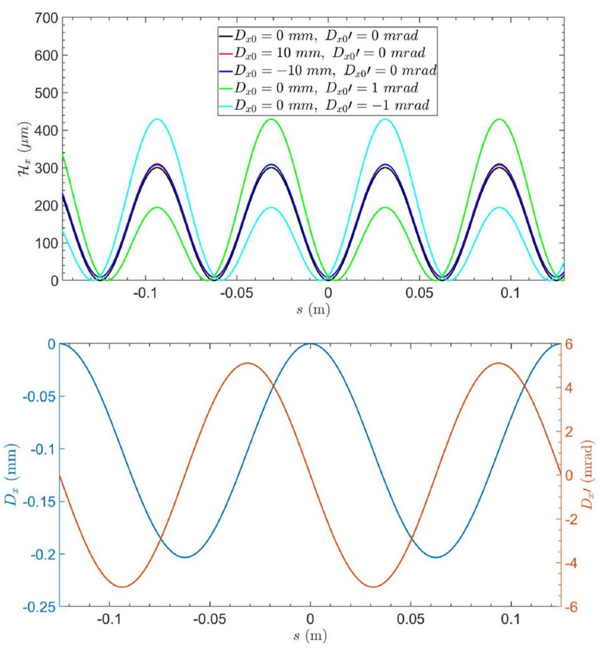


Figure 10: Up: Evolution of $\mathcal{H}_x$ with different (D_{x0}, D'_{x0}). Bottom: Evolution of D_x and D'_x with $(D_{x0}, D'_{x0}) = (0\ \mathrm{mm}, 0\ \mathrm{mrad})$.

- Fig. 4: Is the reduction in radiation power shown consistent with theoretical predictions? This could provide important support for the future SSMB proposal.

Thanks for this important question. Qualitatively, the reduction in radiation power as shown in the Fig.4 agrees with theory. For example, when the factional part of vertical tune is close to 1/5, the coherent signal of 1,2,3,4 turns are all suppressed by this coupling effect, since sin(pi*1/5*[1,2,3,4]) are non-zero. Only when the revolution number is a multiple times of 5, will this coupling effect vanish since sin(pi*1/5*5)=0. Physically, this is because the path lengths of particles with different betatron oscillation amplitudes each turn are different. But every 5 turns, the overall path length of different particles are all $5C_0$ where C_0 is the circumference, such that all the particles starting from a longitudinal slice align in the longitudinal dimension again, which means the impact of betatron motion to microbunching formation should vanish, and this is exactly what we observe in experiment. Quantitatively, it is more difficult to reconcile the exact pattern of higher turn emission with theory, as there are multiple effects in play. We would now like to also refer to the revised evaluation of the emittance scan (previously Fig. 5, now Fig. 6), which shows that the same value of H_y can explain the evolution of all revolutions vs. emittance.

- L200: have the sources of this coupling been identified? Can the procedure described here be used to quantify the coupling, or the values of the vertical dispersion and angle?

Thanks for this interesting question. There are multiple sources for horizontal-vertical coupling at the MLS. These include slightly tilted dipoles and transversely offset sextupoles. It is quite difficult to quantify, however from the revised evaluation of the dependence on vertical emittance, we can now at least give a value of H_y = 8.1 um (for the “corrected” case in Fig. 4d, now 5d).

- L205: the term “resonance” is jargon that will not mean much to a non-expert. How is the resonance changed?

Thanks. We have removed the jargon “resonance” now, and just use the vertical tune.

- L209: does the enhancement of the signal at the 4th and 5th revolutions suggest that the x-y coupling can play a positive role in preserving microbunches?

The result is not the enhancement of signals at the 4^{th} or 5^{th} revolutions, but the suppression of signals of the other revolutions. In our present experiment, we hope the coupling from x and y dimension to z dimension as small as possible. Once there is such coupling, depending on the transverse emittance, chromatic H_x,y and betatron tune as given in Eq. (7), it could degrade the microbunching. But this conclusion only holds when the coupling is a bunch lengthening effect. As explained, if we manipulate the coupling in a more subtle way, like that in emittance exchange, we can actually compress the bunch length using a small energy chirp strength thus small laser power by taking advantage of the ultrasmall vertical emittance in a planar ring. This is currently the scheme the SSMB research group adopts. This is also the reason why we believe the present experiment results are very important for the future development of SSMB. Although here we want to mitigate coupling, while in final SSMB we want to utilizing coupling, the key point here is that we are demonstrating the ability of a real machine to fine control these coupling effect. In other words, these experiment results demonstrate that the ultrahigh precision turn-by-turn control of electron beam phase space as required by SSMB can indeed be realized in a real machine.

- L210: how was it determined that the reduction of coupling was the sole reason for the change in the pattern of coherent emission? Again, a quantification of the effect could be helpful here.

The only change made between Fig. 4c and d is a very small excitation of skew quadrupole magnets (< 0.1 A), so effects on other beam parameters should be small. There may be a small impact on phase slippage, however we had seen that changing phase slippage alone via quadrupoles cannot remove the pattern seen in Fig. 4c. We can roughly confirm the correction of coupling by observing the beam image, before correction, the beam is slightly tilted. After correction, the beam is more upright, which indicates x-y coupling becomes smaller, and then we observe that the coherent signal evolution is as in Fig. 4d, instead of Fig. 4c.

- L220: the mechanism behind this technique, which beam properties they alter to increase the emittance, and how this relates to vertical-longitudinal coupling, should be explained. And if the effect on the emittance could be quantified, that would be even better. Presumably a fit could also be done to the analytical expression Eq. (7), even if the exact values of the emittance are not known.

The explanation of the technique has been expanded. To get a more direct correspondence of the data with the emittance, we have changed the horizontal axis in Fig. 5 (now Fig. 6) to a measured vertical beam size, which is proportional to the square root of the emittance. Now one does not need to know how exactly the white noise excitation impacts the emittance. We have also attempted a comparison with theoretical curves that shows good agreement (shown and explained in the manuscript). Thank you for this excellent suggestion.

- L225: Is there an explanation for the preservation of microbunching in the fourth and fifth revolutions as a function of vertical emittance increase?

This can be understood from Eq. (7). Since sin(2.23*pi*[4,5]) are much smaller than sin(2.23*pi*[1,2,3]), the impact of y-z coupling effect on the coherent signals of the 4^{th} and 5^{th} turn is weaker compared to that in the 1^{st}, 2^{nd} and 3^{rd} turn. Therefore, with the increase of vertical emittance, the relative impact on the 4^{th} and 5^{th} turn is weaker compared to that of other revolutions.

- L246: are there any “smoking guns” to explain the large variability in coherent power?

Yes, the fluctuation of momentum compaction arising from the magnet current ripple could be a candidate explanation. A brief comment has been added.

Referee #2

This paper experimentally verifies the transverse-longitudinal coupling motion effects the microbunching in a storage ring in a clear way, which is the basic beam dynamics for storage ring based microbunching scheme. It is worth publish on Communications Physics. The radiation emitted by SSMB is not entirely embraced by the semiconductor industry; characteristics such as coherence and highly collimated radiation are not preferred. So both in the Abstract and in Introduction, I recommend to claim the EUV power can reaches the level required for industrial applications.

We sincerely thank you for your appreciation of our work and support of publication, and also very helpful suggestions. Yes, the main advantage of an SSMB-EUV source is the high average power, which can benefit the EUV lithography industry. We have revised the Abstract and Introduction correspondingly.

Equations (10-11) are challenging to comprehend; a clearer representation is provided in Fig.3. The figure illustrates the dependence of radiation power on Dx and D'x. To enhance clarity, consider adding a theoretical line in Fig.3 for comparison instead of Eq(10-11) . In this context, the authors concentrate on the accuracy of D, D'x, assuming emittance x is precise. However, if emittance x is not 30nm, it can also introduce discrepancies between theoretical and measured data as per Eq. (7). Therefore, I recommend reorganizing this section.

Thanks for this excellent comment and suggestion. We have changed the horizontal axis of Fig. 3 (now Fig. 4) to directly reflect the dispersion/dispersion angle to increase clarity and made several revisions to the section to improve understandability. But we hope to get your understanding for not including the theoretical curve in the plot since it seems to decrease the readability of the figure. Yes, emittance x could be one candidate explanation of theory and measurement, and thanks for this very good suggestion. However, emittance x alone seems still not enough, since the discrepancy is more significant in Dx'. If it is from emittance x alone, then we would expect the impact on Dx and Dx' scan should be the same. We have incorporated this candidate explanation in this revised manuscript and we hope this is okay. Thanks.

Referee #3

Comments:

The manuscript presents experimental findings from systematic studies in the ongoing proof-of-principle experiment of SSMB. The authors report on the confirmation of the theoretical expectations in multiple aspects, particularly focusing on the influence of transverse-longitudinal coupling dynamics on the microbunching process. The experimental findings are well-documented and supported by systematic studies.

We thank for your appreciation of our work.

However, I have identified several areas that require further attention and clarification:

1. The manuscript focuses on the experimental investigation of transverse-longitudinal coupling, but the description of the experimental setup is insufficient to provide a clear understanding of the experimental process. I recommend including a schematic diagram of the magnetic lattice layout in the Methods section to illustrate how dispersion, dispersion angle, and the coupling between horizontal and vertical were adjusted during the experiment. This addition will enhance the clarity of the experimental setup for readers.

Thanks for this helpful suggestion. A diagram showing the magnetic layout and optical functions has been added to the Methods section.

2. In Figure 3, the experimental results appear to be significantly larger than the theoretical predictions, particularly under the dispersion angle condition. The manuscript should provide a quantitative analysis of the potential effects of bunching tilt and the finite opening angle of the coherent radiation. Please explain which effect is the primary factor contributing to the observed discrepancy and clarify why the error impact under the dispersion angle condition is significantly larger than under the dispersion condition.

Thanks for this important question. Indeed this large discrepancy is one of the key issues we need to understand in our following study. All the theoretical model in the present manuscript is based on the definition of 1D bunching factor as shown in the figure below which depends only on the longitudinal current distribution of the beam. However, in reality, the transverse distribution of the beam can also affect the coherent radiation. [For more discussion on this, refer to *J. Synchrotron Rad.* 30, 35–50 (2023).] In particular in our experiment, when there is dispersion or dispersion angle at the undulator, then one turn or several turns after the laser modulation, the formed microbunching at the undulator actually has a tilted wavefront as shown in the figure below. If we calculate the 1 D bunching factor, then the drop of bunching factor is significant, which means a significant drop of coherent radiation power. But actually, such tilted microbunches can generate strong off-axis coherent radiation, since the off-axis projected microbunch length is smaller than that projected on-axis. Considering the radiation actually has a finite opening angle, the overall radiation power decrease is not as strong as predicted by 1D bunching factor, if we use the 3D bunching factor for the calculation. We have conducted some analysis and calculation as shown below and find that this observation can indeed partly explain the discrepancy between the 1D bunching factor theory and the measured results. But we should admit we still do not have a satisfactory explanation to fully explain the large discrepancy. Study in this respect is still ongoing and progress will be reported in the future, for which we hope to get your understanding and meet with your approval.

1D bunching factor:

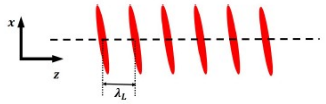
$$b(\omega) = \frac{1}{N_e}\sum_{j=1}^{N_e} e^{-i\omega\frac{z_j}{\beta c}}.$$

3D bunching factor:

$$b(\theta,\varphi,\omega) = \frac{1}{N_e}\sum_{j=1}^{N_e} e^{-i\omega\left(\frac{x_j\sin\theta\cos\varphi + y_j\sin\theta\sin\varphi}{c} + \frac{z_j}{\beta c}\right)}.$$

Coherent radiation considering the impact of transverse dimension of electron beam:

$$\left.\frac{d^2W}{d\omega d\Omega}(\theta,\varphi,\omega)\right|_{\text{beam}} = N_e^2|b(\theta,\varphi,\omega)|^2\left.\frac{d^2W}{d\omega d\Omega}(\theta,\varphi,\omega)\right|_{\text{point}}.$$



When $\mathcal{H}_x \ll \beta_x$, the titled angle of the microbunching in the x-z plane is

$$\theta_{xz} \approx \frac{\langle xz\rangle}{\langle x^2\rangle} = \left[-(1-\cos(m2\pi\nu_x))\frac{\alpha_x D_x + \beta_x D'_x}{\beta_x} - \sin(m2\pi\nu_x)\frac{D_x}{\beta_x}\right]$$

If $\alpha_x = 0$, then

$$\theta_{xz} \approx -(1-\cos(m2\pi\nu_x))D'_x - \sin(m2\pi\nu_x)\frac{D_x}{\beta_x}$$

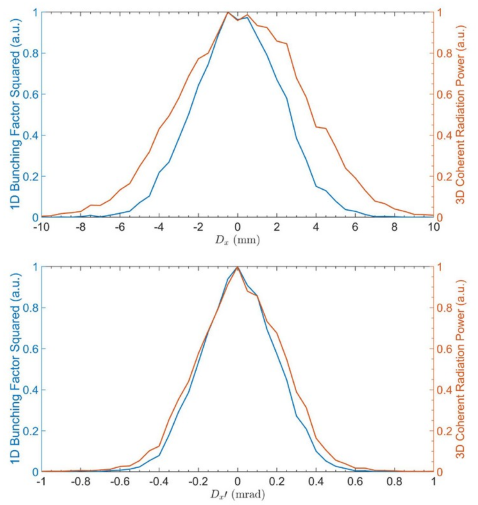


Figure 6: 1D bunching factor squared and coherent radiation power v.s. D_x and D'_x at the undulator. 1st Revolution after Modulation. Gaussian fit is 1D bunching factor squared is $\sigma_{Dx} = 2.2$ mm, $\sigma_{Dx'} = 0.19$ mrad. Gaussian fit of 3D coherent radiation power is $\sigma_{Dx} = 3.2$ mm, $\sigma_{Dx'} = 0.22$ mrad.

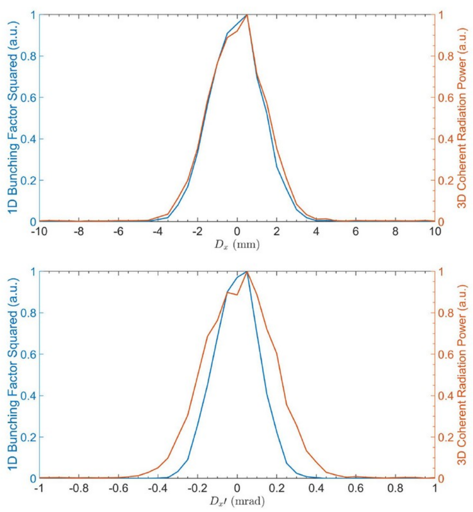
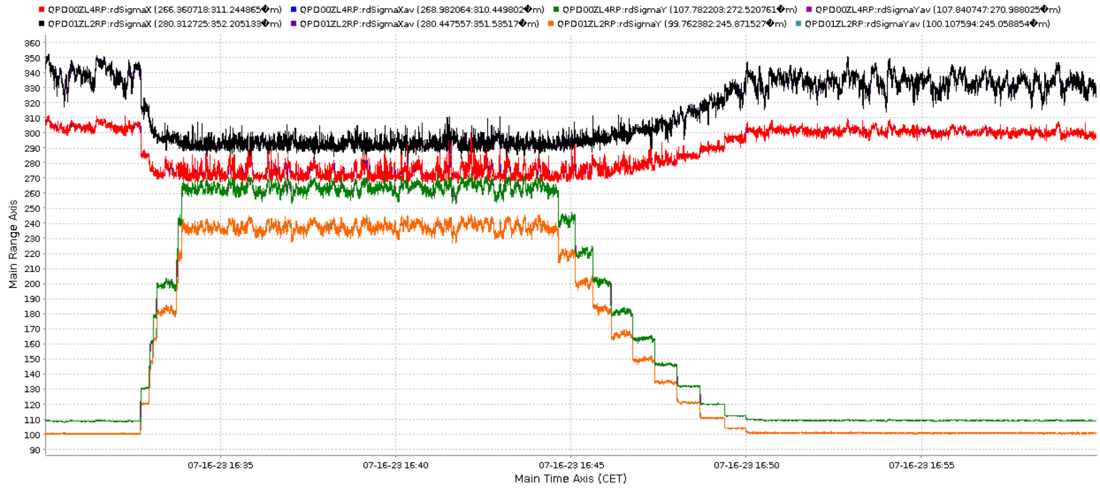


Figure 7: 1D bunching factor squared and coherent radiation power v.s. D_x and D'_x at the undulator. 2nd Revolution after Modulation. Gaussian fit is 1D bunching factor squared is $\sigma_{Dx} = 1.3$ mm, $\sigma_{Dx'} = 0.115$ mrad. Gaussian fit of 3D coherent radiation power is $\sigma_{Dx} = 1.4$ mm, $\sigma_{Dx'} = 0.18$ mrad.

3. Figure 5 shows a trend in the radiation power with the white noise excitation amplitude, where the radiation power initially increases and then decreases, especially for turn 1-3. Please provide an explanation for this phenomenon.

One possible reason is that there are still some collective effects like intra-beam scattering in our experiment, and when we blow up the vertical emittance at the beginning, the energy spread or bunch length shrink, which could result in an increase of bunching factor or bunch charge density, thus the increase of coherent radiation power. This is supported by the observation that the horizontal beam size shrinks slightly (see red and black in the figure below) as the vertical beam emittance is increased (green and orange in the figure), even though the white noise excitation is limited to the vertical plane. We have added a comment to the manuscript.



Finally, we thank all the referees and editors sincerely again for all the time and efforts you have expended to review our manuscript and to help us improve the presentation.

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

Dear authors,

I would like to thank you for your very comprehensive answers to my questions and comments. The explanation of the significance of these results for the future SSMB proposal was particularly illuminating, and much appreciated. I also understand that explaining these concepts while also trying to minimize the use of technical terms can be tricky, so thank you for including some of the more technical details.
In my opinion, the revised manuscript is considerably improved, and the paper can be accepted for publication.
Good luck for the future!

Best regards

Reviewer #2 (Remarks to the Author):

The comments are propaly appropriately responsed.

Reviewer #3 (Remarks to the Author):

Most of my previous concerns have been addressed, and the manuscript has been appropriately modified. I believe that this work represents an important step in the development of SSMB, and I recommend the publication of this article. I suggest that the authors should continue their experiments in the future to further confirm the relevant physical issues.

Response to Referees' comments

Re: **Next steps towards steady-state microbunching: confirming the theoretical foundation**
by Arnold Kruschinski et al.

Dear referees,

We would like to thank each one of you again for your efforts expended to review our manuscript. We are happy to hear that we were able to address all concerns and are met with positive decisions from each of you. We very much appreciated your comments which have proven very insightful to improve the manuscript and further our scientific discourse.

Yours sincerely,
Arnold Kruschinski
on behalf of all co-authors

Individual referees comments:

Referee #1

Comment:

I would like to thank you for your very comprehensive answers to my questions and comments. The explanation of the significance of these results for the future SSMB proposal was particularly illuminating, and much appreciated. I also understand that explaining these concepts while also trying to minimize the use of technical terms can be tricky, so thank you for including some of the more technical details.
In my opinion, the revised manuscript is considerably improved, and the paper can be accepted for publication.
Good luck for the future!

We would like to thank referee #1 especially for raising these very relevant and important questions, which have enabled a significant improvement of the manuscript. Thank you for advancing our scientific discussions, your appreciation and your good wishes.

Referee #2

Comment:

The comments are propaly appropriately responsed.

Thank you for your assessment and appreciation of the manuscript.

Referee #3

Comment:

Most of my previous concerns have been addressed, and the manuscript has been appropriately modified. I believe that this work represents an important step in the development of SSMB, and I recommend the publication of this article. I suggest that the authors should continue their experiments in the future to further confirm the relevant physical issues.

Thank you for your assessment. We will definitely continue the experiments, and we thank you for your support and appreciation.