

The influence of timescales and data injection schemes for reservoir computing using spin-VCSELs

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

This manuscript has been previously reviewed at another journal. This document only contains information relating to versions considered at Communications Physics.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

I think the authors have done a good job addressing the concerns of the referees and the manuscript is now suitable for publication in Communications Physics.

Reviewer #2

(Remarks to the Author)

The paper reports on a theoretical study of a reservoir computing system using spin-VCSELs, delayed feedback and either optical or carrier injection of data.

The paper has already been reviewed in detail by 4 reviewers, all being rather positive about the work. The authors have given clear answers to the questions raised by the reviewers. Also, they have made appropriate changes in the manuscript to deal with the comments raised by the reviewers. Several paragraphs have been rewritten and new figures have been added to further strengthen the content of the paper. One of the main points of criticism was that the article is rather technical and more suited for a specialized journal, and the transfer to Communication Physics is a logical and suitable choice. Therefore, I recommend the paper to be published in its present form in Communication Physics.

Reviewer #3

(Remarks to the Author)

The authors did nothing in the revised manuscript with respect to my previous comments. The following issues need to be addressed to make the paper suitable for a scientific journal:

1 (major) In essence, this work is a detailed bifurcation study of a class of time-delayed nonlinear dynamical systems. Different data injection schemes correspond to different system settings, so it is not surprising that they can lead to different bifurcation structures. Likewise, for a system with multiple parameters, different values of some parameters would lead to different bifurcation diagrams with respect to the other parameters. While the authors stated repeatedly that a comprehensive understanding of the bifurcation structure of the dynamical system being used as the reservoir is necessary to use the dynamical system's full potential for reservoir computing, this is a rather general and certainly not-wrong statement. What is lacking in the paper is specifics. For example, suppose we wish to predict the spatiotemporal chaotic dynamics in the Kuramoto–Sivashinsky equation, what kind of bifurcation structure of the spin-VCSEL reservoir computing system should be chosen? Suppose we wish to use the spin-VCSEL system for detecting weak signals by filtering out strong noise background, what kind of bifurcation structure is optimal? etc.

The authors should provide numerical support and discussions to address these questions.

2 (major) From the standpoint of numerical simulations, the governing equations of the spin-VCSEL system bear certain similarities to those describing external cavity semiconductor lasers in which a time delay also arises in the electrical field

and is induced by an optical feedback through an external cavity: the Lang and Kobayashi equations. The nonlinear dynamics of external cavity semiconductor lasers have been investigated actively for more than four decades theoretically, computationally, and experimentally. From the point of view of studying the nonlinear dynamical behaviors such as bifurcations, chaos, basins of attraction, crises and critical transitions in systems with time-delayed electrical fields, the present work is rather narrow and routine.

The authors should provide a through discussion of the similarities of their spin-VCSEL system to external cavity semiconductor lasers, citing representative papers in the field.

3 (minor) For the spin-VCSEL reservoir computing system, the dimensions of the input and output signals are two and fixed. This may severely limit the practical utility, as the target systems to be predicted and/or controlled in applications can be diverse.

The authors stated in their rebuttal that the "setup is not limited to that and every higher dimensional input can be used after transforming it into a time series (sequentially feeding in all components of the vector and the next vector etc.)" Numerical evidence/support should be provided.

Reviewer #4

(Remarks to the Author)

In their revised manuscript the authors have answered my previous questions satisfactorily:

- 'edge of chaos' has been replaced by 'edge of bifurcations'

- The topic of the echo state property (fading memory) has been addressed indirectly by discussion the different time scales of the response system. In their reply the authors state that the ESP is given in their system due to the losses in photons and carriers. I didn't see a similar statement in the manuscript, or even a more detailed "proof" using model equations. This is not crucial for accepting the manuscript, because all results presented indicate that the ESP is fulfilled.

- In response to my suggestion to use also more ambitious prediction tasks the authors added simulations for cross predicting Lorenz-63 variables (x to z). The results confirm the observations obtained with small time step prediction of x. This is o.k. for the current manuscript, but I would like to encourage the authors to use more difficult tasks as benchmarks in their future studies. (Cross) Predicting Lorenz-63 dynamics is not very challenging, because it is too low dimensional.

I support the authors' decision to submit this manuscript to a more specialized journal and in my opinion it should be accepted there as it is.

made.

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Dear Camilla Pellegrini,

we want to thank you and the referees for the time and effort that was put into carefully reading our manuscript. Please find below the detailed answers to referees #3. The other three referees suggested publish as is. A red lines version with all the changes indicated was also submitted via the web portal.

We hope our paper now meets the criteria for publication in Communication physics.

Sincerely

Kathy Lüdge (for all authors)

Reviewer #3 (Remarks to the Author):

The authors did nothing in the revised manuscript with respect to my previous comments. The following issues need to be addressed to make the paper suitable for a scientific journal:

Reply: *We apologies that we did not make our responses clear enough. We now indicated the changes give detailed answers:*

1 (major) In essence, this work is a detailed bifurcation study of a class of time-delayed nonlinear dynamical systems. Different data injection schemes correspond to different system settings, so it is not surprising that they can lead to different bifurcation structures.

We politely disagree here. The data injection is a small perturbation and only impacts the bifurcation structure if the field is affected directly (for phase injection). In both cases the overall bifurcation structure stays the same (apart from the edges) and, surprisingly, still the performance is different. Our results show that, besides being aware of the bifurcation structure, it is important to consider how information is fed into the reservoir in order it find regions of optimal performance. We changed the wording of manuscript to make that more clear.

Likewise, for a system with multiple parameters, different values of some parameters would lead to different bifurcation diagrams with respect to the other parameters. While the authors stated repeatedly that a comprehensive understanding of the bifurcation structure of the dynamical system being used as the reservoir is necessary to use the dynamical system's full potential for reservoir computing, this is a rather general and certainly not-wrong statement. What is lacking in the paper is specifics. For example, suppose we wish to predict the spatiotemporal chaotic dynamics in the Kuramoto–Sivashinsky equation, what kind of bifurcation structure of the spin-VCSEL reservoir computing system should be chosen? Suppose we wish to use to spin-VCSEL system for detecting weak signals by filtering out strong noise background, what kind of bifurcation structure is optimal? etc. The authors should provide numerical support and discussions to address these questions.

Reply: *We think there was a misunderstanding about the message of our paper. Our results show that it is important to consider how information is fed into the reservoir in order it find regions of optimal performance. We have shown that, the regions of optimal performance for one task change depending on the input method. **We don't claim** to be able to predict what the regions of optimal performance for arbitrary task are. We have shown in previous publications that every task has different requirements (in terms of*

nonlinearity and memory) and the reservoir has to be adjusted to those. Thus we cannot give an answer to the specific questions about the two specific tasks mentioned by the referee. However, **we shed light on another piece of the puzzle:** There are two main ingredients to solving reservoir computing tasks; nonlinearity and memory. We have shown in numerous papers, that the memory needs can be, for the most part, relegated to the input or output layers, and that the most important function of the reservoir is to perform the nonlinear transform of the input data. The results in the current manuscript highlight that these nonlinear transform capabilities are strongly influence by the methods with which data is fed into the reservoir. This is due to the nonlinear dynamics of the system and is **a task independent result**.

What we show in the present paper is the strength in exploiting the bifurcation structure of a given dynamical system to achieve this tunability if used as a reservoir setup.

We have added the following text to page 2 of the manuscript.

"We have chosen this Lorenz task as it does not require much memory [54] and therefore allows us to investigate the influence of the spin-VCSEL dynamics on the nonlinear transform capabilities of the reservoir. For tasks requiring memory on time scales of multiple clock-cycles, memory inducing effects can be incorporated in the input or output layers [51, 54, 56, 60]."

We have added the following text to the manuscript.

"We emphasize that the change in the optimal parameter region arises due to the difference in the dynamical response of the reservoir when data is injected either via the phase or the charge carrier difference, not due to the task we have used to demonstrate our results. "

We also added results for a different task to the supplementary to support our claims.

2 (major) From the standpoint of numerical simulations, the governing equations of the spin-VCSEL system bear certain similarities to those describing external cavity semiconductor lasers in which a time delay also arises in the electrical field and is induced by an optical feedback through an external cavity: the Lang and Kobayashi equations. The nonlinear dynamics of external cavity semiconductor lasers have been investigated actively for more than four decades theoretically, computationally, and experimentally. From the point of view of studying the nonlinear dynamical behaviors such as bifurcations, chaos, basins of attraction, crises and critical transitions in systems with time-delayed electrical fields, the present work is rather narrow and routine. The authors should provide a **through discussion of the similarities of their spin-VCSEL system to external cavity semiconductor lasers**, citing representative papers in the field.

Reply: We added explanations about the similarities of the terms included to model the feedback and the injection.

-On page 8 within the methods section we added "This term is equivalent to the feedback term originally introduced by Lang and Kobayashi [61]. Its effect on the dynamics of a single mode semiconductor laser has been widely explored [64,68, 86, 87]. For our two-mode VCSEL we feedback both modes back into the cavity."

-about the injection term (page 8 Methods section) we state that this setup was widely used before:" The effect of a combination of feedback and injection on the bifurcation

structure is less studied but works exist for conventional semiconductor lasers [62, 76] and quantum-dot lasers [65]."

-in the main text we state that the additional carrier dynamics sets the spin-VCSEL apart from regular laser systems

The message of the paper is that it is important to consider how the nonlinear dynamics or a reservoir can be utilised and that the data injection scheme plays a significant role in this. This is stated, clearly in our abstract and discussion:

"We present numerical evidence of the critical impact of different data injection schemes and internal timescales. A central finding is that the internal dynamics of all dynamical degrees of freedom can only be utilized if an appropriate perturbation via the input is chosen as data injection scheme."

"In order to optimally use the dynamical degrees of freedom of a physical reservoir, the specific response to perturbations from data input has to be taken into account, especially when there are processes within a reservoir which evolve at very different timescales."

We also added a paragraph at the end of the Introduction to emphasis the **main message**.

3 (minor) For the spin-VCSEL reservoir computing system, the dimensions of the input and output signals are two and fixed. This may severely limit the practical utility, as the target systems to be predicted and/or controlled in applications can be diverse.

The authors stated in their rebuttal that the "setup is not limited to that and every higher dimensional input can be used after transforming it into a time series (sequentially feeding in all components of the vector and the next vector etc.)" Numerical evidence/support should be provided.

*In the context of time-multiplexed reservoir computing, this aspect is indeed irrelevant. The data that is fed into the reservoir is a 1 dimensional sequence of numbers. This sequence can be generated by adding together an arbitrary number of weighted (masked) input features. This is analogous to a network node being fed multiple weighted inputs. To make this clearer we have **reformulated the relevant section** of the methods (see Input data and prediction task) and **stated explicitly** what needs to be done for higher dimensional inputs.*

We also added references (from our group and others) where higher dimensional input has been used.