nature portfolio

Peer Review File

Nonparametric inference of higher order interaction patterns in networks

REVIEWERS' COMMENTS:

Reviewer #1 (Remarks to the Author):

Higher-order networks are raising significant interest as they encode the many-body interactions in complex-systems among two or more nodes.

The two most widely diffuse framework to represent higher-order interactions are hypegraphs and simplicial complexes which have a network skeleton formed by a set of cliques.

However in many cases ranging from protein-interaction network to social networks many body interaction can occur among a group of nodes that are not always connected by all possible pairwise interactions.

In applied topology this leads to the formulation of cell complexes whose building blocks are regular polytopes.

However from the statistics and statistical mechanics point of view it is also possible to construct higher-order networks from more general building blocks, or motifs.

This manuscript proposed a rigorous approach to construct generative models of higher-order networks formed by motifs distributed homogeneously or even more interestingly, inhomogeneously among a set of nodes.

The generative model is then used to perform Bayesian inferences and is applied to a variety of interesting dataset.

I think this work is well written relevant and correct and will attract the attention of the readership of Communication Physics, so I suggest publication of the work as it is.

Eventually the authors might consider to mention cell complexes and cite some reference in the field to mention this alternative approach.

Reviewer #2 (Remarks to the Author):

This manuscript brings a solid contribution to the active field of "higher-order networks". I have enjoyed reading it and do not have any reservation about its possible publication in Communications Physics.

Reviewer #3 (Remarks to the Author):

This paper propose a method for obtaining parsimonious decompositions of networks into higher order interactions which can take the form of arbitrary motifs. The method includes "degree-corrected" models that better match real-world networks by accounting for their degree distribution. Results show high accuracy on simulated data and identifies significant subgraphs in real networks, providing an explicit higher-order representation and fit to analytically tractable models.

The approach is quantitatively rigorous, the methods are clear, and the results highlight some interesting topological motifs in real-world networks. I think this work makes an interesting contribution. I don't have major comments, only one minor suggestion. Since the authors talk about methods for extracting higher order interactions from graph data, it could be good to reference this paper in the Introduction: Santoro et al., Higher-order organization of multivariate time series, Nature Physics 2023.

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Response: We thank the reviewer for their positive and constructive comments. In the end we decided not to mention cell complexes as it is difficult to relate the presented approach to cell complexes without a lengthy and technical introduction to cell complexes. But we agree with the reviewer that cell complexes offer a complementary perspective and we intend to investigate the relation between higher order representations and cell complexes in the future.

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Response: We thank the reviewer for their constructive and supportive comments. Following the reviewers suggestion we have now included references to methods that seek to extract higher order interactions from multivariate time series and network dynamics. (End of paragraph 2 on page 2)

Remarks to editor:

In addition to the changes requested in the editorial request table we have added references to methods that seek to extract higher order interactions from multivariate time series and network dynamics (End of paragraph 2 on page 2) which now reads: '....Consequently, methods for extracting higher order interactions from graphs [11] and other types of data [12, 13] are of much current interest.' Otherwise the manuscript remains unchanged.

We have also provided a suggested feature image which is Supplementary Figure 1 in the SI. However we were unable to access the licence to publish from the link in the Editorial Request Table. Needless to say we would be happy to provide the licence if you could provide us with a template.

Finally, we would like to take the opportunity to thank the reviewers and your team at Communication Physics for the professionalism and efficiency in handling our manuscript.

Kind regards,
Anatol E Wegner