Supplementary Information for "Performance of Quantum Annealing Inspired Algorithms for Combinatorial Optimization Problems"

Qing-Guo Zeng,[1,2,3,4,5*] Xiao-Peng Cui,[2] Bowen Liu,[2] Yao Wang,[2]
Pavel Mosharev,[2] and Man-Hong Yung[1,2,3,4,5*]

[1]Shenzen institute for Quantum Science and Engineering,
Southern University of Science and Technology, Shenzhen, China
[2]2012 lab, Huawei Technologies Co., Ltd, Shenzhen, China
[3]International Quantum Academy, Shenzhen, China
[4]Guangdong Provincial Key Laboratory of Quantum Science and Engineering,
Southern University of Science and Technology, Shenzhen, China
[5]Shenzhen Key Laboratory of Quantum Science and Engineering,
Southern University of Science and Technology, Shenzhen, China
* Email: zengqg2022@mail.sustech.edu.cn, yung@sustech.edu.cn

June 16, 2024

Supplementary Figure 1 – The potential landscape of QAIAs

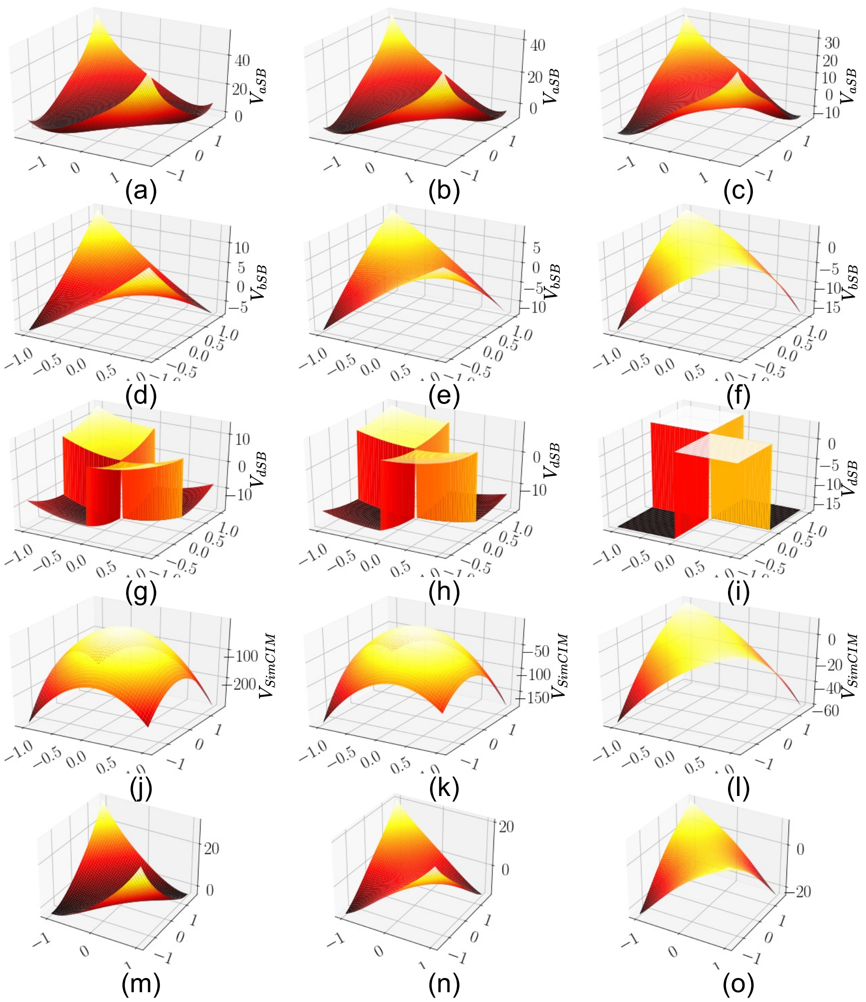
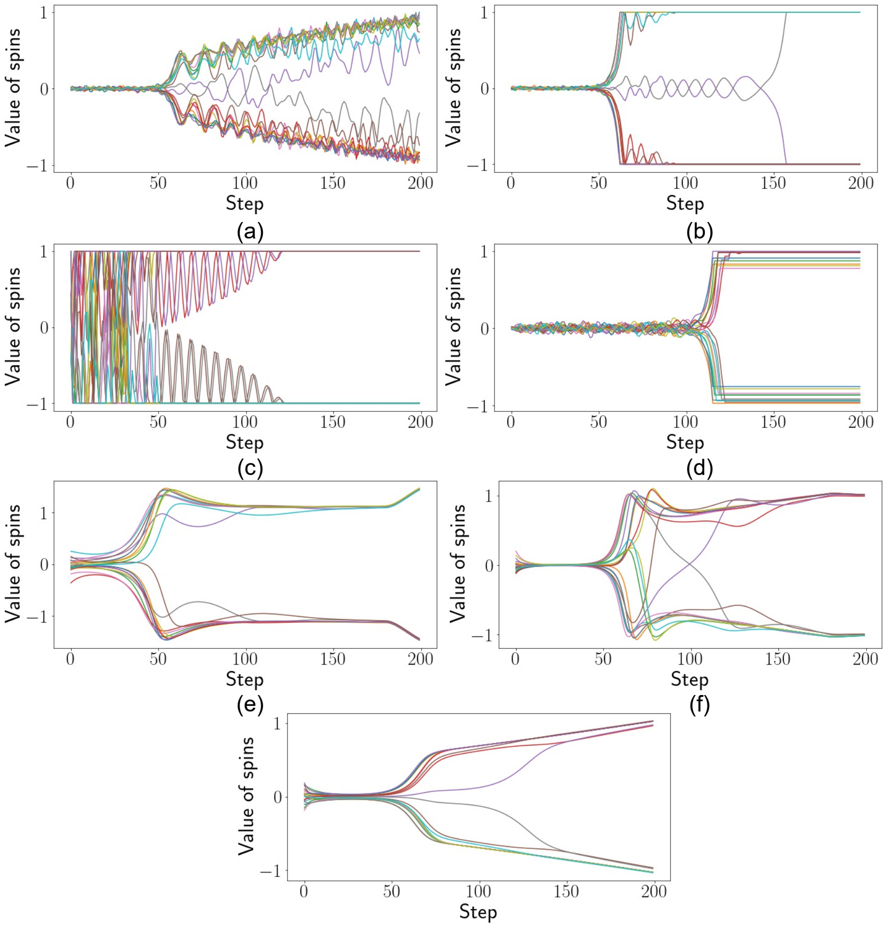


Figure 1: The potential landscape of V_{aSB} (a to c), V_{bSB} (d to f), V_{dSB} (g to i) , V_{SimCIM} (j to l) and (m to o), it is should be noted that the potential landscape of CAC, CFC and SFC are the same) at the beginning (left), intermediate (center) and final (right) time. Twenty spins are randomly divided into two groups and changed simultaneously according to the scale value of the coordinate axis since we cannot show the potential landscape in the high dimension.

Supplementary Figure 2 – The potential landscape of QAIAs



(a) (b) (c) (d) (e) (f) (g)

Figure 2: Trajectories of individual spins along with the annealing time on three-regular Ising problem with twenty spins for aSB(a), bSB(b), dSB(c), SimCIM(d), CAC(e), CFC(f) and SFC(g). With the same configuration, all the algorithms bifurcate at almost the same iteration except for SimCIM. Among them, dSB, CAC and SFC converge fastest.

Supplementary Figure 3 – The comparison of QA-IAs on Gset

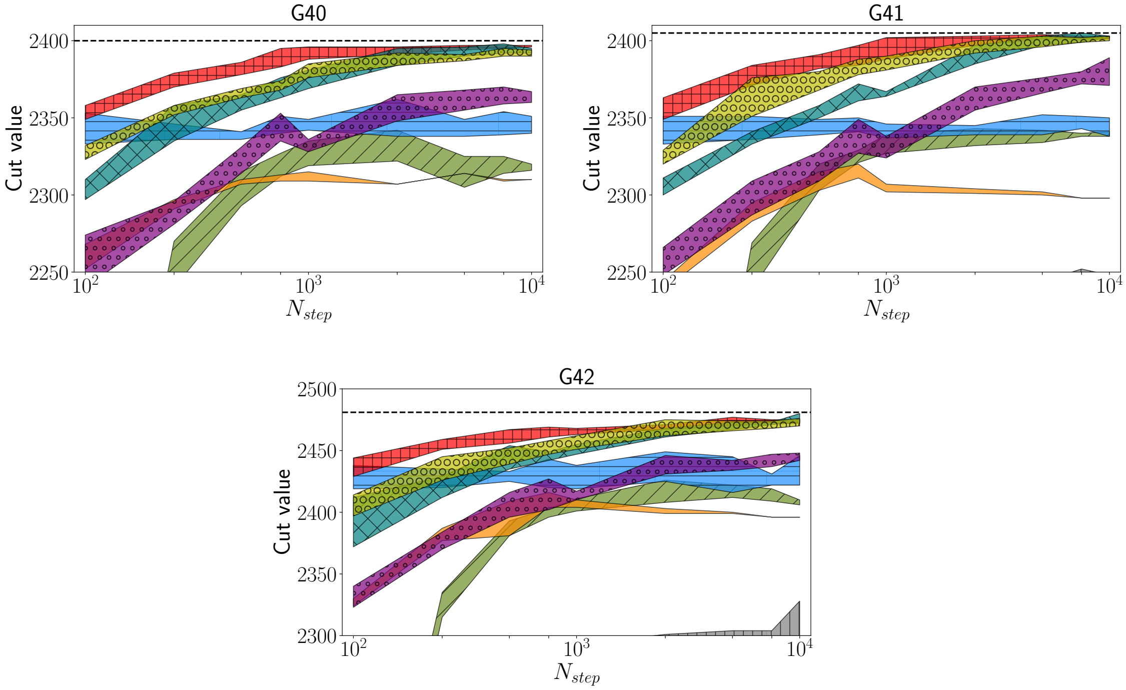


Figure 3: Comparison between quantum annealing inspired algorithms for Gset (G40, G41, G42). The polygons show the top-1% cut value for different algorithms. The present best-known cuts (black dashed line) for Gset are given by BLS algorithm.

Supplementary Methods – Pseudocode of Quamtum Annealing Inspired Algorithms

Algorithm 1 SimCIM

Input: Coupling coefficients $\boldsymbol{J}$, maximum number of iterations T, pump-loss factor $\{v_t\}_{t=1}^{T}$, coupling strength ζ, standard deviation of noise δ, time step μ, momentum β

Output: Ising spins $\hat{\boldsymbol{x}}$

```
for $i = 1$ to $N$ do
    $x_i^0 := 0$
    $m_i^0 := 0$
end for
for $t = 1$ to T do
    for $i = 1$ to $N$ do
        $\Delta x_i := \mu(v_t x_i^{(t-1)} + \zeta \sum_j J_{ij} x_j^{(t-1)}) + \mathcal{N}(0, \delta)$
        $m_i^{(t)} := \beta m_i^{(t-1)} + (1 - \beta)\Delta x_i$
        $x_i^{(t)} := x_i^{(t-1)} + m_i^{(t)} \cdot \mathbf{1}_{\{x||x+m_i|\leq 1\}}(x_i^{(t-1)})$, where $\mathbf{1}_A(x) = \begin{cases} 1, x \in A \\ 0, x \notin A \end{cases}$
    end for
end for
for $i = 1$ to $N$ do
    $\hat{x}_i := \mathrm{sign}(x_i^{(T)})$
end for
```

Algorithm 2 CAC

Input: Coupling coefficients $\boldsymbol{J}$, gain parameter $p(t)_{t=1}^{T}$, coupling strength ζ, target amplitude $\alpha(t)_{t=1}^{T}$, coefficient of error variable β, maximum number of iterations T

Output: Ising spins $\hat{\boldsymbol{x}}$

```
for $i = 1$ to $N$ do
    $x_i^{(0)} \sim \mathcal{N}(0, 10^{-4})$
    $e_i^{(0)} := 1$
end for
for $t = 1$ to T do
    for $i = 1$ to $N$ do
        $x_i^{(t)} := x_i^{(t-1)} + [-{x_i^{(t-1)}}^3 + (p(t) - 1)x_i^{(t-1)} + e_i^{(t-1)} \sum_{j=1}^{N} \zeta J_{ij} x_j^{(t-1)}]\Delta_t$
        $e_i^{(t)} := e_i^{(t-1)} - \beta e_i({x_i^{(t)}}^2 - \alpha(t))\Delta_t$
        if abs($x_i^{(t)}$)$>$ $1.5\sqrt{\alpha(t)}$ then
            $x_i^{(t)} := 1.5\sqrt{\alpha(t)}\text{sign}(x_i^{(t)})$
        end if
    end for
end for
for $i = 1$ to $N$ do
    $\hat{x}_i := \text{sign}(x_i^{(T)})$
end for
```

Algorithm 3 CFC

Input: Coupling coefficients $\boldsymbol{J}$, gain parameter $p(t)_{t=1}^{T}$, coupling strength ζ, target amplitude $\alpha(t)_{t=1}^{T}$, coefficient of error variable β, maximum number of iterations T

Output: Ising spins $\hat{\boldsymbol{x}}$

```
for $i = 1$ to $N$ do
    $x_i^{(0)} \sim \mathcal{N}(0, 0.1)$
    $e_i^{(0)} := 1$
end for
for $t = 1$ to T do
    for $i = 1$ to $N$ do
        $z_i := -e_i^{(t-1)} \sum_j \zeta J_{ij} x_j^{(t-1)}$
        $x_i^{(t)} := x_i^{(t-1)} + [-x_i^{(t-1)^3} + (p(t) - 1)x_i^{(t-1)} - z_i]\Delta_t$
        $e_i^{(t)} := e_i^{(t-1)} - \beta e_i(z_i^2 - \alpha(t))\Delta_t$
        if abs($x_i^{(t)}$)> 1.5 then
            $x_i^{(t)} := 1.5 \cdot \mathrm{sign}(x_i^{(t})$
        end if
        if $e_i^{(t)} < 0.01$ then
            $e_i^{(t)} = 0.01$
        end if
    end for
end for
for $i = 1$ to $N$ do
    $\hat{x}_i := \mathrm{sign}(x_i^{(T)})$
end for
```

Algorithm 4 SFC

Input: Coupling coefficients $\boldsymbol{J}$, gain parameter $p(t)_{t=1}^{T}$, coupling strength ζ, coefficient of error variable β, maximum number of iterations T, system parameters c and k

Output: Ising spins $\hat{\boldsymbol{x}}$

```
for $i = 1$ to $N$ do
    $x_i^{(0)} \sim \mathcal{N}(0, 0.1)$
    $e_i^{(0)} := 0$
end for
for $t = 1$ to T do
    for $i = 1$ to $N$ do
        $z_i := -\sum_j \zeta J_{ij} x_j^{(t-1)}$
        $x_i^{(t)} := x_i^{(t-1)} + [-{x_i^{(t-1)}}^3 + (p(t) - 1)x_i^{(t-1)} - \tanh cz_i - k(z_i - e_i^{(t-1)}]\Delta_t$
        $e_i^{(t)} := e_i^{(t-1)} - \beta(e_i^{(t-1)} - z_i)\Delta_t$
    end for
end for
for $i = 1$ to $N$ do
    $\hat{x}_i := \mathrm{sign}(x_i^{(T)})$
end for
```

Algorithm 5 aSB with modified explicit symplectic Euler method

Input: Coupling coefficients $\boldsymbol{J}$, maximum number of iterations T, time step Δ_t, $a(t)_{t=1}^{T}$, c_0, a_0, positive integer M

Output: Ising spins $\hat{\boldsymbol{x}}$

```
for $i = 1$ to $N$ do
    $x_i^0 := 0$
    $y_i^0 \sim \mathcal{N}(0, 0.01)$
end for
$\delta_t = \frac{\Delta_t}{M}$
for $t = 1$ to T do
    for $m = 1$ to M do
        for $i = 1$ to $N$ do
            $x_i^{(t-1,m)} := x_i^{(t-1,m-1)} + a_0 y_i^{(t-1,m-1)} \delta_t$
            $y_i^{(t-1,m)} := y_i^{(t-1,m-1)} - [x_i^{(t-1,m)^3} + (a_0 - a(t))]\delta_t$
        end for
        $x_i^{(t,0)} := x_i^{(t-1,M)}$
    end for
    for $i = 1$ to $N$ do
        $y_i^{(t,0)} := y_i^{(t-1,M)} + c_0 \sum_{j=1}^{N} J_{ij} x_j^{(t,0)} \Delta_t$
    end for
end for
for $i = 1$ to $N$ do
    $\hat{x}_i := \text{sign}(x_i^{(T,0)})$
end for
```

Algorithm 6 bSB and dSB with explicit symplectic Euler method

Input: Coupling coefficients $\boldsymbol{J}$, maximum number of iterations T, time step Δ_t, $a(t)_{t=1}^{T}$, c_0, a_0

Output: Ising spins $\hat{\boldsymbol{x}}$

```
for $i = 1$ to $N$ do
    $x_i^0 := 0$
    $y_i^0 \sim \mathcal{N}(0, 0.01)$
end for
for $t = 1$ to T do
    for $i = 1$ to $N$ do
        $y_i^{(t)} = y_i^{(t-1)} + [-(a_0 - a(t))x_i^{(t-1)} + c_0 \sum_{j=1}^{N} J_{ij} x_j^{(t-1)}]\Delta_t \longrightarrow$ bSB
        $y_i^{(t)} = y_i^{(t-1)} + [-(a_0 - a(t))x_i^{(t-1)} + c_0 \sum_{j=1}^{N} J_{ij} \text{sign}(x_j^{(t-1)})]\Delta_t \longrightarrow$ dSB
        $x_i^{(t)} := x_i^{(t-1)} + a_0 y_i^{(t)} \Delta_t$
        if $|x_i^{(t)}| > 1$ then
            $x_i^{(t)} = \text{sign}(x_i^{(t)})$
            $y_i^{(t)} = 0$
        end if
    end for
end for
for $i = 1$ to $N$ do
    $\hat{x}_i := \text{sign}(x_i^{(T)})$
end for
```

Algorithm 7 NMFA

Input: Coupling coefficients $\boldsymbol{J}$, external magnetic field $\boldsymbol{h}$, maximum number of iterations T, inverse temperature $\{\beta_t\}_{t=1}^{T}$, standard deviation of noise σ, momentum α

Output: Ising spins $\tilde{\boldsymbol{s}}$

```
for $i = 1$ to $N$ do
    $s_i^{(0)} := 0$
end for
for $t = 1$ to T do
    for $i = 1$ to $N$ do
        $\phi_i := \frac{(h_i + \sum_j J_{ij} s_j^{(t-1)})}{\sqrt{h_i^2 + \sum_j J_{ij}^2}} + \mathcal{N}(0, \sigma)$
        $\hat{s}_i = -\tanh(\Phi_i * \beta_t)$
        $s_i^{(t)} := \alpha \hat{s}_i + (1 - \alpha) s_i^{(t-1)}$
    end for
end for
for $i = 1$ to $N$ do
    $\tilde{s}_i := \frac{s_i^{(T)}}{|s_i^{(T)}|}$
end for
```

Supplementary Table I – Parameter setup of SimCIM

Table 1: The parameter setup of SimCIM.

Parameter	Value
Initial pump-loss factor v_0	$\{-5.5, -6.5, -7.5\}$
Pump-loss factor v_t	increase in tanh law
standard deviation of noise δ	$\{0.01, 0.1, 1\}$
time step μ	$\{0.01, 0.1, 1\}$
Momentum β	0.99

*Note: $\{\cdot\}$ means that the value is determined by grid search within this set.

Supplementary Table II – Parameter setup of SB

Table 2: The parameter setup of SB.

Parameter	Value
The number of inner product M	2
Kerr coefficient a_0	0
Pumping amplitude $a(t)$	[0, 1] increase linearly
Coefficient of Ising problem c_0	$\frac{a_0}{\lambda_{\max}}$ for positive weight, $2\sqrt{N}\sqrt{\frac{\sum_{ij} J_{ij}^2}{N(N-1)}}$ for signed weight
time step Δt	1

Supplementary Table III – Parameter setup of NMFA

Table 3: The parameter setup of NMFA.

Parameter	Value
Initial inverse temperature β_0	$1/T$
Increment rate of inverse temperature $\Delta\beta$	$1/T$
Standard deviation of noise σ	0.15
Momentum α	0.15

*Note: T is the maximum number of iterations.