

Nested Chirality and Multimode Transfer Around Exceptional Points Driven by External Forces in Electric Circuit Settings

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

This manuscript has been previously reviewed at another journal. This document only contains information relating to versions considered at Communications Physics.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The paper by Naqiao Pan et al. is an experimental work on multimode transfer around exceptional points (EPs) in a circuit system. By applying external forces to a 2D non-Hermitian higher-order topological insulator model, the authors have realized dynamically encircling two degenerate order-2 EPs. Different evolutions and nested chiral behavior have been investigated throughout, and then a circuit experiment is performed to demonstrate all the theoretical predictions. The results are interesting, and I recommend its publication in Communication Physics. The following are two concerns and suggestions:

(1) The authors claim that the twofold degenerate order-2 EPs are the fourth-order EPs, which I am afraid I have to disagree with. Besides the eigenfrequencies, the fourth-order EP also requires three defective states, but the twofold degenerate order-2 EPs only have two defective states. Therefore, I highly suggest the authors carefully consider revising this term, and the term of the twofold degenerate order-2 EPs does not destroy any interesting results here.

(2) The nested chiral behavior itself has been demonstrated clearly both experimentally and theoretically, but the word “nested” shall be further clarified. I believe the authors use this word inspired by concepts, such as the nested Fermi surface, which means one segment of a Fermi surface can be connected to another segment in the Fermi surface via the reciprocal lattice vector. Here, the “nested” seemingly means the two loops can be treated as effectively connected based on the EPs. Please clarify this in the main text.

Reviewer #2

(Remarks to the Author)

The manuscript titled “Nested chirality and multimode transfer around exceptional point driven by external forces” by N. Pan et al. presents an intriguing study on nested chirality in a two-dimensional non-Hermitian lattice. The implementation of these results on an electric circuit platform is novel and merits consideration for publication. However, I have several concerns that I hope the authors will address before I can make a final recommendation.

Regarding Fig. 2 and the main conclusion of this manuscript, a key parameter is the evolution of $|c_j(t)|^2$. The comments 1-4 below are related to this.

1. In line 119, the manuscript mentions “a superposition of the eigenstates.” Please clarify that the $\psi_i(t)$ here refer to the instantaneous eigenstates, rather than the eigenstates at the initial time.

2. Did the authors consider the static eigenstate switching when encircling an EP while labeling ψ_i in Fig. 2 and throughout the manuscript? To clarify, consider a 2-by-2 Hamiltonian as an example. When encircling a second-order EP, the relationship between instantaneous eigenstates can result in $\psi_2(t=T)=\psi_1(t=0)$, $\psi_1(t=T)=\psi_2(t=0)$ (neglecting the geometric phase) if the labelling 1 and 2 are chosen properly. Therefore, if the initial input state is $\psi(t=0)=\psi_1(t=0)$ ($|c_1(t=0)|^2=1$, $|c_2(t=0)|^2=0$), and after encircling the EP, it evolves to $\psi_2(t=T)$ ($|c_1(t=0)|^2=0$, $|c_2(t=0)|^2=1$) due to a non-adiabatic transition. It might seem like a transition from $\psi_1 \rightarrow \psi_2$ has occurred. However, since the final output state is identical to the initial state $\psi_1(t=0)$, the notation might cause confusion. Please clarify the meaning of ψ_4 , ψ_2 , in

contexts such as line 156.

3. In Fig. 2, the color discontinuities at $k(t)=\pi$ (or $-\pi$) for example, the green to yellow in the lower left panel of Fig. 2(a) and the red to blue in the lower left panel of Fig. 2(b)—are described as an “adiabatic state flip.” My understanding is that an adiabatic state flip typically occurs after completing one loop, when the system returns to its initial Hamiltonian, as discussed in comment 2. Additionally, I believe the color discontinuity results from the labeling of the subscript i when monitoring the instantaneous eigenstates, which is a human-imposed factor and does not reveal any physical phenomenon.

4. Regarding the nested chirality discussed in Fig. 3, such as the first example in Fig. 3(a) where ψ_1 transitions to ψ_2 after one loop and back to ψ_1 after two loops, there may be labeling confusion. After addressing the previous comments, could the authors provide the numerical values of the final states after one and two loops in the response letter to confirm the nested chirality?

5. In line 103, what is the eigenstate at such EP? Could the authors confirm and provide evidence that this is indeed a fourth-order EP?

6. Concerning the non-adiabatic coupling detailed in Supplementary Note 2, the authors refer to references [3,4] and calculate A_{ij} using Eq. (S5). Could the authors clarify the relationship between this method and the one described in Phys. Rev. A 92, 052124 (2015), specifically Eqs. (7) and (8) in this paper?

7. In line 262, could the authors clarify whether the Hamiltonian H_e is time-independent? If it is, does this imply that the encircling occurs in momentum-space rather than real-space?

8. In the top middle panel of Fig. 4(a), could the authors clarify the difference between the dashed-line arrows and the solid-line arrows (in the colors red, blue, and yellow)?

Reviewer #3

(Remarks to the Author)

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of a Communications Physics initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Reviewer #4

(Remarks to the Author)

The paper, titled “Nested chirality and multimode transfer around exceptional point driven by external forces,” investigates controlling exceptional points (EPs) in non-Hermitian systems. The authors propose generating EPs and realizing multimode transfer by applying external forces to a lattice system. They demonstrate that the dynamical encircling of a fourth-order EP can be controlled by applying external forces in different directions in a 2D non-Hermitian higher-order topological insulator implemented in electric circuit settings.

The authors studied this theoretically by solving the eigenequation to find the energy bands and analyzing their behavior under perturbation of “external force”. They also explored the dynamical behavior of the system on the path encircling the fourth-order EP. The main finding is that the largest occupied state experiences a non-adiabatic transition (NAT) and an adiabatic state flip during counterclockwise encircling, while the results are different for clockwise encircling. Their findings were confirmed experimentally in a finite real-space lattice system using a designed classical electric circuit network.

The paper is well-written and structured. However, some clarification is needed regarding the following statements:

“Due to the complex non-trivial topology of Riemann surfaces associated with non-Hermitian Hamiltonians, the dynamical encirclement or proximity of EPs in parameter space has been shown to lead to some novel physical phenomena. However, in the previous studies on dynamically encircling EPs, tuning the phenomena for applications requires modulation of the system parameters, which is very difficult to be realized as the systems have been constructed. Here, we propose theoretically and demonstrate experimentally a new way to generate EPs and realize asymmetric multimode switching by tuning external forces without modifying the system parameters.”

The authors claim that modulating system parameters for dynamical encircling in previous studies was difficult. However, the works that deals with similar problems but in photonics platforms, for example:

1. Regensburger, A., Bersch, C., Miri, M.A. et al. Parity–time synthetic photonic lattices. *Nature* 488, 167–171 (2012). <https://doi.org/10.1038/nature11298>

2. Nasari, H., Lopez-Galmiche, G., Lopez-Aviles, H.E. et al. Observation of chiral state transfer without encircling an exceptional point. *Nature* 605, 256–261 (2022). <https://doi.org/10.1038/s41586-022-04542-2>

3. Leefmans, C.R., Parto, M., Williams, J. et al. Topological temporally mode-locked laser. *Nat. Phys.* 20, 852–858 (2024). <https://doi.org/10.1038/s41567-024-02420-4>

demonstrate successful parameter modulation using external forces (e.g., intensity, phase modulators, variable couplers) in photonic systems. These methods effectively modulate the Hamiltonian operator, similar to changing the on-site potential “external forces” in the electric circuit system proposed by the authors of the paper under the review.

In essence, both approaches modify the system parameters (Hamiltonian) to achieve dynamical encircling. While the platform (electric circuit vs. photonics) differs, the underlying principle remains the same. The key novelty of this work in my opinion lies in demonstrating control over EPs and multimode switching using external forces in a specific platform (electric circuits).

Moreover, the statement “Here, we propose theoretically and demonstrate experimentally a new way to generate EPs, and realize asymmetric multimode switching by tuning external forces without modifying the system parameters.” Is also not precise and needs to be revised. By applying the external force F authors indeed are changing the system parameters because they modify the total system Hamiltonian operator H_e - the circuit Hamiltonian (line 262 in the manuscript) as authors precisely describe in their work:

Moreover, under the light of Regensburger et al., Nasari et al., Leefmans et al. papers, the statement “... requires modulation of the system parameters, which is very difficult to be realized” is not necessarily precise and in my opinion needs to be revised.

It would be also beneficial if the authors could elaborate more on the potential applications with specific examples of non-Hermitian systems, as mentioned in their conclusions:
“Thus, our work paves a new way to explore non-Hermitian dynamics and develop applications of non-Hermitian systems.”

Finally, in my opinion, the authors should clarify also title to be more precise, for example:
“Nested Chirality and Multimode Transfer Around a Second-Order Exceptional Point Driven by External Forces in Electric Circuit Settings”

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The authors have addressed my concerns, and I now recommend its publication in its current form.

Reviewer #2

(Remarks to the Author)

The authors have responded to my questions and made corresponding revisions to the manuscript. I agree with its acceptance by Communications Physics.

Reviewer #4

(Remarks to the Author)

I am satisfied with the revised version and the responses to my comments.

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Reply to Reviewer 1

First, we would like to thank the reviewer for bringing us useful comments and suggestions to improve the manuscript. All suggestions have been implemented in the revised manuscript. Here we would like to give reply to these comments in detail.

Overall Comment: *The paper by Naiqiao Pan et al. is an experimental work on multimode transfer around exceptional points (EPs) in a circuit system. By applying external forces to a 2D non-Hermitian higher-order topological insulator model, the authors have realized dynamically encircling two degenerate order-2 EPs. Different evolutions and nested chiral behavior have been investigated throughout, and then a circuit experiment is performed to demonstrate all the theoretical predictions. The results are interesting, and I recommend its publication in Communication Physics. The following are two concerns and suggestions:*

Reply: Many thanks to the reviewer for the positive comments. Next, we will give one-to-one replies to the questions raised by the reviewer.

Comment 1: *The authors claim that the twofold degenerate order-2 EPs are the fourth-order EPs, which I am afraid I have to disagree with. Besides the eigenfrequencies, the fourth-order EP also requires three defective states, but the twofold degenerate order-2 EPs only have two defective states. Therefore, I highly suggest the authors carefully consider revising this term, and the term of the twofold degenerate order-2 EPs does not destroy any interesting results here.*

Reply: Thanks for the suggestions. After we carefully calculate the eigenstates of system under the parameter $k_x=k_y=0$ and $q=\gamma-\lambda$, and find that the eigenstates are $v_1=(0;0;-1;1)$, $v_2=(0;1;0;0)$ and $v_3=v_4=(0;0;0;0)$. So there are only two defective states and the EPs should be called “the twofold degenerate second-order EPs”. We have corrected the relevant descriptions in the revised manuscript. Relevant calculation results are included in Supplement Note 1.

Comment 2: *The nested chiral behavior itself has been demonstrated clearly both experimentally and theoretically, but the word “nested” shall be further clarified. I believe the authors use this word inspired by concepts, such as the nested Fermi surface, which means one*

segment of a Fermi surface can be connected to another segment in the Fermi surface via the reciprocal lattice vector. Here, the “nested” seemingly means the two loops can be treated as effectively connected based on the EPs. Please clarify this in the main text.

Reply: Thanks for the suggestions. The word “nested” in the study of the nested Fermi surface is often used to describe a connection between one segment of Fermi surface and another segment of the Fermi surface via the reciprocal lattice vector [53-55]. Here, in our work, the two-loops circling the EP display the chiral evolution. This two-loops evolution is composed of two segments of chiral evolutions in which each segment contains one-loop circling the EP. We summarize the corresponding chiral behaviors in Table 1.

Table 1

Initial State	$t = T$		$t = 2T$	
				
$ \psi_1\rangle$	$ \psi_2\rangle$	$ \psi_3\rangle$	$ \psi_1\rangle$	$ \psi_4\rangle$
$ \psi_2\rangle$	$ \psi_1\rangle$	$ \psi_4\rangle$	$ \psi_2\rangle$	$ \psi_3\rangle$
$ \psi_3\rangle$	$ \psi_1\rangle$	$ \psi_4\rangle$	$ \psi_2\rangle$	$ \psi_3\rangle$
$ \psi_4\rangle$	$ \psi_2\rangle$	$ \psi_3\rangle$	$ \psi_1\rangle$	$ \psi_4\rangle$

Table 1 and related descriptions are added to page 10 of the revised manuscript.

Within the one-loop ($0 < t < T$) circling the EP, for initial state $|\psi_1\rangle$ ($|\psi_4\rangle$), the output states after counter-clockwise circling the EP and clockwise circling the EP are $|\psi_2\rangle$ and $|\psi_3\rangle$, respectively. Therefore, no matter the initial state $|\psi_1\rangle$ or $|\psi_4\rangle$, the output states only depend on the encircling direction after one-loop circling the EP. Similar results can be found for initial states $|\psi_2\rangle$ and $|\psi_3\rangle$. As a comparison, when treating the two-loop ($0 < t < 2T$) as one period, the chirality also can be found. For initial state $|\psi_1\rangle$ ($|\psi_4\rangle$), the output states after counter-clockwise circling the EP and clockwise circling the EP are $|\psi_1\rangle$ and $|\psi_4\rangle$, respectively. Similar results can be also found for initial states $|\psi_2\rangle$ and $|\psi_3\rangle$. In the revised manuscript, we have added Table 1 and corresponding descriptions.

Reply to Reviewer 2

First, we would like to thank the reviewer for bringing us useful comments and suggestions to improve the manuscript. All suggestions have been implemented in the revised manuscript. Here we would like to give reply to these comments in detail.

Overall Comment: *The manuscript titled “Nested chirality and multimode transfer around exceptional point driven by external forces” by N. Pan et al. presents an intriguing study on nested chirality in a two-dimensional non-Hermitian lattice. The implementation of these results on an electric circuit platform is novel and merits consideration for publication. However, I have several concerns that I hope the authors will address before I can make a final recommendation.*

Reply: Many thanks to the reviewer for the positive comments. Next, we will give one-to-one replies to the questions raised by the reviewer.

Comment 1: *Regarding Fig. 2 and the main conclusion of this manuscript, a key parameter is the evolution of $|c_j(t)|^2$. The comments 1-4 below are related to this.*

In line 119, the manuscript mentions “a superposition of the eigenstates.” Please clarify that the $|\psi_i(t)$ here refer to the instantaneous eigenstates, rather than the eigenstates at the initial time.

Reply: Thanks for the suggestion. In the end of Page 5 of the revised manuscript, we have added a clarification on the $|\psi_i\rangle$ as “In this paper, $|\psi_i\rangle$ ($i = 1, 2, 3, 4$) refer to the instantaneous eigenstates, rather than the eigenstates at the initial time.”.

Comment 2: *Did the authors consider the static eigenstate switching when encircling an EP while labeling ψ_i in Fig. 2 and throughout the manuscript? To clarify, consider a 2-by-2 Hamiltonian as an example. When encircling a second-order EP, the relationship between instantaneous eigenstates can result in $\psi_2(t=T)=\psi_1(t=0)$, $\psi_1(t=T)=\psi_2(t=0)$ (neglecting the geometric phase) if the labelling 1 and 2 are chosen properly. Therefore, if the*

initial input state is $\psi(t=0) = \psi_1(t=0)$ ($|c_1(t=0)|^2=1$, $|c_2(t=0)|^2=0$), and after encircling the EP, it evolves to $\psi_2(t=T)$ ($|c_1(t=0)|^2=0$, $|c_2(t=0)|^2=1$) due to a non-adiabatic transition. It might seem like a transition from $\psi_1 \rightarrow \psi_2$ has occurred. However, since the final output state is identical to the initial state $\psi_1(t=0)$, the notation might cause confusion. Please clarify the meaning of ψ_1 , ψ_2 , in contexts such as line 156.

Reply: We consider the dynamical encircling results throughout the manuscript. In fact, since the instantaneous eigenstates are only dependent on the parameters, they will not change under the same parameters ($t=0$ and $t=T$). So the statement “ $\psi_2(t=T)=\psi_1(t=0)$, $\psi_1(t=T)=\psi_2(t=0)$ ” is not accurate.

Taking a 2-by-2 Hamiltonian as an example, the initial state $|\psi(t=0)\rangle = |\psi_1\rangle$ ($|c_1(t=0)|^2=1$, $|c_2(t=0)|^2=0$) will evolve to $|\psi(t=T)\rangle = |\psi_2\rangle$ ($|c_1(t=T)|^2=0$, $|c_2(t=T)|^2=1$) after encircling the EP if the non-adiabatic transition does not happen. Here, the final state $|\psi(t)\rangle$ changes from $|\psi_1\rangle$ to $|\psi_2\rangle$ due to the adiabatic evolution and walking across the branch cut. If the non-adiabatic transition happens, the initial state $|\psi(t=0)\rangle = |\psi_1\rangle$ ($|c_1(t=0)|^2=1$, $|c_2(t=0)|^2=0$) will first experience a non-adiabatic transition due to the non-adiabatic coupling, which leads to $|c_1(t=t_1)|^2 < |c_2(t=t_1)|^2$ at a time t_1 . Then, $|\psi(t)\rangle$ experiences the adiabatic evolution, and traverses the branch cut. The final evolutionary result is $|\psi(t=T)\rangle = |\psi_1\rangle$ ($|c_1(t=0)|^2=1$, $|c_2(t=0)|^2=0$). In our manuscript, the evolutionary process and the labeling are consistent with the above processes. We have added more detailed descriptions in Page 7 of the revised manuscript.

Comment 3: In Fig. 2, the color discontinuities at $k(t)=\pi$ (or $-\pi$) for example, the green to yellow in the lower left panel of Fig. 2(a) and the red to blue in the lower left panel of Fig. 2(b)—are described as an “adiabatic state flip.” My understanding is that an adiabatic state flip typically occurs after completing one loop, when the system returns to its initial Hamiltonian, as discussed in comment 2. Additionally, I believe the color discontinuity results

from the labeling of the subscript i when monitoring the instantaneous eigenstates, which is a human-imposed factor and does not reveal any physical phenomenon.

Reply: We thank the reviewer for the suggestion. In our definition for the instantaneous eigenstates, the subscript changes when the momentum crosses the branch cut of the bands. So we use the description of “adiabatic state flip” to distinguish it from the non-adiabatic transitions in other positions. In order to avoid misunderstanding, we change the description of the corresponding process in Page 7 of the revised manuscript as “Then, the wave packet changes from $|\psi_4\rangle$ to $|\psi_2\rangle$ (yellow dotted line) when walking across the branch cut. The later evolution process is adiabatic and the state is kept to be $|\psi_2\rangle$.”.

Comment 4: *Regarding the nested chirality discussed in Fig. 3, such as the first example in Fig. 3(a) where ψ_1 transitions to ψ_2 after one loop and back to ψ_1 after two loops, there may be labeling confusion. After addressing the previous comments, could the authors provide the numerical values of the final states after one and two loops in the response letter to confirm the nested chirality?*

Reply: Here are the numerical values of the final states after one and two loops with the initial state $|\psi(t=0)\rangle = |\psi_1\rangle = (0.1508; -1.6583; 0; 0.7071)$. When encircling counterclockwise, the final state at the time $t=T$ is $|\psi(t=T)\rangle \approx (10394; 114987; 48891; 140)$ which is approximated as the second instantaneous eigenstate $|\psi_2\rangle = (0.1508; -1.6583; 0.7071; 0)$ at the time $t=T$; the final state at the time $t=2T$ is $|\psi(t=2T)\rangle \approx i(-19136; 211708; 257; -90015)$ which is approximated as the first instantaneous eigenstate $|\psi_1\rangle = (0.1508; -1.6583; 0; 0.7071)$ at the time $t=2T$.

When starting from the initial state $|\psi(t=0)\rangle = |\psi_1\rangle = (0.1508; -1.6583; 0; 0.7071)$ and encircling clockwise, the final state at the time $t=T$ is $|\psi(t=T)\rangle \approx (3634182 + 107441i; -40205312 - 1181843i; -48891; 17094715 + 503940i)$ which is approximated as the third instantaneous eigenstate

$|\psi_3\rangle = (0.1508; -1.6583; 0; 0.7071)$ at time $t = T$; the final state at the time $t = 2T$ is

$$|\psi(t = 2T)\rangle \approx (-197814 + 6691081i; -2175952 + 74024104i; -927830 + 31473969i; -90015i)$$

which is approximated as the fourth instantaneous eigenstate

$$|\psi_4\rangle = (0.1508; 1.6583; 0.7071; 0) \text{ at the time } t = 2T.$$

Comment 5: *In line 103, what is the eigenstate at such EP? Could the authors confirm and provide evidence that this is indeed a fourth-order EP?*

Reply: Thanks for the suggestion. We have calculated the eigenstates at the EP under the parameter $k_x = k_y = 0$ and $q = \gamma - \lambda$, and found that the eigenstates are $v_1 = (0; 0; -1; 1)$, $v_2 = (0; 1; 0; 0)$ and $v_3 = v_4 = (0; 0; 0; 0)$. So there are only two defective states and the EPs should be called “the twofold degenerate second-order EPs”. We have corrected the relevant descriptions in the revised manuscript. In addition, the results of our work will not be affected by this. Relevant calculation results are included in Supplement Note 1.

Comment 6: *Concerning the non-adiabatic coupling detailed in Supplementary Note 2, the authors refer to references [3,4] and calculate A_{ij} using Eq. (S5). Could the authors clarify the relationship between this method and the one described in Phys. Rev. A 92, 052124 (2015), specifically Eqs. (7) and (8) in this paper?*

Reply: We take the two eigenstates (Eq. (5)) of the paper (Phys. Rev. A 92, 052124 (2015)) into our Eq. (S5) to calculate the non-adiabatic coupling, as

$$A_{+,-} = \left\langle l_+ \left| \frac{\partial}{\partial t} \right| r_- \right\rangle = \frac{1}{2} \dot{v}$$

with $\tan v = -g / (\omega + i\gamma / 2)$. Further calculation shows that

$$\begin{aligned} A_{+,-} &= \frac{1}{2} \cdot \frac{1}{1 + [-g / (\omega + i\gamma / 2)]^2} \cdot \frac{-\dot{g}(\omega + i\gamma / 2) + g(\dot{\omega} + i\dot{\gamma} / 2)}{(\omega + i\gamma / 2)^2} \\ &= \frac{-\dot{g}(\omega + i\gamma / 2) + g(\dot{\omega} + i\dot{\gamma} / 2)}{2\lambda^2} \end{aligned}$$

which is just the result of Eq. (8) in this paper. So we used the same method as this paper. We have added Phys. Rev. A 92, 052124 (2015) to the references of the Supplementary Materials.

Comment 7: *In line 262, could the authors clarify whether the Hamiltonian H_e is time-independent? If it is, does this imply that the encircling occurs in momentum-space rather than real-space?*

Reply: Yes. The Hamiltonian H_e is time-independent. The encircling process takes place in momentum-space, as we describe in the theory section. But we observe the corresponding phenomenon through the evolution of the real-space wave packet. We have emphasized such a point in Page 11 of the revised manuscript.

Comment 8: *In the top middle panel of Fig. 4(a), could the authors clarify the difference between the dashed-line arrows and the solid-line arrows (in the colors red, blue, and yellow)?*

Reply: In the top middle panel of Fig. 4(a), which illustrates the circuit design for the HOTI model, the dashed-line arrows and the solid-line arrows represent circuit connections with positive and negative hopping terms, respectively. Specifically, for two hopping terms with the same parameters (same colors) but opposite signs, they are achieved by circuit modules composed of the same components but connected in opposite directions. We have made clarifications in Page 12 of the revised manuscript.

Reply to Reviewer 3

Overall Comment: *I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of a Communications Physics initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.*

Reply: we would like to thank the reviewer #3 co-reviewed this manuscript with one of the reviewers

Reply to Reviewer 4

First, we would like to thank the reviewer for bringing us useful comments and suggestions to improve the manuscript. All suggestions have been implemented in the revised manuscript. Here we would like to give reply to these comments in detail.

Overall Comment: *The paper, titled "Nested chirality and multimode transfer around exceptional point driven by external forces," investigates controlling exceptional points (EPs) in non-Hermitian systems. The authors propose generating EPs and realizing multimode transfer by applying external forces to a lattice system. They demonstrate that the dynamical encircling of a fourth-order EP can be controlled by applying external forces in different directions in a 2D non-Hermitian higher-order topological insulator implemented in electric circuit settings.*

The authors studied this theoretically by solving the eigenequation to find the energy bands and analyzing their behavior under perturbation of "external force". They also explored the dynamical behavior of the system on the path encircling the fourth-order EP. The main finding is that the largest occupied state experiences a non-adiabatic transition (NAT) and an adiabatic state flip during counterclockwise encircling, while the results are different for clockwise encircling. Their findings were confirmed experimentally in a finite real-space lattice system using a designed classical electric circuit network.

The paper is well-written and structured. However, some clarification is needed regarding the following statements:

Reply: Many thanks to the reviewer for the positive comments. Next, we will give one-to-one replies to the questions raised by the reviewer.

Comment 1: *"Due to the complex non-trivial topology of Riemann surfaces associated with non-Hermitian Hamiltonians, the dynamical encirclement or proximity of EPs in parameter space has been shown to lead to some novel physical phenomena. However, in the previous studies on dynamically encircling EPs, tuning the phenomena for applications requires modulation of the system parameters, which is very difficult to be realized as the systems have been constructed. Here, we propose theoretically and demonstrate experimentally a new way*

to generate EPs and realize asymmetric multimode switching by tuning external forces without modifying the system parameters."

The authors claim that modulating system parameters for dynamical encircling in previous studies was difficult. However, the works that deals with similar problems but in photonics platforms, for example:

1. Regensburger, A., Bersch, C., Miri, MA. et al. Parity–time synthetic photonic lattices. Nature 488, 167–171 (2012). <https://doi.org/10.1038/nature11298>

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demonstrate successful parameter modulation using external forces (e.g., intensity, phase modulators, variable couplers) in photonic systems. These methods effectively modulate the Hamiltonian operator, similar to changing the on-site potential "external forces " in the electric circuit system proposed by the authors of the paper under the review.

In essence, both approaches modify the system parameters (Hamiltonian) to achieve dynamical encircling. While the platform (electric circuit vs. photonics) differs, the underlying principle remains the same. The key novelty of this work in my opinion lies in demonstrating control over EPs and multimode switching using external forces in a specific platform (electric circuits).

Reply: Thanks for the suggestions and the references provided. We have read the references in detail. This is true that some parameters (e.g., intensity, phase modulators, variable couplers) of non-Hermitian systems with EPs can be modulated to achieve dynamical encircling. For example, in optical platforms, some modulations have been realized in large-scale temporal lattices that are parity–time symmetric [Nature 488, 167–171 (2012)] and a topological temporally mode-locked laser [Nat. Phys. 20, 852–858 (2024)], chiral state transfer without encircling the EP has also been observed [Nature 605, 256-261 (2022)].

However, our work provides another way of modifying system parameters by applying a constant external force instead of periodic tuning, which realizes dynamical encircling of the

EPs in the lattice models. It should be noted that such a method is not limited to non-Hermitian HOTI models, but is applicable to general non-Hermitian lattice systems. Furthermore, we have designed and fabricated the corresponding circuit networks, and experimentally observed the phenomena. In the revised manuscript, we have modified some inaccurate descriptions from “Abstract” to “main text”, and the references provided have also been cited.

Comment 2: *Moreover, the statement “Here, we propose theoretically and demonstrate experimentally a new way to generate EPs, and realize asymmetric multimode switching by tuning external forces without modifying the system parameters.” Is also not precise and needs to be revised. By applying the external force F authors indeed are changing the system parameters because they modify the total system Hamiltonian operator H_e - the circuit Hamiltonian (line 262 in the manuscript) as authors precisely describe in their work:*

Reply: This is true that the application of the external force F in our work achieves the effect of modifying the system parameters as on other platforms. In fact, our work provides another way of modifying system parameters by applying a constant external force instead of periodic tuning, which realizes dynamical encircling of the EPs in the lattice models. We have also made changes in Abstract section of the revised manuscript as “Here, we propose theoretically and demonstrate experimentally a new way to generate EPs, and realize asymmetric multimode switching by tuning external forces. The application of the external force achieves the effect of modifying the system parameters, so that the dynamical encircling of the EPs can be easily realized.”

Comment 3: *Moreover, under the light of Regensburger et al., Nasari et al., Leefmans et al. papers, the statement “... requires modulation of the system parameters, which is very difficult to be realized” is not necessarily precise and in my opinion needs to be revised.*

Reply: In the Abstract section and the last paragraph of Conclusion section, we have removed the words “difficult to be realized” and modified the description of our method to “a new way of modifying system parameters by applying a constant external force instead of periodic tuning”.

Comment 4: *It would be also beneficial if the authors could elaborate more on the potential applications with specific examples of non-Hermitian systems, as mentioned in their conclusions: "Thus, our work paves a new way to explore non-Hermitian dynamics and develop applications of non-Hermitian systems."*

Reply: Thanks for the suggestion. We have added some descriptions of the specific application of our method to the last paragraph of the revised manuscript as "our design can also be applied in nonreciprocal transmission, tunable topological switching, mode transfer and selectivity for electrical or other signals".

Comment 5: *Finally, in my opinion, the authors should clarify also title to be more precise, for example: "Nested Chirality and Multimode Transfer Around a Second-Order Exceptional Point Driven by External Forces in Electric Circuit Settings"*

Reply: Thanks for the suggestion. In the revised manuscript, we have modified the title as "Nested Chirality and Multimode Transfer Around Exceptional Points Driven by External Forces in Electric Circuit Settings"

Reply to Reviewer 1

Comment: *The authors have addressed my concerns, and I now recommend its publication in its current form.*

Reply: We thank the reviewer for the recognition of our work.

Reply to Reviewer 2

Comment: *The authors have responded to my questions and made corresponding revisions to the manuscript. I agree with its acceptance by Communications Physics.*

Reply: We thank the reviewer for the recognition of our work.

Reply to Reviewer 4

Comment: *I am satisfied with the revised version and the responses to my comments.*

Reply: We thank the reviewer for the recognition of our work.