

A scaling relation of vortex-induced rectification effect in a superconducting thin-film heterostructure

Corresponding Author: Dr Junichi Shiogai

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

This manuscript reports a series of vortex-induced nonreciprocal behaviors in the superconducting Fe(Se,Te)/FeTe heterostructure, demonstrating the Rashba spin-orbit interaction as an origin of the asymmetric vortex dynamics. The research on nonreciprocal superconducting transport properties is hot and interesting in recent years. However, there are several questions I would like to bring out before this manuscript can be recommended for publication in Communications Physics.

There are many writing errors in the manuscript. All of the main text, figures and formulas need to be carefully proofread. 'A contour plot of normalized $R_{xx}(T,B)$ is shown on the basal plane of Fig. 1d to construct a vortex phase diagram.' Here, author mentioned 'Fig. 1d', which I cannot find in the whole manuscript.

The description of heterostructure in the manuscript is mixed with 'Fe(Se,Te)/FeTe' and 'Se/Fe(Se,Te)/FeTe', making it confusing to read.

'Although the theoretical analysis is indispensable for interpretation of the suppression of η at high magnetic field and low temperature.' The suppression effect of low temperature on η cannot be seen from Figure 2.

It is better to add axes in Fig. 1a.

The optical image and the measurement schematic diagram of the device need to be provided here, and whether the measurement results come from the interface of the heterostructure.

It is difficult to see that the Fe(Se,Te)/FeTe heterostructure has upper critical field $B_{c2}(T)$ in Fig. 1c. A more intuitive comparison diagram or numerical ratio is needed.

'The value of γ rapidly decreases with further decreasing T toward zero resistance. This corresponds to the freezing of the vortices.' The rapid decrease of γ seems to correspond well with the superconducting transition temperature. Further discussion should be made here about the relation between superconductivity and vortices.

Comparing Figs 3b and 3c, the linear scaling relation seems to fit well only in a small part. More discussion is required here about the deviation.

The coordinates in Fig. 3c are logarithmic, in which the expression of its 'linear' relationship requires caution.

Reviewer #2

(Remarks to the Author)

This paper reports the superconducting diode effect in thin films of iron-based superconductor Fe(Se,Te). They found an intriguing scaling relation between the two parameters η and γ , from the measurements of I-V characteristics and second-harmonic resistivity components. They provide a phenomenological model based on the vortex motion with asymmetric potential, which reasonably explains the observed behavior. The origins of the superconducting diode effect are currently under intense debate and the present results will have merit in these discussions. I found that discussions, including the heating issues in the Methods section, are well written. Thus I can recommend publication in Communication Physics in its current form.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The authors have adequately addressed my previous concerns. This manuscript has been substantially improved and makes a noteworthy contribution to the field. The reported superconducting diode effect and nonreciprocal resistance in the Fe(Se,Te)/FeTe heterostructure highlight the role of the spin-momentum-locked state on the asymmetric pinning potential, offering a unified picture of superconducting rectification that paves the way for the functionalization of superconducting diode devices. Therefore, I recommend the manuscript for publication.

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A. Our response to Reviewers #1

Manuscript ID: MS#COMMSPHYS-24-1566-T

Reviewer #1

This manuscript reports a series of vortex-induced nonreciprocal behaviors in the superconducting Fe(Se,Te)/FeTe heterostructure, demonstrating the Rashba spin-orbit interaction as an origin of the asymmetric vortex dynamics. The research on nonreciprocal superconducting transport properties is hot and interesting in recent years. However, there are several questions I would like to bring out before this manuscript can be recommended for publication in Communications Physics.

Our Response

We would like to thank Reviewer #1 very much for their time devoted to carefully reviewing our manuscript. All their insightful comments are constructive and encourage us to improve the manuscript. Our point-by-point responses to those comments are described below.

Reviewer Comment #1-1

There are many writing errors in the manuscript. All of the main text, figures and formulas need to be carefully proofread.

Our Response #1-1

We apologize that we overlooked several typos in the manuscript, which made you confused and spend more time reviewing than usual. We have proofread the revised manuscript and corrected it accordingly.

Reviewer Comment #1-2

‘A contour plot of normalized $R_{xx}(T,B)$ is shown on the basal plane of Fig. 1d to construct a vortex phase diagram.’ Here, author mentioned ‘Fig. 1d’, which I cannot find in the whole manuscript.

Our Response #1-2

We apologize that Fig. 1d was missing in the previous manuscript. We intended to send the readers to Fig. 1c, not Fig. 1d. We have checked and corrected the figure number to cite Fig. 1c throughout the main manuscript and supplementary information.

Reviewer Comment #1-3

The description of heterostructure in the manuscript is mixed with ‘Fe(Se,Te)/FeTe’ and ‘Se/Fe(Se,Te)/FeTe’, making it confusing to read.

Our Response #1-3

Reviewer is right in that the term describing the heterostructure was not well fixed throughout the previous manuscript. In the revised manuscript, we decided to use Fe(Se,Te)/FeTe instead of Se/Fe(Se,Te)/FeTe.

Reviewer Comment #1-4

‘Although the theoretical analysis is indispensable for interpretation of the suppression of η at high magnetic field and low temperature.’ The suppression effect of low temperature on η cannot be seen from Figure 2.

Our Response #1-4

We admit that this sentence contains the typo, and we are sorry again for making you confused by this error. We intended to express that $\eta(B)$ is suppressed at high magnetic field above 5 T in low temperature. However, when we were carefully revising the manuscript, we recognized that the suppression of $\eta(B)$ occurs in any investigated temperatures. Thus, we made the following amendment in the revised manuscript and would like to express thank Reviewer.

Page 9, Line 8:

“Although the theoretical analysis is indispensable for interpretation of the suppression of η at high magnetic field *and low temperature*,...”

was revised to

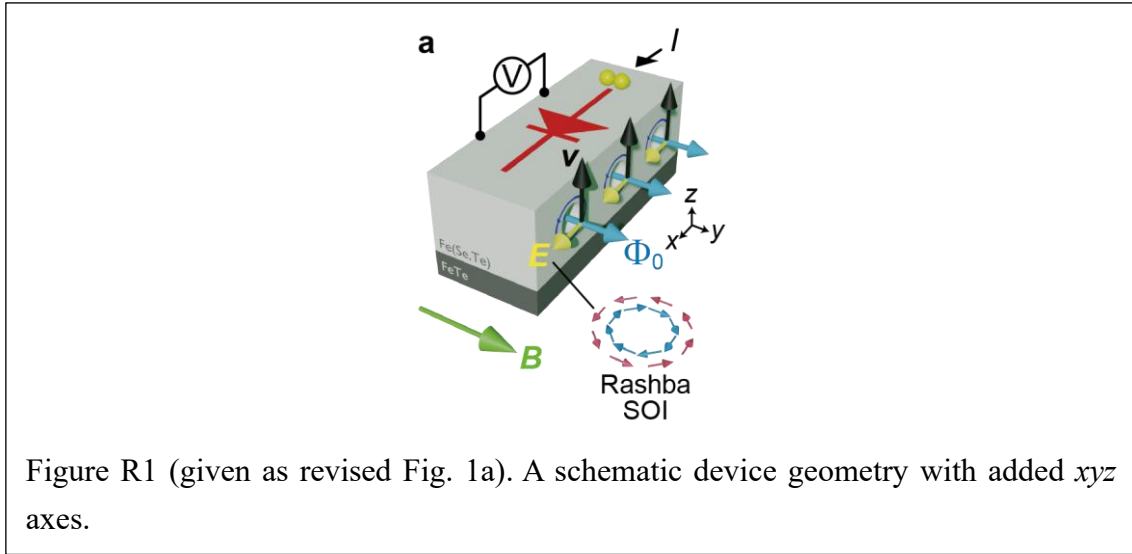
“Although the theoretical analysis is indispensable for interpretation of the suppression of $\eta(B)$ at high magnetic field **above 5 T**,...”

Reviewer Comment #1-5

It is better to add axes in Fig. 1a.

Our Response #1-5

We would like to thank Reviewer for this comment. According to Reviewer's suggestion, xyz axes were added in revised Fig. 1a as shown in Fig. R1.

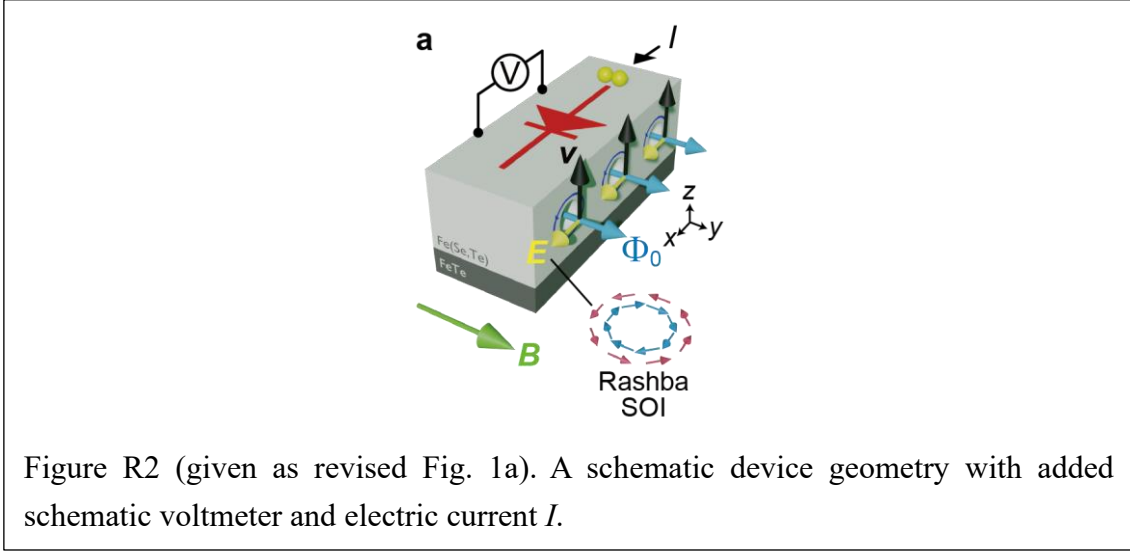


Reviewer Comment #1-6

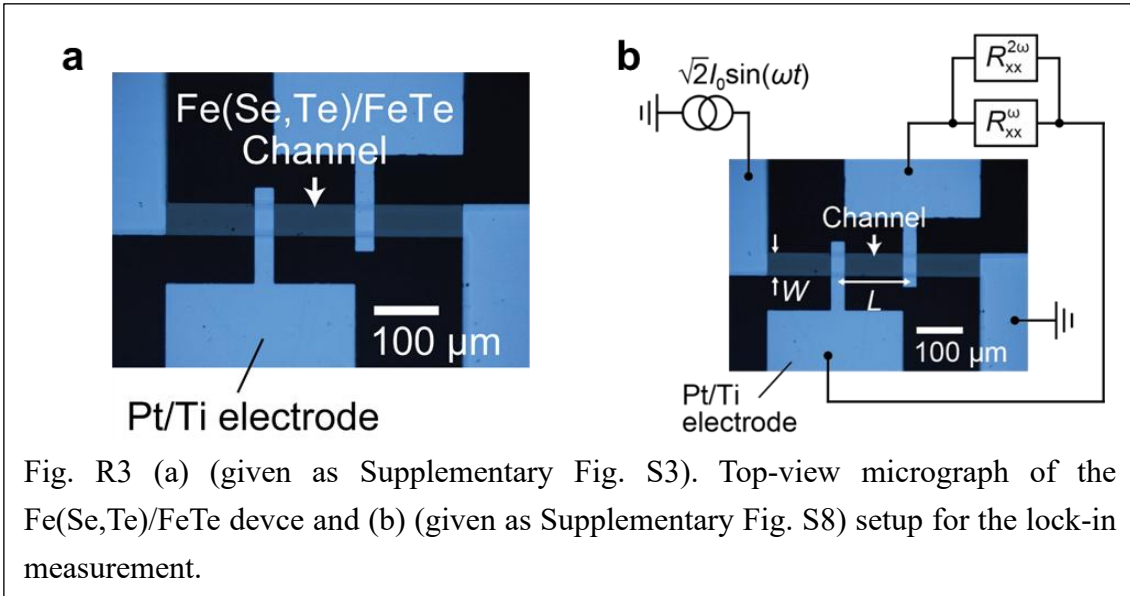
The optical image and the measurement schematic diagram of the device need to be provided here, and whether the measurement results come from the interface of the heterostructure.

Our Response #1-6

We agree with Reviewer pointing out that the measurement schematic diagram is needed in Fig. 1a. We added schematic voltmeter and electric current direction in revised Fig. 1a as shown in Fig. R2.



The optical image of the investigated device and measurement schematic diagram based on the image are shown in Figs. R3a and R3b, respectively. We agree with Reviewer that optical image of the device should be provided in the vicinity of the sentence describing Fig. 1a.



According to Reviewer's suggestion, we added Fig. R3a as Supplementary Fig. S3 in the revised Supplementary Information and revised the following sentence as follows.

Page 5, Line 12:

“Here, we studied the nonreciprocal superconducting transport properties in a

Fe(Se,Te)/FeTe heterostructure (Fig. 1a), where the inversion symmetry is broken along the out-of-plane direction.”

was revised to

“Here, we studied the nonreciprocal superconducting transport properties in a Fe(Se,Te)/FeTe heterostructure **device** (Fig. 1a and **Supplementary Fig.S3**), where the inversion symmetry is broken along the out-of-plane direction (**see Materials and Methods section, and Supplementary Sections 1 and 2 for structural characterization and sample preparation**).”

In addition, we added the following sentence in Materials and Methods section in the revised manuscript to send the readers to the optical micrograph of the device and measurement schematic diagram.

In Page 17, Line 13:

“...characteristics by water lift-off technique (see Supplementary Section 2). **The optical micrograph of the fabricated device and schematic setup for the lock-in measurements are shown in Supplementary Fig. S3 and S8, respectively.** The current-voltage (I - V) characteristics...”

We consider Reviewer’s “interface” as the thickness region which is influenced by adjacent layer such as the FeTe or Se cap layer through the band modification. In this sense, the entire Fe(Se,Te) layer of 23.3 nm thick is still in the interface. We deposited the Pt/Ti contact onto the Se-caped Fe(Se,Te) layer. Because the thickness of the Se cap layer (a few nm) is thin enough to allow the electric current and voltage probe pass through, the Pt/Ti can serve as Ohmic contact to the Fe(Se,Te) layer. Thus, the Pt/Ti electrode can measure the properties of the 23.3-nm-thick Fe(Se,Te) layer as the interface. This assumption is well supported by coincidence of normal-state four-terminal resistance measured by IV curve ($R_{IV}^N = V/I$) at $T = 4.2$ K with a higher current than critical current and that measured by at $T = 20$ K well above T_c with a low excitation current. To address this point, we revised the following sentence as follows in Materials and Methods section in the revised manuscript.

Page 17, Line 17:

“This value is consistent with the four-terminal resistance (L/W) $R_{xx}^N = 273 \Omega$ at $T = 20$ K measured with $10 \mu A$, ensuring that the effect of Joule heating is excluded.”

was revised to

“This value is consistent with the four-terminal resistance (L/W) $R_{xx}^N = 273 \, \Omega$ at $T = 20$ K measured with $10 \, \mu\text{A}$, ensuring that the effect of Joule heating is excluded and the Pt/Ti electrodes serve as Ohmic contacts. This implies that our transport measurements reflect the properties of the 23.3-nm-thick Fe(Se,Te) layer.”

Reviewer Comment #1-7

It is difficult to see that the Fe(Se,Te)/FeTe heterostructure has upper critical field $B_{c2}(T)$ in Fig. 1c. A more intuitive comparison diagram or numerical ratio is needed.

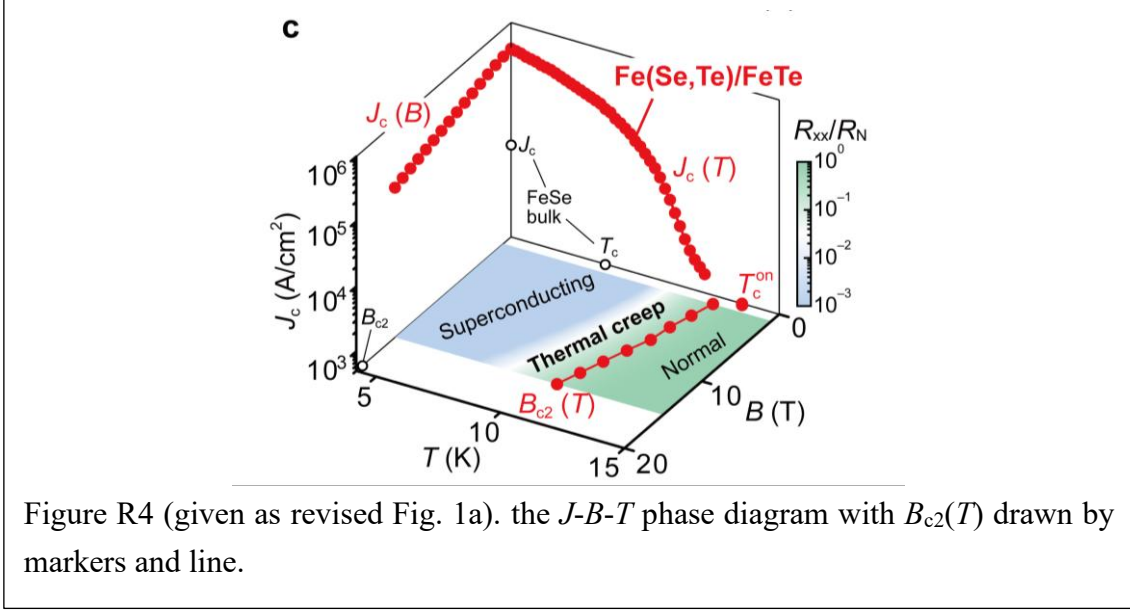
Our Response #1-7

If we consider Reviewer’s “comparison” as the comparison between our heterostructure and FeSe bulk single crystal, we admit that it was not straightforward to recognize that the Fe(Se,Te)/FeTe heterostructure exhibits larger B_{c2} than FeSe bulk single crystal in the previous version of manuscript.

Firstly, the temperature dependence of the upper critical field $B_{c2}(T)$ in Fig. 1c was experimentally determined by temperature dependence of the sheet resistance R_{xx} under the in-plane magnetic field B . As explained in Supplementary Section 3, $B_{c2}(T)$ was obtained as the inverse functions of magnetic-field dependence of midpoint superconducting critical temperature $T_c(B)$ that corresponds to the temperature crossing the half value of normal state resistance in the $R_{xx}(T)$ curves under B . To express that the $B_{c2}(T)$ are experimentally-determined values, we draw $B_{c2}(T)$ curve by markers and line in the revised Fig. 1c instead of simple line.

Second, thanks to the Reviewer’s comment, we noticed that the reference for FeSe bulk used in Fig. 1c were inappropriate. The B_{c2} value plotted in the previous version of Fig. 1c and Supplementary Fig. S4c was extracted from polycrystalline bulk [F. C. Hsu et al., Proc. Natl. Acad. Sci. USA **105**, 14262 (2008).]. The J_c value plotted in the previous Fig. 1c was extracted from single crystalline FeSe but measured at $T = 2$ K [H. Lei et al., Phys. Rev. B **84**, 014520 (2011).]. For fair comparison in the revised manuscript, we use values for single crystalline samples measured at $T \sim 4.2$ K extracted from [new Ref. S7: S. I. Vedenev et al., Phys. Rev. B **87**, 134512 (2013)] and [new Ref. 33 and Ref. S8: M. Bristow et al., Phys. Rev. B **108**, 184507 (2023)] for B_{c2} and from [new Ref. 34 and Ref. S9: Y. Sun et al., Phys. Rev. B **92**, 144509 (2015).] for J_c , respectively. In addition, we deleted black shaded area to clarify the superconducting critical parameters J_c , T_c , and B_{c2} of the FeSe reference.

We incorporated these revisions in the revised Fig. 1c as shown in Fig. R4.



In Table R1, we list the values of B_{c2} at $T \sim 0.9T_c$ of the Fe(Se,Te)/FeTe heterostructure and the FeSe bulks in addition to T_c^{zero} and J_c . Because we think that addition of Table R1 to the main text may alter the mainstream of our discussion, Table R1 was added as Supplementary Table 1 in Supplementary Section 3.

Table R1 (given as Supplementary Table S1). Comparison of superconducting critical parameters of our Fe(Se,Te)/FeTe heterostructure and FeSe bulk single crystal.

Materials and structures	T_c^{zero} (K)	$B_{c2}(T \sim 0.9T_c)$ (T)	$J_c(0)$ (A/cm ²) at 4.2 K
Fe(Se,Te)/FeTe heterostructure	11.6	13 ($\perp c$ axis), 7.0 ($\parallel c$ axis)	3.9×10^5
FeSe bulk	8 [S6]	5.0 ($\perp c$ axis), 4.1 ($\parallel c$ axis) [S7] 5.0 ($\perp c$ axis), 1.0 ($\parallel c$ axis) [S8]	1.3×10^4 [S9]

In addition, we revised the following sentence as follows in the revised manuscript.

Page 7, Line 4

“We achieved a larger critical current density $J_c(B, T)$ and upper critical field $B_{c2}(T)$ than

the values reported in FeSe bulk single crystal^{32,34} (black circles) owing to Te doping (see Supplementary Figures S3 and S4 for raw data).”

was revised to

“We achieved a larger critical current density J_c and upper critical field B_{c2} than the values reported in FeSe bulk single crystal³²⁻³⁴ (black **open** circles) owing to Te doping (see Supplementary Figures S4 and S5 for raw data, **and Table S1 for the comparison of critical parameters**).”

The following sentences were added in Supplementary Section 3 in the revised Supplementary Information.

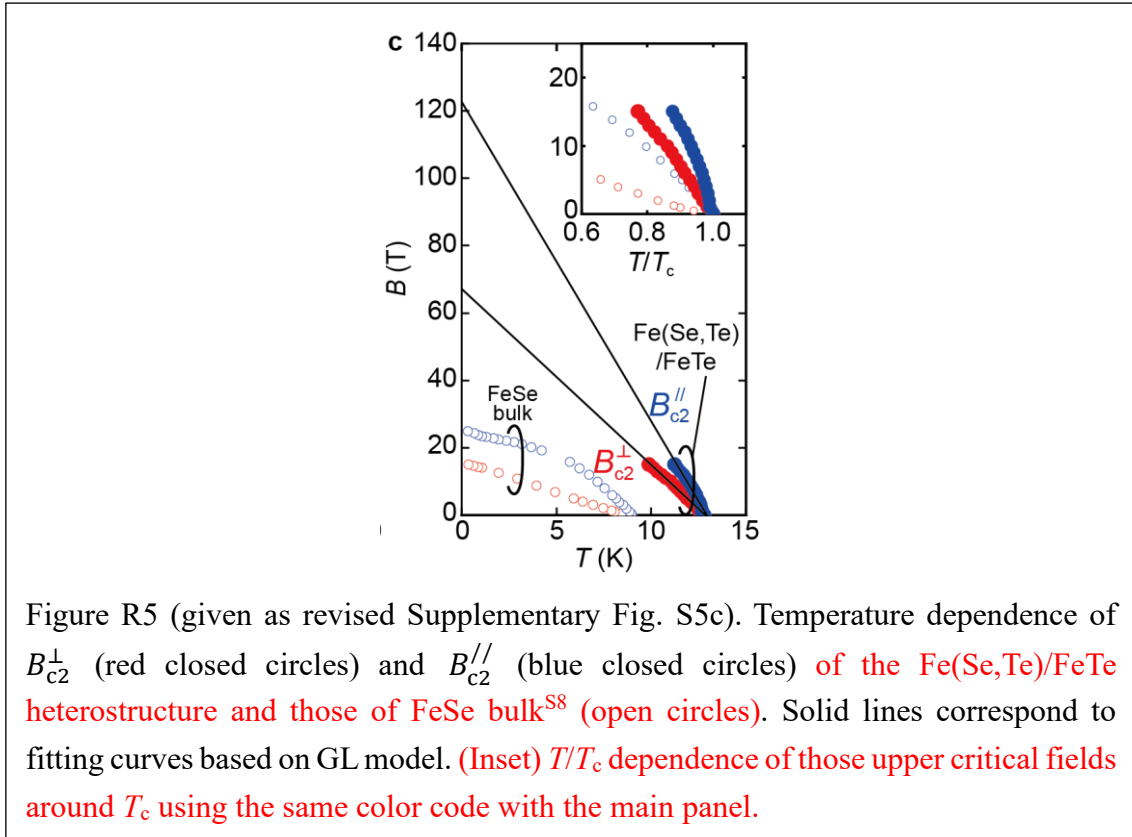
“**Superconducting critical parameters of our Fe(Se,Te)/FeTe heterostructure and FeSe bulks are summarized in Table S1. The critical current density J_c and the upper critical field B_{c2} , which is larger than the values reported in FeSe bulk single crystals^{S6,S7,S8,S9} are realized in the Fe(Se,Te)/FeTe heterostructure, as expected from the increased T_c in Te-doped FeSe superconductors^{S10,S11}.**”

Accordingly, we added References 33 and 34 in the revised reference list and Supplementary References S6 – S11 in the revised supplementary reference list:

33. Bristow, M., Gower, A., Prentice, J. C. A., Watson, M. D., Zajicek, Z., Blundell, S. J., Haghighirad, A. A., McCollam, A., & Coldea, A. I., Multiband description of the upper critical field of bulk FeSe. *Phys. Rev. B* **108**, 184507 (2023).
34. Sun, Y., Pyon, S., Tamegai, T., Kobayashi, R., Watashige, T., Kasahara, S., Matsuda, Y., & Shibauchi, T., Critical current density, vortex dynamics, and phase diagram of single-crystal FeSe. *Phys. Rev. B* **92**, 144509 (2015).
- S6. Hsu, F. C., Luo, J. Y., Yeh, K. W., Chen, T. K., Huang, T. W., Wu, P. M., Lee, Y. C., Huang, Y. L., Chu, Y. Y., Yan, D. C., & Wu, M. K., Superconductivity in the PbO-type structure α -FeSe. *Proc. Natl. Acad. Sci. USA* **105**, 14262-14264 (2008).
- S7. Vedenev, S. I., Piot, B. A., Maude, D. K., & Sadakov, A. V., Temperature dependence of the upper critical field of FeSe single crystals. *Phys. Rev. B* **87**, 134512 (2013).
- S8. Bristow, M., Gower, A., Prentice, J. C. A., Watson, M. D., Zajicek, Z., Blundell, S. J., Haghighirad, A. A., McCollam, A., & Coldea, A. I., Multiband description of the upper critical field of bulk FeSe. *Phys. Rev. B* **108**, 184507 (2023).
- S9. Sun, Y., Pyon, S., Tamegai, T., Kobayashi, R., Watashige, T., Kasahara, S., Matsuda, Y., & Shibauchi, T., Critical current density, vortex dynamics, and phase diagram of

- single-crystal FeSe. Phys. Rev. B 92, 144509 (2015).
- S10. Imai, Y., Sawada, Y., Nabeshima, F., & Maeda, A., Suppression of phase separation and giant enhancement of superconducting transition temperature in FeSe_{1-x}Te_x thin films. Proc. Natl. Acad. Sci. USA **112**, 1937 (2015).
- S11. Ishida, K., Onishi, Y., Tsujii, M., Mukasa, K., Qiu, M., Saito, M., Sugimura, Y., Matsuura, K., Mizukami, Y., Hashimoto, K., & Shibauchi, T., Pure nematic quantum critical point accompanied by a superconducting dome. Proc. Natl. Acad. Sci. USA **119**, e2110501119 (2022).

Furthermore, we added the in-plane and out-of-plane upper critical fields of a FeSe single crystal as reference in the revised Supplementary Fig. S5c as shown in Fig. R5 for direct (and intuitive) comparison of temperature dependence of B_{c2} between Fe(Se,Te)/FeTe and FeSe bulk.



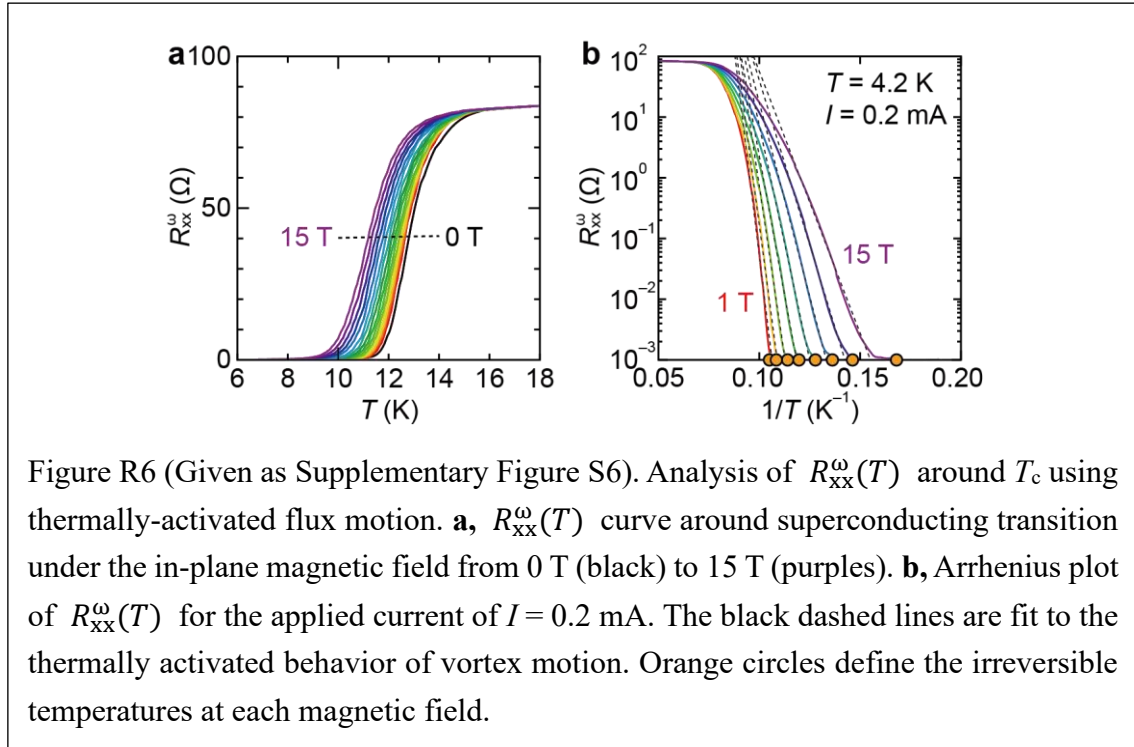
In addition, we noticed that “ $\mu_0 H_{c2}$ ” was used to express the upper critical field in the previous Supplementary Information instead of “ B_{c2} ” as used in main manuscript. In this revision, we use B_{c2} throughout the main manuscript and Supplementary Information.

Reviewer Comment #1-8

‘The value of γ rapidly decreases with further decreasing T toward zero resistance. This corresponds to the freezing of the vortices.’ The rapid decrease of γ seems to correspond well with the superconducting transition temperature. Further discussion should be made here about the relation between superconductivity and vortices.

Our Response #1-8

As Reviewer pointed out, the relation between superconductivity and vortices is not clearly described in the previous manuscript. According to the Josephson relation, when the vortices move with velocity $\mathbf{v}$ under magnetic field $\mathbf{B}$ along the direction of the Lorentz force $\mathbf{J} \times \mathbf{B}$, the motion generates the electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}$, which is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ (thus, parallel to $\mathbf{J}$). Thus, the symmetric and asymmetric motion of vortices against ac current (Lorentz force) results in generation of linear resistance R_{xx}^ω and non-reciprocal resistance $R_{xx}^{2\omega}$, respectively. As shown in Fig. R6, $R_{xx}^\omega(T)$ in the superconducting transition region can be described by the thermally activated flux flow (TAFF) model. This implies that the decrease in R_{xx}^ω accompanied by the enhanced γ is a consequence of the activated motion of vortices across the asymmetric pinning potential.



On the other hand, considering the emergence of γ originates from activated motion of vortices, the sudden decrease in γ with further decreasing T suggests the disappearance of the activated motion itself which causes zero resistance in the low current limit. This corresponds to the freezing of vortices from a liquid (TAFF) state to a solid state. Indeed, as discussed in second paragraph on Page 11 of main text and Supplementary Section 7, we found later the TAFF model is reasonable for explaining the increase in γ with decreasing temperature, but not for the decrease. This is because the calculated R_{xx}^ω in TAFF model has non-zero value at finite temperature even if it is negligibly small. Therefore, the concept of the freezing transition of vortices with zero resistance is necessary

As first, to make it clear that the transport properties in the superconducting transition region mainly originate from the motion of vortices, we revised the following sentences in the revised manuscript as follows.

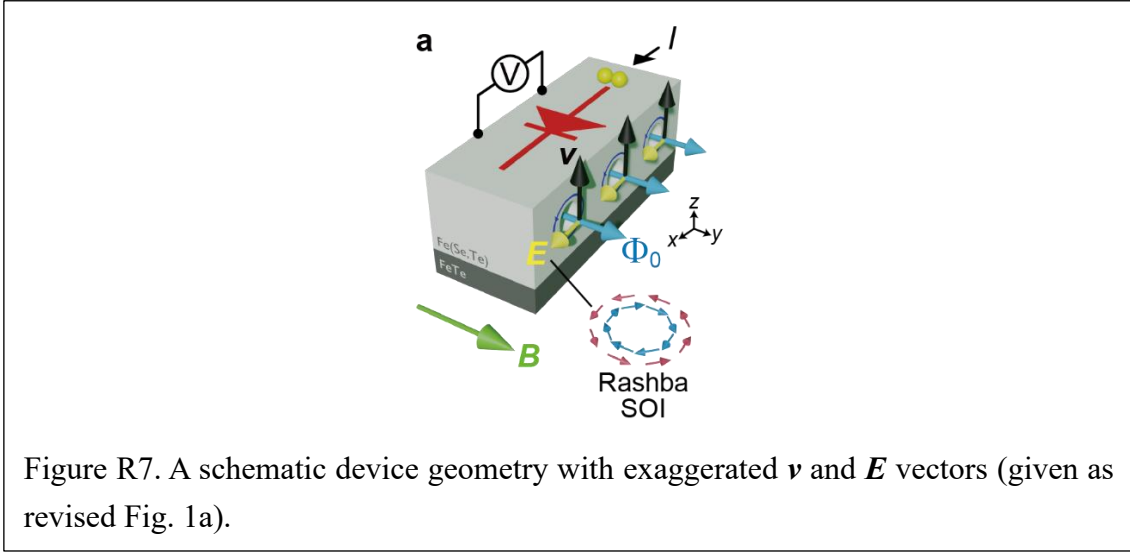
Page 4 Line 9:

“One of the proposed mechanisms for the supercurrent rectification effect in earlier works was the ratchet motion of vortices caused by macroscopically asymmetric pinning potential, which was introduced artificially in trivial superconductors²³⁻²⁷. Recently, a variety of other origins of the ratchet mechanism, such as...”

was revised to

“One of the proposed mechanisms for the supercurrent rectification effect is the ratchet motion of vortices caused by asymmetric pinning potential. According to the Josephson’s relation, the motion of vortices (hopping between pinning potentials) with a velocity $\mathbf{v}$ along the Lorentz force $\mathbf{J} \times \mathbf{B}$ generates the electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}$, which is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ (thus, parallel to $\mathbf{J}$) as schematically shown in Fig. 1a. In the case that the pinning potential is spatially asymmetric, the generated $\mathbf{E}$ leads to rectification effect around T_c . In earlier works²³⁻²⁷, the vortex ratchet motion was confirmed in trivial superconductors with artificially-introduced macroscopic pinning potentials. Recently, a variety of other origins of the ratchet mechanism, such as...”

In addition, we exaggerated the vectors representing $\mathbf{E}$ and $\mathbf{v}$ (revised from F_L : Lorentz force) in the revised Fig. 1a as shown in Fig. R7.



As second, to explain the relation between the superconductivity (zero resistance) and the motion of vortices, we revised the following sentences in the revised manuscript as follows.

Page 11, Line 1

“(II) The value of γ rapidly decreases with further decreasing T toward zero resistance. *This corresponds to the freezing of the vortices, resulting in the suppression of vortex motion at the present low excitation current.* The concomitant occurrence of the enhancement of γ and the $R_{xx}(T)$ drop described by TAFF model in $T_c^{\text{zero}} < T < T_c^{\text{on}}$ implicates that the activation energy (pinning potential) is effectively asymmetric against the polarity of current and magnetic field, which is the origin of the nonreciprocal resistance.”

was revised to

“(II) The value of γ rapidly decreases with further decreasing T toward zero resistance. The concomitant occurrence of the enhancement of γ and the $R_{xx}(T)$ drop described by TAFF model **for (I)** implicates that the activation energy (pinning potential) is effectively asymmetric against the polarity of current and magnetic field, which is the origin of the nonreciprocal resistance. **On the other hand, the sudden decrease in γ for (II) suggests the disappearance of the activated motion of the vortices itself which causes zero resistance in the low current limit. This corresponds to the freezing of vortices from a liquid (TAFF) state to a solid state.**”

Reviewer Comment #1-9

Comparing Figs 3b and 3c, the linear scaling relation seems to fit well only in a small part. More discussion is required here about the deviation.

Our Response #1-9

We thank the reviewer for the appropriate comment. We admit that the linear scaling relation between γ and η/BT is satisfied only in the temperature regime where the resistance is governed by thermally activated creep of vortices. There are two extreme regimes of high- and low-temperatures where the data does not obey the γ - η/BT scaling relation. In high-temperature regime (left side in Fig. R8) around onset T_c (T_c^{on}), $R_{xx}^\omega(T)$ deviate from the liner function in the Arrhenius plot (dashed lines in Fig. R9b). This implies that the transport properties in the regime are no longer dominated by thermal creep of vortices but the superconducting fluctuation of order parameter amplitude, leading to the deviation from the γ - η/BT scaling relation. In low-temperature regime (right side in Fig. R8), γ rapidly decreases because the motion of vortices are frozen as explained in our Response #1-8. This is also outside the scope of the thermal creep model.

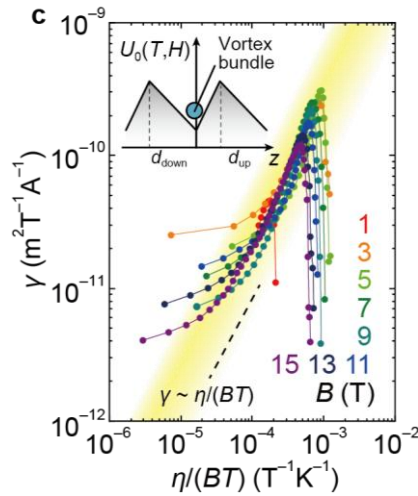


Figure R8 (Given as revised Fig. 3c). The relationship between γ and η/BT obtained for magnetic field $B = 1$ to 15 T. A dashed line represents a linear relation. (Inset) Schematic of asymmetric pinning potential around the vortex core.

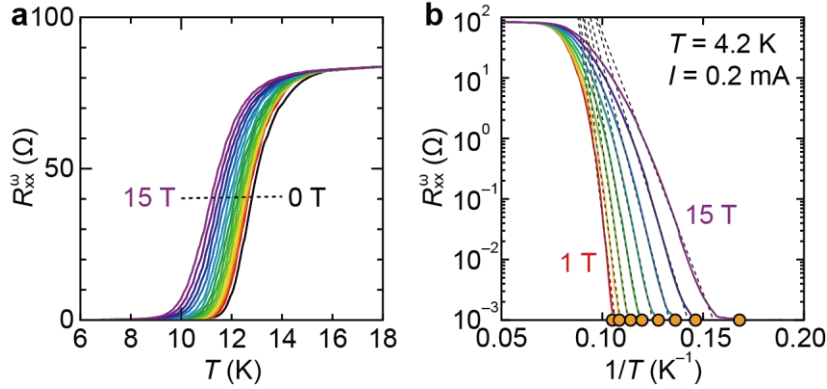


Figure R9 (Given as Supplementary Figure S6). Analysis of $R_{xx}^{\omega}(T)$ around T_c using thermally-activated flux motion. **a**, $R_{xx}^{\omega}(T)$ curve around superconducting transition under the in-plane magnetic field from 0 T (black) to 15 T (purples). **b**, Arrhenius plot of $R_{xx}^{\omega}(T)$ for the applied current of $I = 0.2$ mA. The black dashed lines are fit to the thermally activated behavior of vortex motion. Orange circles define the irreversible temperatures at each magnetic field.

To discuss these points, we added the following sentences in the revised manuscript and Supplementary Information:

Page 13, Line 14 in the revised manuscript:

“The comparable value of d_0 to the superconducting coherence lengths ($\xi_{ab} = 2.2$ nm and $\xi_c = 1.2$ nm, Supplementary Section 3) ensures the validity of our model. **The universal scaling in the γ - η / BT relation breaks down in high and low-temperature regimes. In these regimes, $R_{xx}(T)$ is not governed by thermal creep; instead, the transport properties are dominated by superconducting fluctuation of order parameter amplitude at high temperatures (Supplementary Section 4) and the freezing of vortex motion at low temperatures, respectively.**”

Supplementary Section 4 in Supplementary Information:

“We defined $T_{irr}(H)$ (orange circles in Fig. S5b) when $R_{xx}(T)$ intersects the detectable limit in this work (~ 10 m Ω). The $H_{irr}(T)$ plotted in Fig. 1d is the irreversible magnetic field derived as the inverse function of $T_{irr}(H)$.”

was revised to

“The deviation from the Arrhenius plot in high-temperature regime around onset T_c indicates that $R_{xx}(T)$ is no longer described by the activated motion of vortices but is

dominated by superconducting fluctuation of order parameter amplitude. We defined irreversible temperature $T_{\text{irr}}(H)$ (orange circles in Fig. S6b) when $R_{\text{xx}}(T)$ intersects the detectable limit in this work ($\sim 10 \text{ m}\Omega$), which is close to the freezing temperature of vortex system.”

Reviewer Comment #1-10

The coordinates in Fig. 3c are logarithmic, in which the expression of its ‘linear’ relationship requires caution.

Our Response #1-10

We would like to thank the reviewer for this critical point. Firstly, we put a dashed line representing γ being proportional to η/BT as shown in Fig. R10. As pointed out by Reviewer, the γ - η/BT relation exhibits slightly larger slope than unity particularly when the γ value reaches maximum at each magnetic field. According to Eq. (4) in the main manuscript, the expression of γ is described as

$$\gamma = \frac{V_c d_0}{k_B} \frac{\eta}{T} = \frac{\Phi_0 \lambda d_0}{k_B} \frac{\eta}{BT}.$$

Here, λ and d_0 are the penetration depth and a size of the pinning potential. The slight deviation from the linear relation can be ascribed to temperature dependence of these parameters. Qualitative estimation of temperature dependent λ and d_0 is, however, difficult especially in thin-film samples. We think that inclusion of the temperature dependence of λ and d_0 in our analysis does not alter our finding of scaling relation between γ and η/BT .

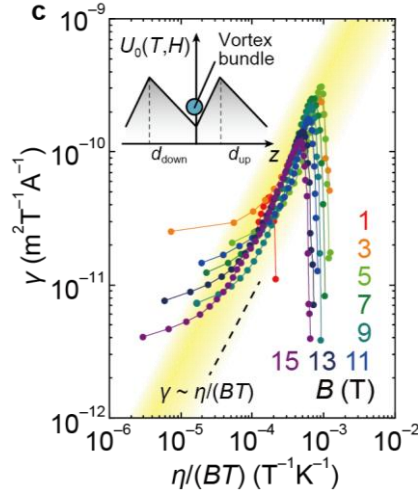


Figure R10 (Given as revised Fig. 3c). The relationship between γ and η/BT obtained for magnetic field $B = 1$ to 15 T. A dashed line represents a linear relation. (Inset) Schematic of asymmetric pinning potential around the vortex core.

Nevertheless, we admit that we put too much stress on “the linear relation”. Therefore, we revised the following sentence in the revised manuscript as follows:

Page 11, Line 17:

“...in the yellow shaded region of Fig. 3c. This linear scaling relation indicates that...”
was revised to

“...in the yellow shaded region of Fig. 3c. This **almost** linear scaling relation indicates that...”

Page 13 Line 10:

“The *linear* relation of $\gamma\eta/BT$ yields a size of the pinning potential $d_0 \sim 2.3$ nm by assuming the penetration depth $\lambda \sim 580$ nm in our sample (Supplementary Section 3).”
was revised to

“The relation of $\gamma\eta/BT$ yields a size of the pinning potential $d_0 \sim 2.3$ nm by assuming the penetration depth of our sample $\lambda \sim 580$ nm **as a temperature independent parameter** (Supplementary Section 3).”

B. Our response to the reviewers #2

Manuscript ID: MS#COMMSPHYS-24-1566-T

Reviewer #2

This paper reports the superconducting diode effect in thin films of iron-based superconductor Fe(Se,Te). They found an intriguing scaling relation between the two parameters η and γ , from the measurements of I-V characteristics and second-harmonic resistivity components. They provide a phenomenological model based on the vortex motion with asymmetric potential, which reasonably explains the observed behavior. The origins of the superconducting diode effect are currently under intense debate and the present results will have merit in these discussions. I found that discussions, including the heating issues in the Methods section, are well written. Thus I can recommend publication in Communication Physics in its current form.

Our Response

We would like to thank Reviewer #2 very much for their time devoted to carefully reviewing our manuscript. We are pleased with Reviewer's recommendation of our manuscript for publication in Communication Physics.

C. List of revisions

Manuscript ID: MS#COMMSPHYS-24-1566-T

Main Text

1. Page 4, Line 9: The sentences were revised as our Response #1-8
2. Page 5, Line 12: The sentences were revised as our Response #1-6
3. Page 7, Line 4: The sentence was revised as our Response #1-7
4. Page 9, Line 8: The sentence was revised as our Response #1-4
5. Page 11, Line 1: The sentences were revised as our Response #1-8
6. Page 11, Line 17: The sentences were revised as our Response #1-10
7. Page 13, Line 10: The sentence was revised as our Response #1-10
8. Page 13, Line 14: The sentences were added as our Response #1-9
9. Page 17, Line 13 (Materials and Methods section): The sentence was added as our Response #1-6
10. Page 17, Line 17 (Materials and Methods section): The sentences were revised as our Response #1-6
11. Page 29, Line 10 (caption of Fig. 3c): The sentence was added as our Response #1-10

Main Figure

12. Figure 1a: The figure was modified as our Response #1-5, 1-6, and 1-8
13. Figure 1c: $B_{c2}(T)$ curve was drawn by makers and line as our Response #1-7
14. Figure 3c: The dashed line was added as our Response #1-10

References

15. References 33 and 34 were added as our Response #1-7

Supplementary Information

16. Supplementary Section 3: The sentence was added as our Response #1-7
17. Supplementary Section 4: The sentences were revised as our Response #1-9

Supplementary Figure and Table

18. Supplementary Fig. S3: Top-view micrograph of the Fe(Se,Tc)/FeTe device was added as our Response #1-6

19. Supplementary Fig. S5c: Upper critical fields of FeSe bulk reference were added as our Response #1-7
20. Supplementary Table S1: The table was added as our Response #1-7

Supplementary References

21. References S6 – S11 were added as our Response #1-7

Miscellaneous revisions

In addition to these revisions as our respond to the reviewers' comments, the following revisions have been made to improve the correctness of explanations. These changes do not affect any conclusion presented in the manuscript.

22. Page 12, Line 14:

“Here, d_{up} and d_{down} represent the sizes of $U_0(B,T)$ from its minimum point...”
was revised to

“Here, d_{up} and d_{down} represent the sizes of **the pinning potential** from its minimum point...”

23. Page 13, Line 9:

“The substitution of it to V_c in Eq. (4)...”
was revised to

“The substitution of $\lambda\alpha^2$ to V_c in Eq. (4)...”