

Supplementary Information

Polarization-diverse soliton transitions and deterministic switching dynamics in strongly-coupled and self-stabilized microresonator frequency combs

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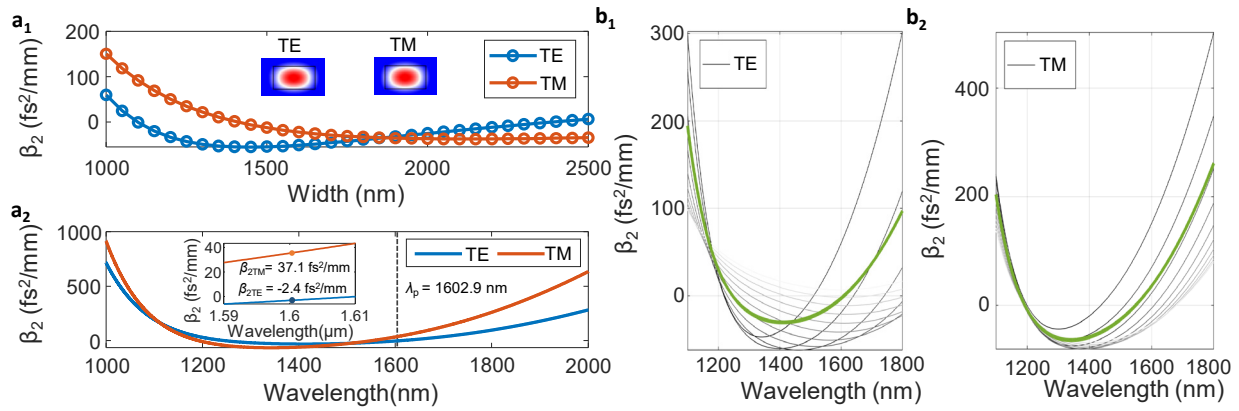
This Supplementary Information consists of the following sections:

- I.** The dispersion-managed nitride microresonator design and characterization
- II.** Thermally-induced cavity mode pulling and the intracavity power evolution
- III.** Additional coupled LLE simulations for different conditions
- IV.** Approximating third order dispersion using dispersive wave and LLE modeling
- V.** The characterization of the parametric time magnifier

Supplementary Note 1. The dispersion-managed nitride microresonator design and characterization

Supplementary Figure 1a₁ shows the calculated group velocity dispersion (GVD) based on the finite element analysis method considering geometric and material dispersion of the tapered nitride waveguide. When the waveguide height is fixed at 0.8 μm , we vary waveguide width from 1 to 2.5 μm leading to the oscillation of GVD from -54 to 59 fs^2/mm and from -34 to 150 fs^2/mm in the

TE and TM polarization. Insets are the TE and TM electrical field profiles. The calculation is carried out based on a triangular grid with 50 nm spatial resolution. Subsequently, the path-averaged GVD is calculated based on the simulated dispersion from the swept waveguide widths as shown in Supplementary Fig. 1a₂ over the spectral range from 1 to 2 μm . The calculated path-averaged GVD in the TE and TM polarization are $\beta_{2\text{TE}} = -2.4 \text{ fs}^2/\text{mm}$ and $\beta_{2\text{TM}} = 37.1 \text{ fs}^2/\text{mm}$ at the experimental pump wavelength of 1602.9 nm. Supplementary Figure 1b₁ and 1b₂ are the calculated path-averaged GVD depicted with the light green solid curves at the TE and TM polarization with the relation $\beta_2 = \frac{\beta_u \times L_u + 14 \sum \beta_{2L} dL}{L}$, where L_u is the uniform waveguide length, β_u is the corresponding GVD at the uniform waveguide, the β_{2L} is the calculated GVD at the different waveguide width, dL is the waveguide width increment step, and L is the cavity circumference. The color-coded gray curves are the calculated GVD with different waveguide widths.

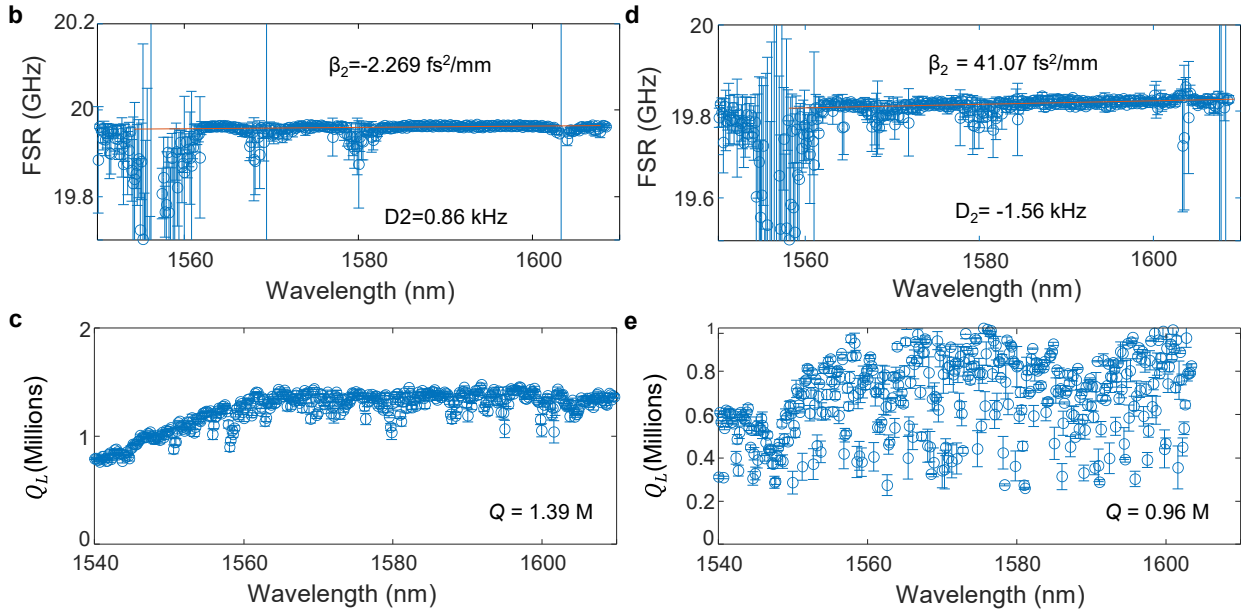
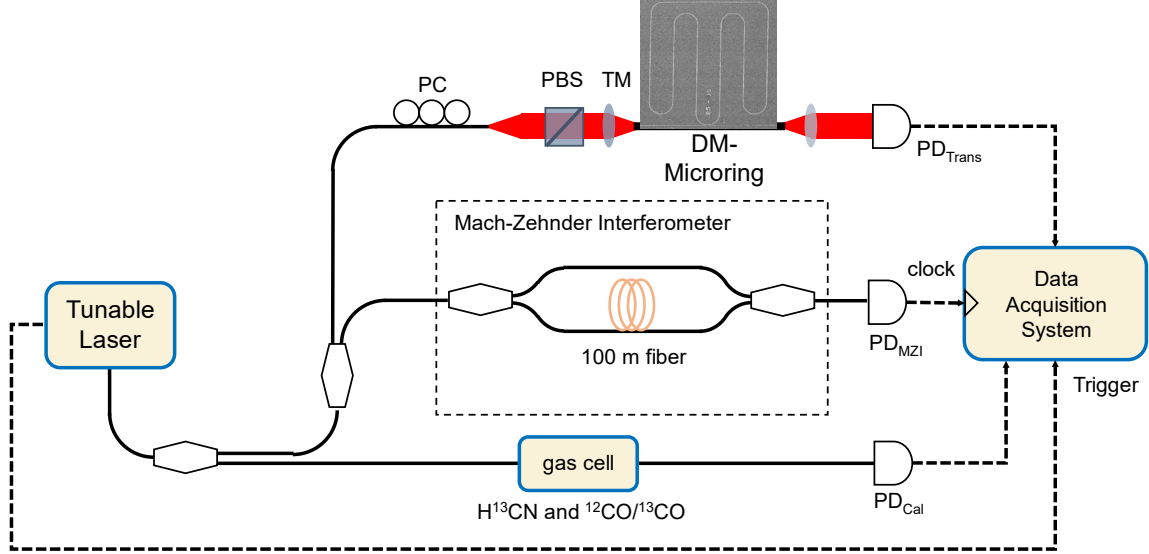


Supplementary Figure 1 | The dispersion-managed nitride microresonator design. **a₁**, The calculated GVD with different waveguide widths at the same waveguide height and pump wavelength for TE and TM polarizations. Insets are the calculated electrical mode profiles. **a₂**, The calculated path-averaged GVD over a spectral range from 1 to 2 μm . The path-averaged GVD at TE and TM polarizations are $-2.4 \text{ fs}^2/\text{mm}$ and $37.1 \text{ fs}^2/\text{mm}$ at a pump wavelength of 1602.9 nm. **b₁** and **b₂**, The calculated GVD with different waveguide widths along with the path-averaged GVD in TE and TM polarizations.

The cavity mode dispersion and the loaded quality factor of the designed microresonator are characterized by a swept wavelength interferometry method. Supplementary Figure 2a shows the schematic of the swept wavelength interferometer setup which includes a tunable laser followed by a fiber coupled hydrogen cyanide gas cell for absolute frequency calibration and a fiber Mach-

Zehnder interferometer with 100 m unbalanced fiber length to provide a clock signal. A part of the laser light is coupled into the designed microresonator to measure the wavelength-dependent cavity mode spectra.

a



Supplementary Figure 2 | Swept wavelength interferometry for cold cavity mode dispersion and loaded quality factor (Q) characterization. **a**, The experimental setup of the swept wavelength interferometer which is triggered by the unbalanced linear interferometer and frequency-calibrated by the high precision gas cell. PC, polarization controller, PBS, polarization beam splitter, PD, photodetector. **b** and **c**, The extracted cold cavity mode dispersion and the loaded cavity quality factor with respect to wavelength in the TE polarization. The measured GVD and

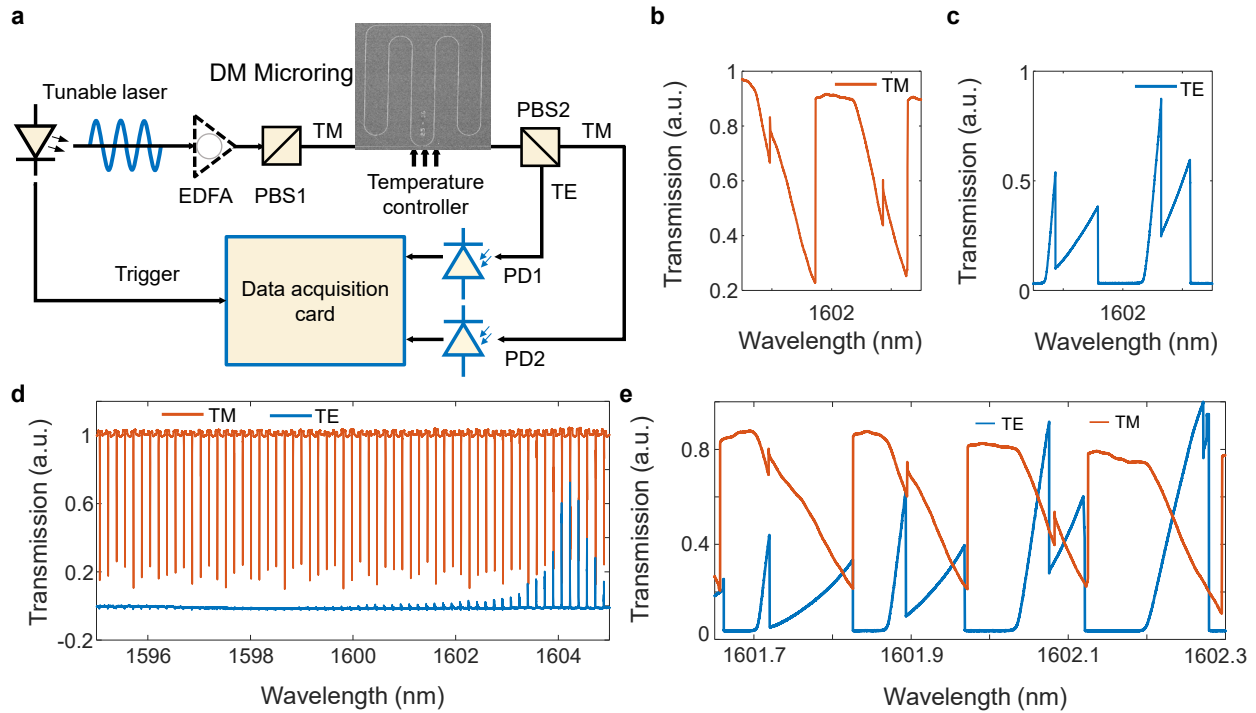
quality factor are $-2.27 \text{ fs}^2/\text{mm}$ and 1.39×10^6 for the TE-polarized mode family at the pump wavelength. **d** and **e**, The extracted cold cavity mode dispersion and the loaded cavity quality factor at the TM polarization. The measured GVD and quality factor are $41.07 \text{ fs}^2/\text{mm}$ and 960,000 for the TM-polarized mode family at the pump wavelength.

By sweeping the tunable laser over 60 nm spectral range with a sweeping speed of 100 nm/s, the power transmission of the microresonator is recorded with the interferometer-triggered data acquisition system. The transmission data are calibrated with the high-precision gas cell with 85 absorption peaks over C-L band. The cavity mode dispersion is extracted from the calibrated transmission data with a linear curve fitting. The measured GVD in TE and TM polarizations are $\beta_{2\text{TE}} = -2.3 \text{ fs}^2/\text{mm}$ and $\beta_{2\text{TM}} = 41.1 \text{ fs}^2/\text{mm}$ as shown in Supplementary Figure 2b and 2d which are close to the designed GVD. On the other hand, the loaded quality factors at the two polarizations are measured to be $Q_{\text{TEpump}} = 1.39 \times 10^6$ and $Q_{\text{TEpump}} = 960,000$ at the pump wavelength. The mode family FSR for the two polarizations are 19.82 and 19.96 GHz. The wavelength dependence of the quality factor is observed in the cavity mode quality factor distribution. The standard deviation arises from 5 individual measurements.

Supplementary Note 2. Thermally-induced cavity mode pulling and intracavity power evolution

The avoided mode crossing [1,2] and polarization conversion process are characterized by the modified swept wavelength interferometry setup where the output of the microresonator is split into two parts by a polarization beam splitter as shown in Supplementary Figure 3a. The mode-splitting of the TM and TE mode families is examined by the mode transmission spectra. Figure S3b and S3c show the measured TM-polarized mode spectrum and the converted TE-polarized mode spectrum. The high pump laser power introduces significant thermal effects which shift the resonance wavelength via the relation $\Delta\lambda(T) = \lambda_0 \left(\frac{1}{n} \frac{dn}{dT} + \frac{1}{r} \frac{dr}{dT} \right) \Delta T$, where λ_0 , r , and n are the resonance wavelength, device equivalent radius, refractive index of the nitride microresonator respectively, and dn/dT and dr/dT are the thermal refractive index and thermal expansion coefficients. The TM mode spectrum shows clear mode splitting structures. Thermally-induced resonance broadening is also observed. The TE mode spectrum is converted from the TM mode via polarization conversion. The corresponding cold-cavity mode transmission is measured as shown in Supplementary Figure 3d where the two channels are synchronized. The converted TE

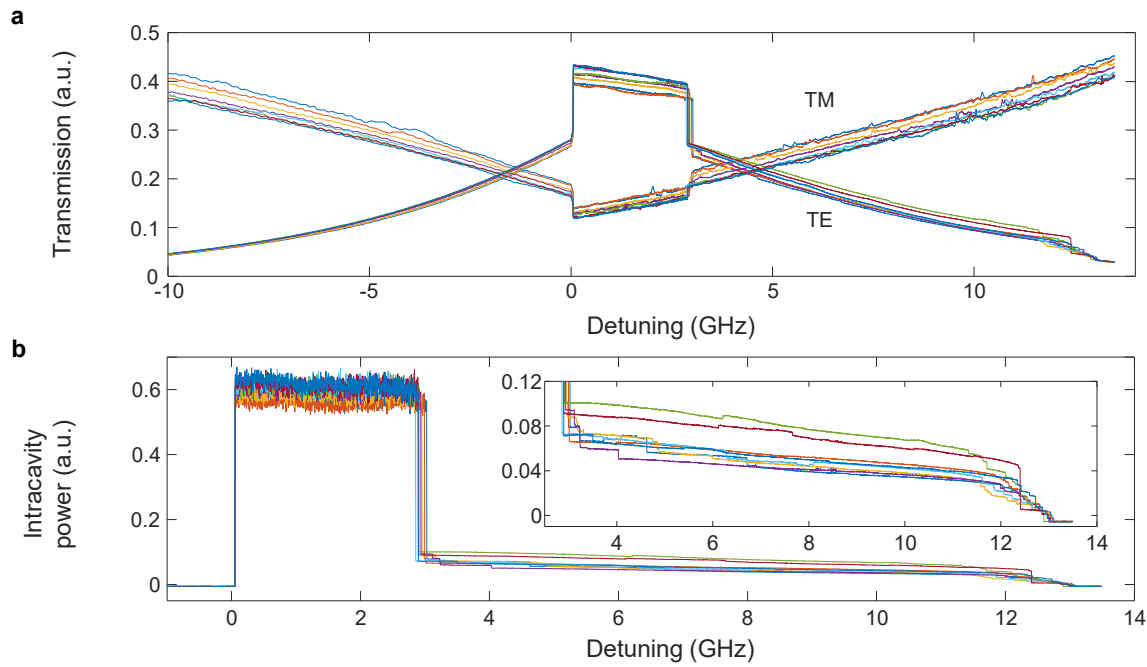
mode spectrum indicates avoided mode crossing and polarization conversion. In contrast to the cold-cavity mode transmission, the hot-cavity mode transmission is measured as shown in Supplementary Figure 3e after pump power amplification with a high power EDFA showing the gradually varied mode-splitting frequency spacing. The varied splitting frequency is related to differential optothermal effects via the relation $\Delta\omega/(2\pi) \approx \mu c/L(T)/(n_{TE}(T) - n_{TM}(T))/2\pi$ where μ is the mode number, c is the light speed in vacuum, $L(T)$ is the physical cavity length, $n_{TE}(T)$ and $n_{TM}(T)$ are the temperature-dependent refractive indexes for TE and TM modes.



Supplementary Figure 3| Thermally-induced mode-pulling measurement. **a**, The experimental setup of the avoided mode coupling and power conversion measurement, EDFA, Erbium-doped fiber amplifier. **b** and **c**, Thermal broadening cavity resonances in TM and TE polarizations indicating the polarization conversion. **d** and **e**, The cold and hot cavity mode transmission indicating the thermally-induced mode-pulling effect.

After the cold and hot cavity mode characterization, the intracavity power of the soliton microcomb is examined by bidirectional pump wavelength tuning in the splitting supermode. The pump laser wavelength is firstly forward tuned to excite modulational instability and chaotic high-noise microcombs. Then, the backward pump laser tuning is utilized to access the thermally-stabilized soliton microcomb states over an extraordinary soliton existence range. Supplementary

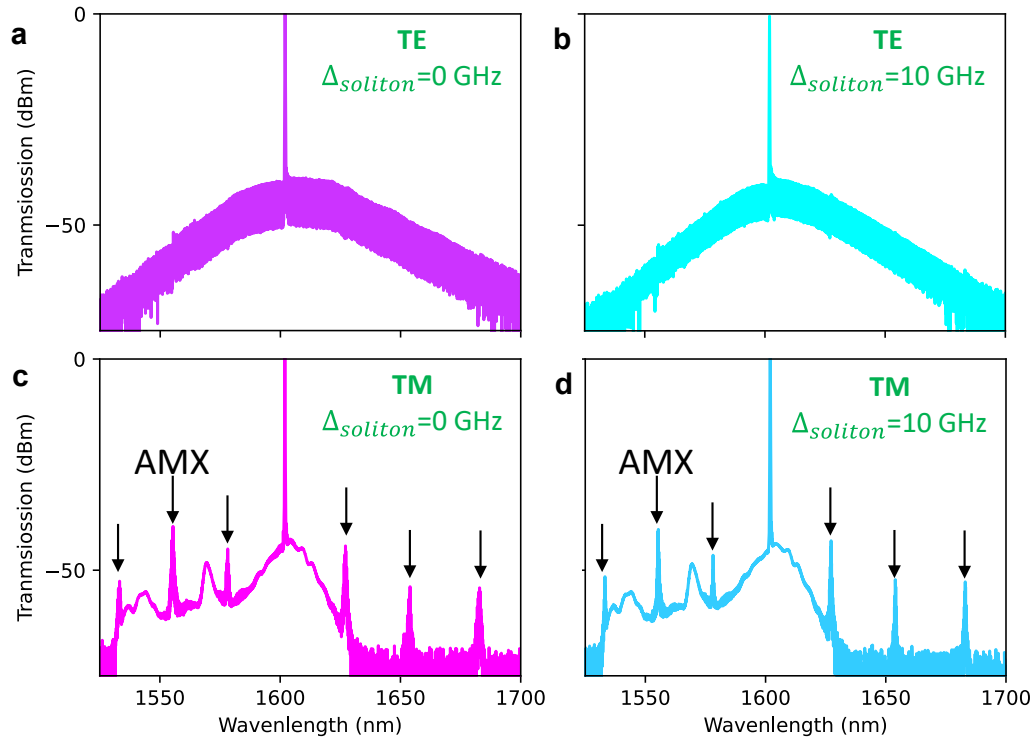
Figure 4a denotes the power transmission in TM and TE polarizations along with pump laser wavelength forward and backward tuning with repeated measurements. The intracavity power switching dynamics are illustrated in Supplementary Figure 4a which is related to thermally-induced polarization conversion processes. The sudden power drop in the TM transmission indicates the strong-coupling between the TM and TE polarizations. Subsequently the pump laser wavelength backward tuning is engaged to decrease the intracavity power in the TM polarization leading to the decrease of the operating temperature of the microresonator. The kicking out power of the TE polarization is associated with the polarization conversion from strong-coupling to weak-coupling. In the soliton microcomb generation, the pump laser stays at the thermally-locked blue detuning side. The regular soliton steps are observed in the TE mode transmission. In addition, the intracavity power evolution in the TE polarization with respect to the pump-resonance detuning is recorded as shown in Supplementary Figure 4b.



Supplementary Figure 4 | Cavity power transmission in TM and TE polarizations and TE-polarized intracavity power evolution. **a**, The power transmission in TE and TM polarizations with respect to the pump-resonance detuning. **b**, The intracavity power in the TE polarization with respect to the pump-resonance detuning showing ≈ 10 GHz soliton existence range.

Furthermore, we observe a formation of periodic pattern at the TM polarization in the TE soliton state which is due to linear power coupling between the two modes where they are frequency degenerate. This is done by monitoring the output of each polarization independently by using a

polarization splitter. Supplementary Figure 5 shows the optical spectrum of both polarizations at the beginning and the end of the single soliton existence range. We observe a periodic power coupling from TE to TM spectrum at different wavelengths that is consistent with the calculated AMX period of ≈ 2.3 THz due to the group index difference between the fundamental TE and TM modes.



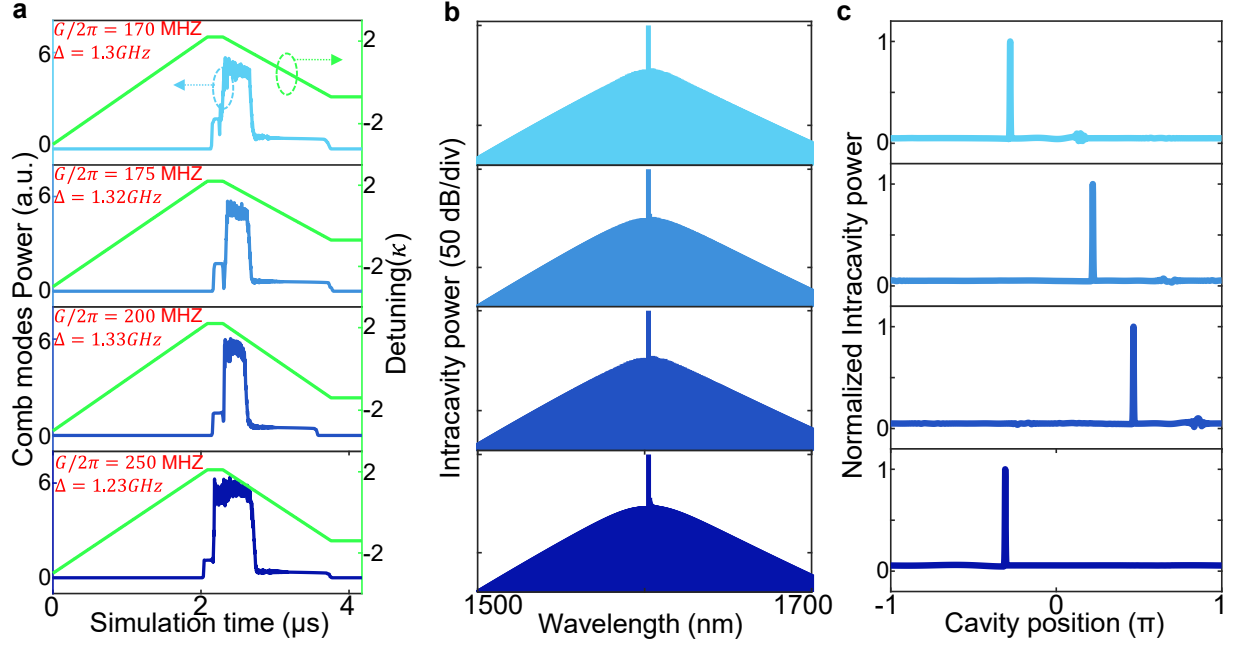
Supplementary Figure 5 | Output spectrum in TE and TM polarizations at different detuning.

Optical spectrums at the beginning of the single soliton existence range for the **a**, TE polarization **b**, TM polarization. Optical spectrums at the end of the single soliton existence range for the **b**, TE polarization **d**, TM polarization. Black arrows indicate the location of the TE-TM AMX in **c** and **d**.

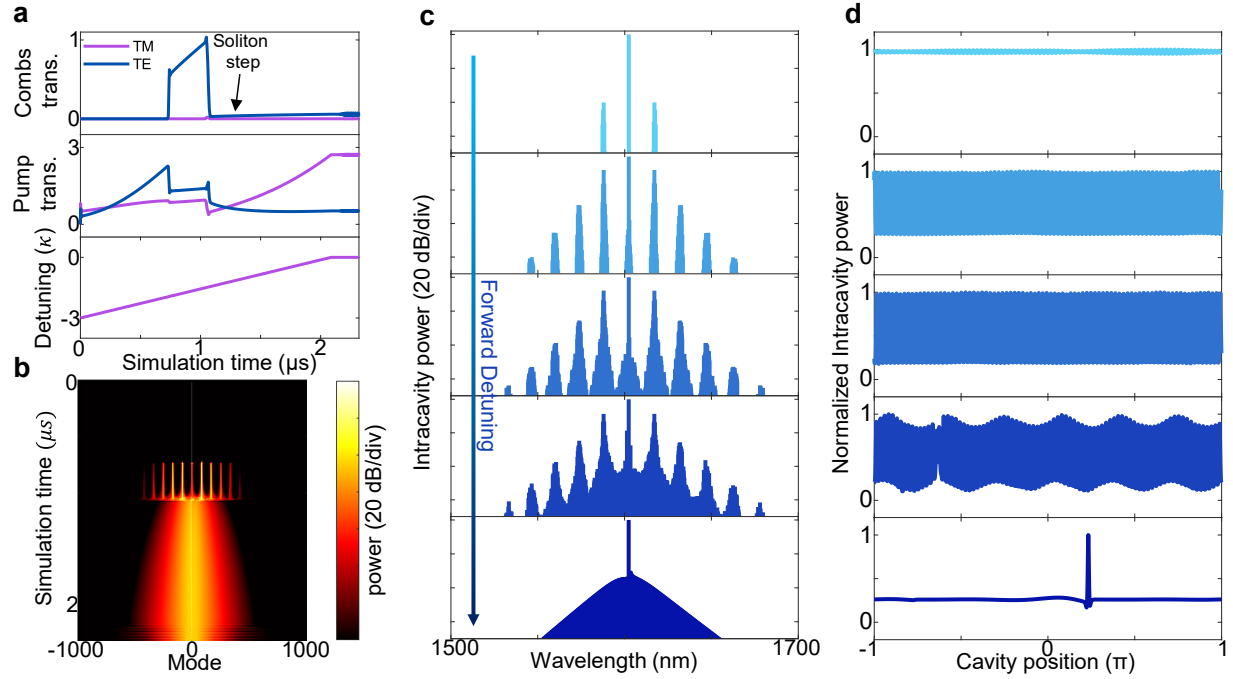
Supplementary Note 3. Additional coupled LLE simulations for different conditions

We investigate soliton generation under different conditions of coupling rate and initial inter-mode detuning with different power that generates a soliton state as demonstrated in Supplementary Figure 6. Single soliton can be achieved for all the conditions. This shows that a single soliton state can be achieved for different coupling rates by choosing the proper power and initial inter-mode detuning.

Moreover, we perform simulations in the absence of thermal effect to investigate the nonlinear behavior only. We observe direct access to single without modulation instability by forward detuning only as illustrated in Supplementary Figure 7. The spectral evolution goes from parametric oscillations to single soliton as shown in Supplementary Figure 7b. Different slices spectrum are shown in Supplementary Figure 7c with the corresponding temporal profile in Figure S7d to verify the absence of chaotic states.



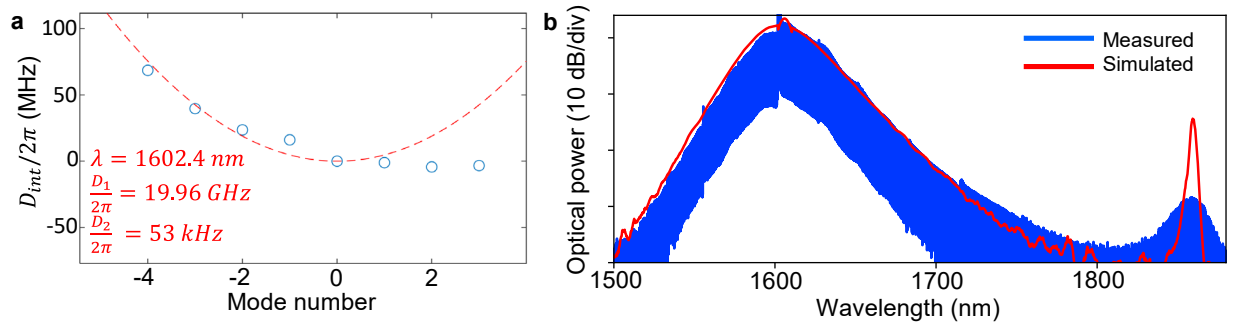
Supplementary Figure 6 | Additional LLE modeling of different coupling rate and TE-TM inter-mode detuning conditions. a, Intracavity transmission and detuning profile. **b,** Optical spectrum of the generated soliton state. **c,** The temporal profile of the generated soliton state in c.



Supplementary Figure 7 | Additional LLE modeling without thermal effects. **a**, TE and TM transmission with the corresponding detuning profile as a function of the simulation time for pump power of 1.4 W and inter-mode detuning of 0.35 GHz. **b**, The evolution of the optical spectrum. **c**, **d**, Spectral and temporal profiles of different simulation times.

Supplementary Note 4. Approximating third-order dispersion using dispersive wave and LLE modeling

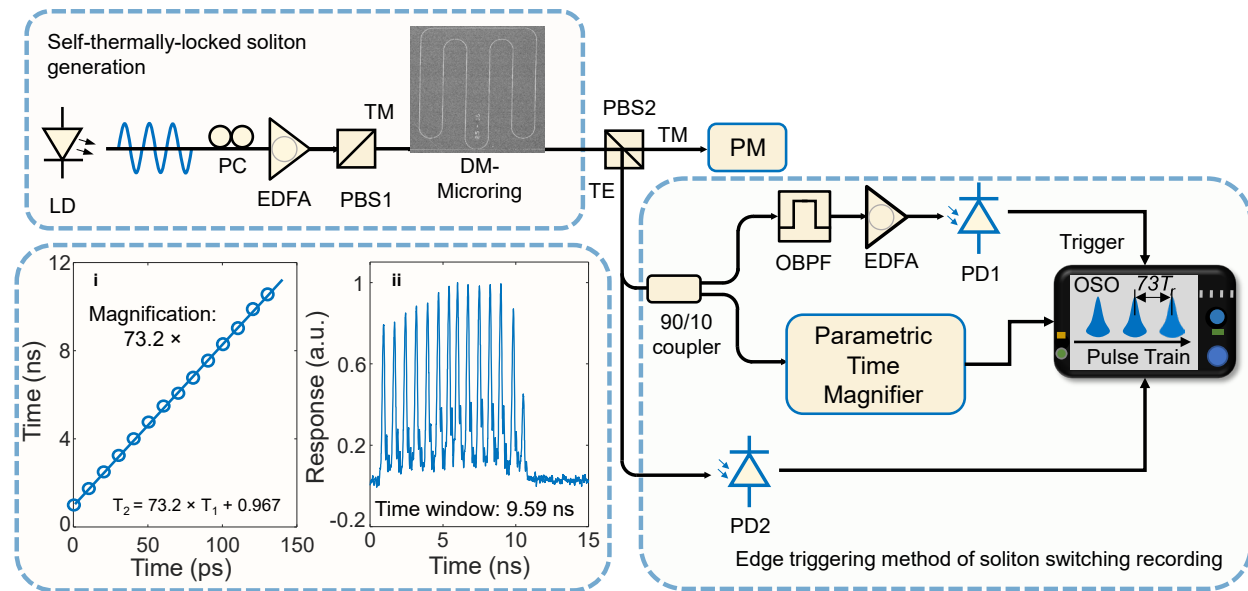
We perform a one-dimensional LLE simulation to approximate the third-order dispersion using the dispersive wave near 1870nm. Supplementary Figure 8a shows the local integrated dispersion around the pump in a strong coupling region where we obtain $\frac{D_2}{2\pi} = 53$ kHz. The obtained simulated spectrum is shown in Supplementary Figure 8a with $\frac{D_2}{2\pi} = 48$ kHz, and $\frac{D_3}{2\pi} = 120$ Hz which represents the best match with the measured optical spectrum.



Supplementary Figure 8 | LLE modeling of a dispersive wave near 1870 nm. a, Local integrated dispersion near the pump mode. **b,** Measured and simulated soliton spectrum using LLE.

Supplementary Note 5. The characterization of the parametric time magnifier

A parametric time magnifier is developed based on four-wave mixing in a highly nonlinear fiber as shown in Supplementary Figure 9. The generated microcomb is selected by a polarization beam splitter and subsequently split into three parts to record average optical power, soliton steps, and real-time soliton transitions. We utilized voltage edges induced by the soliton steps to trigger the parametric time magnifier system to catch the soliton transition dynamics. Insets are the characterization of the magnification and the temporal window of the system measured with a tunable delayed optical pulse. The time magnifier has a magnification of 73.2 and negligible linear or nonlinear time distortion. The effective magnified temporal window can be up to 9.5 ns over each frame.



Supplementary Figure 9 | The experimental setup of real-time soliton transition measurement via a parametric time magnifier. The characteristic intracavity power soliton step triggers the parametric time magnifier to record soliton transitions. Inset **i**: The characterization of the time magnification showing 73 times magnification. Inset **ii**: The magnitude response of the parametric time magnifiers

Supplementary References

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2. X. X. Xue, Y. Xuan, C. Wang, P.-H. Wang, Y. Liu, B. Niu, D. E. Leaird, M. H. Qi, and A. M. Weiner, Thermal tuning of Kerr frequency combs in silicon nitride microring resonators. *Opt. Express* **24**, 687-698 (2016).