

Experimental simulation of Dirac equation in superconducting qubits

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

In this manuscript, the authors design and build a superconducting qubit chip with four qubits to simulate the evolution of a single particle Dirac Hamiltonian in 3+1 dimensions. The work is solid, and worth publishing. I do recommend the authors address some items that are unclear or become more challenging to decipher due to definitions in the main text either showing up on the Methods or SI section.

1. The authors mention the rearranged basis related to H_s , but are brief in the main text. They point to SI, but in the SI they refer to the Methods and SI. The authors should expand lines 111-114 to provide more essential details in the main manuscript, and avoid pointing to multiple places.

2. Lines 195-198 show a new labeling without explanation. One has to go to the SI to understand the expression. This makes this part of the main manuscript difficult to read.

3. Results in Figures 2 and 3. The signals clearly get worse with longer evolution times. The authors should comment on the challenges with decoherence in the chip. This also brings up a separate question: Are the results raw output results or did the authors use error correction or mitigation strategies to increase the tomography results?

4. DO I understand it correct that essentially the authors end up simulating two independent two-qubit systems? As such, would it have been possible to reduce the system to two one-qubit systems?

Som minor issues:

- The authors talk about "this letter" and later on "this article". Please make this consistent.

Reviewer #2

(Remarks to the Author)

This paper follows the ideas of <https://journals.aps.org/pr/abstract/10.1103/PhysRevA.99.042308> very closely, but does not cite it. The acknowledgement to the first author of that paper is not sufficient. Almost all of the theoretical methodology and motivation for this work comes one-to-one from that paper (super-symmetric representation of the Dirac equation and state number re-ordering, bright-dark state, degenerate Zener manifolds, adiabatic ramps and pair-production)

<https://journals.aps.org/pr/abstract/10.1103/PhysRevA.99.042308> is a theory paper, and this manuscript is an experimental paper. However, many experimental details are lacking (also from the supporting material). It is very hard, or even impossible to seriously judge the experimental aspects (that may indeed merit publication).

I recommend rejecting the paper.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

I believe the authors have addressed the issues raised by me in a satisfactory way.

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Response to Report of Referee A

Comment A0: In this manuscript, the authors design and build a superconducting qubit chip with four qubits to simulate the evolution of a single particle Dirac Hamiltonian in 3+1 dimensions. The work is solid, and worth publishing. I do recommend the authors address some items that are unclear or become more challenging to decipher due to definitions in the main text either showing up on the Methods or SI section.

Reply: We sincerely thank the reviewer for the positive feedback and constructive suggestions. We appreciate the recognition of the solid work and the potential for publication.

Comment A1: The authors mention the rearranged basis related to H_s , but are brief in the main text. They point to SI, but in the SI they refer to the Methods and SI. The authors should expand lines 111-114 to provide more essential details in the main manuscript, and avoid pointing to multiple places.

Reply: We appreciate the valuable suggestion and have included an explanation in the main text on how to rearrange the basis.

Comment A2: Lines 195-198 show a new labeling without explanation. One has to go to the SI to understand the expression. This makes this part of the main manuscript difficult to read.

Reply: We acknowledge the helpful suggestion, we have provided explanations for the new labelling in the main manuscript.

Comment A3: Results in Figures 2 and 3. The signals clearly get worse with longer evolution times. The authors should comment on the challenges with decoherence in the chip. This also brings up a separate question: Are the results raw output results or did the authors use error correction or mitigation strategies to increase the tomography results?

Reply: For the Zitterbewegung experiment, we considered the impact of decoherence in the simulation, which leads to oscillation decay. It can be observed that for small masses, the experimental and simulation results are consistent. However, for larger masses, the experimental data deviate at longer evolution times. We attribute this to coherent errors, which are likely caused by inaccuracies in phase calibration.

Regarding the tomography results, our data are redundant, so we averaged the results from multiple tomography directions to mitigate some systematic errors, such as voltage deviations.

We appreciate the suggestion regarding error mitigation. Upon carefully re-examining the raw data, we identified the presence of readout crosstalk. In the revised version, we have explained and corrected the impact of this issue.

Comment A4: DO I understand it correct that essentially the authors end up simulating two independent two-qubit systems? As such, would it have been possible to reduce the system to two one-qubit systems?

Reply: We have used four qubits to construct the Hamiltonian to simulate the free evolution, Zitterbewegung effect, and Schwinger effect, respectively. Due to parameter requirements, we can not directly realize the couplings we want on two qubits, unless there are some special cases. For example, in the Schwinger effect experiment, the system evolution in the simulated electric field can be viewed as two independent Landau-Zener transitions after the unitary transformation. Therefore, the experiments in this part appear to be two-qubit simulations. Still in general, to simulate the 3+1D Dirac equation, it is much more convenient to implement this simulation using our protocol, which use the entire four-qubit system for this mapping of the parameters.

Comment A5: The authors talk about "this letter" and later on "this article". Please make this consistent.

Reply: We sincerely thank the reviewer for pointing this out. We have ensured consistency.

Response to Report of Referee B

Comment B0: This paper follows the ideas of <https://journals.aps.org/pr/abstract/10.1103/PhysRevA.99.042308> very closely, but does not cite it. The acknowledgement to the first author of that paper is not sufficient. Almost all of the theoretical methodology and motivation for this work comes one-to-one from that paper (super-symmetric representation of the Dirac equation and state number re-ordering, bright-dark state, degenerate Zener manifolds, adiabatic ramps and pair-production)

Reply: We sincerely apologize for our oversight in omitting this essential reference. Our experimental work was indeed inspired by that theoretical article. It was included in our earlier version but was accidentally missed in the submitted article. We have included it in the revised version.

We appreciate the reviewer pointing out the similarity between our experimental work and the previous theoretical study. This is not surprising, as much of the

experiment was inspired by the article and discussions with the authors. However, we not only realized this simulation with our own experimental scheme but also obtained some novel results, such as the relationship between the ZBW effect and momentum direction. We analyze its state dependence from both experimental and theoretical perspectives.

The experimental methods proposed in the original article have strict requirements on experimental parameters and are not applicable to more general simulations. Our approach provides a higher-dimensional parameter space and more degrees of freedom, making it more suitable for general quantum simulation. Recently, many labs have used similar schemes to simulate very important physical phenomena such as the fractional Quantum Hall effect (*Science* 384,579-584(2024)) and the AB effect(*Nat. Phys.* 20, 1881–1887 (2024)). This proves that our scheme is a promising tool for future simulation research of more complex physics systems.

The advantages of our experimental scheme include: 1. The Hamiltonian is realized by using the manifold of the first excited state coupled with the parametric modulations, which has good scalability. In addition to the four-level structure implemented in this paper, it can be easily generated to higher parameter dimensions, which is not available in previous schemes. 2. the parameters can be easily calibrated in terms of experimental details, including the AC Stark shifts induced by the flux modulation. The detuning of the parametric modulation frequency and the flux crosstalk can be pretty small, indicating that experiment costs can be saved in this protocol. The former article uses the microwave drive to construct Hamiltonian; one needs to calibrate the AC Stark shift and compensate for different effective strengths.

In conclusion, we believe that the revised article meets the criteria for publication.

Comment B1: <https://journals.aps.org/pr/abstract/10.1103/PhysRevA.99.042308> is a theory paper, and this manuscript is an experimental paper. However, many experimental details are lacking (also from the supporting material). It is very hard, or even impossible to seriously judge the experimental aspects (that may indeed merit publication).

Reply: We appreciate the suggestion. We have added experimental details regarding the calibration of the frequency and amplitude of the modulation parameters, as well as their mapping to the effective coupling strength. We also provide a detailed description of the error mitigation used in the experiment. We believe this will make the article much more complete.

Dear Editors,

Thank you for your message and for consideration in evaluating our manuscript. We would also like to express our gratitude to the two referees for their valuable comments and suggestions. We appreciate their efforts in reviewing our work.

Response to Report of Referee A _____

Comment A0: I believe the authors have addressed the issues raised by me in a satisfactory way.

Reply: We sincerely appreciate the reviewer's recognition of our efforts in addressing their concerns, and we are grateful for their time and expertise throughout this review process.