

Supplementary Information for Quantum origin of anomalous Floquet phases in cavity-QED materials

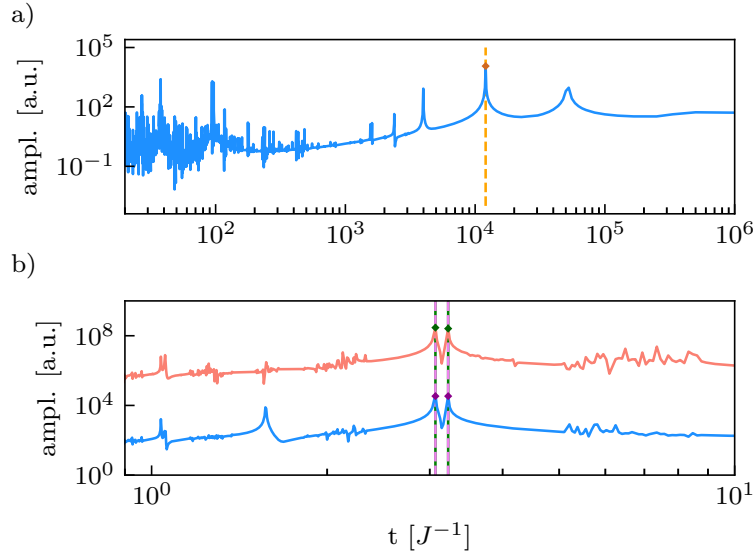
(Dated: November 25, 2024)

Supplementary Note 1: Fourier analysis of light-matter dynamics

To investigate the time-periodic behaviour of the occupation probabilities shown in the main text, we applied a Fourier transform to the data. This allows us to decompose the temporal evolution of $P_1^{\text{ch}}(t)$ and $P_n^{\text{ph}}(t)$ into its constituent frequency components, thereby providing a robust framework for identifying the dominant contributions to the system's dynamics. These frequencies correspond to underlying physical energy scales that are relevant to the problem.

In Supplementary Fig. 1 we depict the power spectrum in time domain for each subplot in Fig 7 in the main text. Each peak corresponds to a dominant frequency associated with a characteristic period $T = 1/f$. The larger peaks correspond to those oscillation periods that have a bigger contribution, and hence dominate the resulting dynamics. They are highlighted through a vertical, dashed line.

We can identify two different oscillation patterns in the dynamics. First, harmonic oscillations, characterized by discrete, well-defined peaks. In the time-dependent signal, this corresponds to periodic processes with regular oscillations. This is the case of Supplementary Fig. 1a), where the largest peak appearing in the power spectrum corresponds to the oscillation of the charge between ending sites of the chain, induced by the energy splitting between single-particle edge states. Second, a beating pattern arise when two close frequencies interfere, creating an oscillatory envelope that modulates the signal. This is in fact shown in Supplementary Fig. 1b), where the slight energy differences between the single-particle edge states and the anomalous edge states hosted in the upper and lower resonant gap give rise to two nearby oscillation periods.



Supplementary Figure 1. Fourier analysis, as given by the power spectrum in time domain, of charge dynamics (blue), represented by $P_1^{\text{ch}}(t)$, and photon dynamics (red), represented by $P_{n=0}^{\text{ph}}(t)$ (the dynamics data are plotted in Fig. 7 of the main text). The position of the main peaks (i.e., dominant frequency in the dynamics) has been indicated with a dashed, vertical line, in both subplots. In subplot a), there is a main oscillation period corresponding to the charge oscillating between ending sites of the chain. In subplot b), there is a double peak, corresponding to two nearby oscillation periods, that are responsible for the beating pattern shown in Fig. 7. All parameters are chosen as in the main text: $\Omega = 4$, $J' = 2$, $J = 1$, $g = 0.35$, $N = 8$, $n_{\text{max}} = 50$