nature portfolio

Peer Review File

Unravel the rotational and translational behavior of a single squirmer in flexible polymer solutions at different Reynolds numbers

Corresponding Author: Professor Qi Kai

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The authors perform coupled Lattice-Boltzmann/Brownian s simulations to investigate the motion of single pusher, puller, and neutral squirmers in a polymer solution in fluids with different Reynold numbers. They find that the presence of polymers reduces the mean swimming velocity of the particles and enhances the rotational diffusion of the particles. The polymer-surface interactions (slip versus no-slip) have a relatively small effect except for strong pushers/pullers and high Reynolds number. The results are consistent with available experiments.

I support publication of this paper after the authors have addressed a few points.

1) Major: From a statistical mechanics point of view, the model has serious deficiencies because it is thermoally inconsistent even in aspects that are not explicitly non-equilibrium. The main problem is that the squirmer-polymer interactions are not reciprocal. The polymers are repelled by the squirmers, but the squirmers are only indirectly affected by the polymers via the coupling between the polymers and the flow. Such non-reciprocal interactions might result in unconventional cooperative phenomena that break the second law of thermos even in "equilibrium" systems where the squirmers are not active (i.e., the case B_1 = 0). Furthermore, in real systems, polymers are often introduced as crowders that mediate effective depletion interactions between other solutes, and this would not be captured correctly by the present model. (To see this, consider a simulation where the coupling to the Lattice Boltzmann fluid is turned off. The squirmers would then be totally oblivious to the presence of the polymers.)

Personally, I dislike this class of thermoally inconsistent models, but I acknowledge that they are commonly used in the active matter field and that the present model is probably well-suited to study the problem at hand, i.e., the influence of polymers on the rotational diffusion of the squirmers. I would just ask the authors to add a section on the limitations of the model in the conclusions, and a caveat regarding its use for simulations of many squirmers.

2) Major: The authors have varied various parameters of the system, but

not the system size (apparently). From other studies, it is known that hydro interactions not only affect the translational diffusion, but also the rotational diffusion of particle, see, e.g., Voegele et al, J. Phys. Chem. B 123, 5099 (2019). It is possible that such effects can be neglected in active systems, but if this is the case, it should be discussed and ideally shown by additional simulations.

3) Minor: Figure 9a suggests a crossover in the behavior of the rotational diffusion constant somewhere below the lowest polymer packing fraction that has been considered, 0.04. Could the authors comment on that? How does this packing fraction relate to the crossover concentration that separates dilute and semidilute polymer systems?

Reviewer #2

(Remarks to the Author)
Comments to the Authors

The paper studies the rotational and translational behavior of a single squirmer in polymer solutions at two specified Reynolds numbers by LB simulations. Eventually, the authors provide remarkable enhancements in the rotation motion and decay in translation motion. On the one hand, the results provide new insight into the influences of Reynolds number and squirmer-polymer boundary conditions on the s, but on the other hand, the reported conclusions are lack of significant and clear findings or deeper physical discussions compared with those of the previous work in the similar topic. Thus I have some points that should be considered before the paper could be considered for publication.

Major Points

1) My major concerns related to the physical contribution of the manuscript are the new findings compared with those of the authors' previous work (i.e., Ref. [63]). I believe that there are many similarities in these studies. The authors should state the conceptual advance offered in the new manuscript and focus on the differences between the present work and this published one.

2) The manuscript discussed the rotational and translational behavior of a single squirmer in polymer solutions at different Reynolds numbers. I think that the rotational behavior has been well reported in Ref. [63] while the translational behavior in the present study has not been discussed in Ref. [63]. The authors considered the mean swimming velocity as the sole translational characteristic, which is relatively weak. As the squirmer experiences 'random' rotation and 'random' walk, the MSD and translational diffusivity of the single squirmer are much more important physical quantities for translational behavior. Could the authors provide new insights?

3) In my opinion, the effect of flow inertia is one of the key aspects that has advanced beyond Ref. [63]. However, the manuscript presents results for only two Reynolds numbers. The impact of Reynolds number is not discussed in the same section, which makes the text more difficult to follow. Additionally, is there a critical Reynolds number at which a transition occurs in the rotational or translational behavior? What is the Reynolds number range where viscous forces dominate or inertial forces dominate in this work? The authors should conduct a direct comparison of the two Reynolds numbers and explore the Re-dependent rotational and translational behaviors across the range of 0.04–0.8.

4) I think the conclusion section is too specific and even longer than the introduction section, making it difficult for readers to grasp the key points. I think this is not suitable as a conclusion for Communications Physics. I suggest the authors provide a more concise conclusion and highlight the key contributions in comparison to Ref. [63]. As the author has written 'To thoroughly elucidate the relevance of fluid viscoelasticity, we have investigated the swimming behavior of swimmers at finite and low Reynolds numbers.', my view is that the study highlights the relevance of fluid viscoelasticity but does not thoroughly elucidate the underlying mechanism. For instance, it is essential to provide a simple model or scaling approach to estimate the diffusivity, one that at the least takes the Reynolds number into account.

5) Fig. 10 cannot provide sufficient information. The authors only investigated two values of R/l0, so it is insufficient to represent such a clear linear trend. I suggest the authors add more data points (e.g., R/l0=4 and 5) in the figures.

Minor Points

1) The authors state 'system heterogeneity plays a crucial role in determining the behavior'. It seems that heterogeneity is a key characteristic. However, which parameter can represent the degree of system heterogeneity? What will happen when a squirmer is in rigid polymer solutions? Additionally, is there any relationship between viscoelasticity and heterogeneity? I'm not sure which part is more important.

2) Could the author explain the similarities/differences between the LB simulation used in the present work and the MPCD in Ref. [63]? Do these methods yield the same results in a low-Re case, such as Re = 0.04 or Re = 0?

3) Does the polymer number affect the results, and is the chosen polymer number sufficient to obtain reliable results?

4) In Fig. 4, how does the author obtain the contours of number density and velocity for the monomers? Are they time-averaged or instantaneous results?

Reviewer #3

(Remarks to the Author)
The authors model the locomotion of a spherical squirmer in a viscoelastic fluid at finite Reynolds numbers using the Lattice-Boltzmann method (LBM) for simulating the solute and an immersed, inertialess bead-spring model for the polymers. The authors note that while the effects of viscoelasticity and fluid inertia on a squirmer have been studied separately in the past, their combined effects on swimming is not well known. The authors review much of the relevant literature to motivate their study and contribute a useful analysis of an inertial squirmer in a polymeric fluid, especially with regards to the consideration of rotational diffusivity.
In particular, the authors demonstrate greatly enhanced rotational diffusion in some situations due to asymmetric squirmer-polymer interactions.
However, I am left with several questions and there are a few points where the applicability or physical explanations of the manuscript are not clear.
I therefore recommend that the authors address the following points prior to publication, with the "major remarks" having higher priority.

Major Remarks

1. I am unsure to what physical systems the authors' results apply. Are there biological or artificial swimmers that fall into the parameter space of active stress and temperature that the authors chose for their study? The choice of temperature / kinetic energy ranges is especially confusing to me. From the labeling of Fig. 2, one can infer the temperatures to be unrealistically hot if the units of k_B T are assumed to be Joules (though no units are given). I think the authors should do some work to relate their results to the physical world, as I am not very familiar with LBM, and the manuscript works exclusively in LBM "units".

2. On a similar note, in what sorts of systems would the no-slip or repulsive boundary conditions best apply? What is the reason for considering both cases?

3. There should be a more thorough comparison of the present work with that of Ref. [63], which seems to be a similar study where the flow is presumably at or very near zero Reynolds number. I would expect that the results of these papers should be especially comparable in the Re = 0.04 case, and this appears to be true at least for the neutral squirmer. One key difference seems to be in the use of a Lennard-Jones polymer-boundary interaction in [63] versus the no-slip and repulsive conditions used in the present work. Does this difference in boundary conditions significantly affect the results? Are there any other significant differences? I suggest the authors note such differences in their resubmission to avoid seeming redundant.

4. I found the explanation of why there is less difference between the no-slip and repulsive boundary conditions at the lower Reynolds number (Re) of 0.04 to be somewhat unsatisfactory. The polymer "motility" is cited, but this is not a great description because the polymers are not self-propelled (motile) in any way. I believe the authors meant to refer to the "mobility" of the polymer, but even this should not be different because the polymers are hydroally inertialess and modeled with Stokeslets independently of Re. I think what is happening has to do with differences in background flow characteristics at one Reynolds number versus the other. In particular, I would expect sharper gradients near the squirmer surface at higher Re, which might lead to increased polymer-boundary interactions. I might be wrong, but I think the authors should reconsider or clarify their explanation of the physical mechanisms behind their results.

5. When investigating the effects of monomer packing fraction, the authors point out that "an increase in polymer concentration enlarges the viscous resistance, which further impairs the transport of squirmers". While I agree that effective viscosity should increase with monomer concentration, this does not necessarily impede the squirmer by itself. In the Newtonian case, squirmer speed only depends on the strength of the first mode of the slip velocity B_1, and it does not depend on viscosity. (The squirmer is mighty and will swim right through honey like it were water, but it will expend much more energy doing so.)

6. When investigating the effects of squirmer-polymer aspect ratio in Sec. 2.4, the squirmer radius is varied and the polymer length is held fixed. However, this choice causes the Reynolds number to change, too. Since the objective is to isolate the effect of aspect ratio, it would seem better to instead change the polymer length while keeping squirmer radius fixed (and keeping Reynolds number constant). As a side note, I suggest the authors use a term other than "aspect ratio". In my experience, aspect ratio almost always refers to the ratio of width to height of the same object, not the lengths of two different objects. Something more generic like "size ratio" might fit better.

7. I found the following article [*] (see below), which also considers an inertial squirmer in a Giesekus fluid. The authors should consider addressing this article in their introduction and potentially comparing trends regarding squirmer swimming speed.

8. I am curious if there is any handling or polymer tangling in the authors' simulations or if polymers are simply assumed to pass through one another if cross-overs occur. If the latter assumption is used, to the authors have a hypothesis for the

effects entanglements might cause? I could imagine polymers getting "tied around" squirmers for relatively long time scales and potentially causing interesting effects.

9. No error bars are shown on plots that give rotational diffusivity enhancement values, though there are rather significant statistical errors in swimming speed enhancements in many cases. The authors should include these errors or otherwise show that they are not significant.

Minor Remarks

10. It seems awkward to have a "Model and simulation methods" subsection under "Results". I would consider moving this subsection into its own standalone section.

11. In Sec. 2.1, the authors state that the swimming speed of a squirmer is "... determined by the first surface mode strength B_1", but this is only true either when Re = 0 or β = 0 in a Newtonian fluid. This fact should be stated for the unfamiliar reader.

12. While I think it makes sense to keep the details of the Lattice Boltzmann model in the supplemental materials (SM), I suggest including squirmer model description in main text to make the definition of the active stress clear to the reader. The authors use often use the term "no-slip" to refer to a boundary condition at the squirmer surface, but it is really a velocity-fixed slip condition as given by equation (9) in the SM.

13. Some notation seems not to be defined in the main text. For example, D_r^{p,ns} and $D_r^{p,ns}$ are used in the main text but the definitions are given in the captions of Figs. 2 and 3, which might not be immediately seen by the reader.

14. The authors might consider using the equivalent scales on comparable plots for easier visual comparison. For example, the vertical scales on Fig. 3a and 3b are slightly different. Similarly, plots in Figs. 3 and 6 have close enough vertical scales to consider making them the same so trends for the two different Reynolds numbers are most easily compared and contrasted.

15. I found a number of English grammatical issues while reading the manuscript (e.g., several incomplete sentences). While they did not severely detract from my understanding of the text, they were prevalent enough to become distracting. The authors should consider editing thoroughly for these issues.

[*] Z. Ouyang, Z. Lin, J. Lin, N. Phan-Thien, and J. Zhu, “Swimming of an inertial squirmer and squirmer dumbbell through a viscoelastic fluid,” Journal of Fluid Mechanics, vol. 969, p. A34, 2023. doi:10.1017/jfm.2023.593

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
I thank the authors for the clarifications regarding the definition of the model and providing a revised SI. My points have been addressed and I can now recommend publication.

Reviewer #2

(Remarks to the Author)
The authors have addressed most of the concerns I raised, and I have gained new insights from the revised manuscript. I recommend this paper for publication.

Upon further consideration, I would like to offer one final, but I believe important, suggestion. If the authors could provide some fitting expressions or functional forms for the results presented in figures (e.g., Figs. 8, 9, and 10), it would significantly enhance the clarity and utility of their findings.

Reviewer #3

(Remarks to the Author)
I appreciate the authors' diligence in addressing my many questions and comments. The revised manuscript's significance is now readily apparent and its clarity is significantly improved. I consider it to be a high quality work that I am happy to recommend for publication in the present form.

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Reviewer #1:

The authors perform coupled Lattice-Boltzmann/Brownian dynamics simulations to investigate the motion of single pusher, puller, and neutral squirmers in a polymer solution in fluids with different Reynold numbers. They find that the presence of polymers reduces the mean swimming velocity of the particles and enhances the rotational diffusion of the particles. The polymer-surface interactions (slip versus no-slip) have a relatively small effect except for strong pushers/pullers and high Reynolds number. The results are consistent with available experiments.

I support publication of this paper after the authors have addressed a few points.

We express our gratitude for your exceedingly positive feedback on our manuscript. We have meticulously reviewed and deliberated upon your valuable comments, addressing each comment accordingly. All modifications to the text in both the revised manuscript and the revised supplemental information are highlighted in red.

Comment 1:
From a statistical mechanics point of view, the model has serious deficiencies because it is thermodynamically inconsistent even in aspects that are not explicitly non-equilibrium. The main problem is that the squirmer-polymer interactions are not reciprocal. The polymers are repelled by the squirmers, but the squirmers are only indirectly affected by the polymers via the coupling between the polymers and the flow. Such non-reciprocal interactions might result in unconventional cooperative phenomena that break the second law of thermodynamics even in "equilibrium" systems where the squirmers are not active (i.e., the case $B_1 = 0$). Furthermore, in real systems, polymers are often introduced as crowders that mediate effective depletion interactions between other solutes, and this would not be captured correctly by the present model. (To see this, consider a simulation where the coupling to the Lattice Boltzmann fluid is turned off. The squirmers would then be totally oblivious to the presence of the polymers.)

Personally, I dislike this class of thermodynamically inconsistent models, but I acknowledge that they are commonly used in the active matter field and that the present model is probably well-suited to study the problem at hand, i.e., the influence of polymers on the rotational diffusion of the squirmers. I would just ask the authors to add a section on the limitations of the model in the conclusions, and a caveat regarding its use for simulations of many squirmers.

We would like to express our sincere gratitude to the referee for their constructive comments. We acknowledge that the methods were not described as thoroughly and clearly as they should have been in the initial version of the manuscript. In our study, the interactions between the polymer and squirmer are governed by two types of boundary conditions: the no-slip boundary condition and soft-sphere repulsion.

In the case of the no-slip boundary condition, when a monomer comes into contact with the squirmer, its translational velocity is reversed, thereby generating a force on the squirmer. To conserve momentum, an equal and opposite force is exerted on the monomer, in addition to the monomer-monomer bond and exclusive repulsion, which influence its motion according to Newton's second law. In this scenario, since the resultant no-slip force is not necessarily directed towards the center of mass of the squirmer, not only is the squirmer's translation affected, but its rotation is also influenced by the resultant torque.

Another type of boundary condition governing the interaction between the squirmer and polymer is the soft-sphere repulsion. In this case, when a monomer approaches the squirmer, it is repelled, generating a force directed towards the center of the squirmer. This force primarily affects the translational motion of the squirmer. In turn, an equal and opposite repulsive force is exerted on the monomer, influencing its motion in accordance with Newton's second law. It is important to note that polymers in the vicinity of the squirmer will alter the local flow field, thereby indirectly affecting both the rotational and translational motion of the squirmer.

In both boundary conditions, the monomer is treated as an inertialess Stokeslet and is updated according to the first Faxén relation. Consequently, the hydrodynamic force acting on the monomer is balanced by the total external force, which includes contributions from the monomer-monomer bond, repulsion, and the monomer-squirmer repulsion/no-slip force. Conversely, the hydrodynamic force is distributed to the fluid nodes following the interpolation scheme proposed by Peskin [Acta Numerica 11, 479–517 (2002)]. As a result, the total momentum of the system is conserved under both boundary conditions.

In summary, the squirmer can directly 'sense' the presence of polymers through either the no-slip boundary condition or the squirmer-polymer repulsion. The resulting force and torque affect its translational and rotational motion, either directly or indirectly. In both scenarios, the momentum of the system is conserved, both locally and globally. To provide further clarity on this point, we have revised the '1 Model and Simulation Method' section in the Supplemental Materials.

A limitation of our model arises when studying systems with multiple squirmers, as it does not allow for simulations at very high packing fractions. It is necessary to ensure sufficient spacing between squirmers to avoid discretization artifacts and issues related to unresolved lubrication forces. Adequate fluid nodes must be maintained between squirmers to preserve accurate hydrodynamic interactions and to prevent the formation of artificial vacuum regions. Another limitation of our model is that our flexible polymer chain representation cannot exhibit a significant elastic effect. Although very large Deborah numbers (256 at finite Reynolds number and 2222 at low Reynolds number) are obtained in our simulations, they actually indicate that the relaxation process of locally deformed polymers is too slow to produce any meaningful elasticity. Therefore, further investigation is needed to determine whether the entangled polymer solution as a whole possesses substantial viscoelasticity.

We have added this comment to the final paragraph of section '4 Summary' in the main manuscript for clarity, with the new text highlighted in red.

Comment 2:
The authors have varied various parameters of the system, but not the system size (apparently). From other studies, it is known that hydrodynamic interactions not only affect the translational diffusion, but also the rotational diffusion of particle, see, e.g., Voegele et al, J. Phys. Chem. B 123, 5099 (2019). It is possible that such effects can be neglected in active systems, but if this is the case, it should be discussed and ideally shown by additional simulations.

We thank the referee for raising this important question. The finite-size effect is a common concern in soft matter simulations, as one must ensure that the system's dynamics are not influenced by interactions with its own periodic images. In our study, we used a squirmer with radius $R = 3a$, and a simulation box of width $L = 72a$ was chosen to provide sufficient separation between the squirmer and its periodic image. To further address concerns regarding finite-size effects, we performed additional simulations with a larger box size of $L = 108a$ for both pullers and pushers at Reynolds number Re=0.04 and temperature $k_BT/\varepsilon = 0.0005$. The resulting rotational diffusion coefficients, translational diffusion coefficients, and swimming speeds show good agreement with those obtained in the smaller system, indicating that finite-size effects do not significantly influence our results. The new results with a system size of $L=108a$ have been incorporated in the Supplemental Materials (sections 2.7) and highlighted in red for clarity.

Comment 3:
Figure 9a suggests a crossover in the behavior of the rotational diffusion constant somewhere below the lowest polymer packing fraction that has been considered, 0.04. Could the authors comment on that? How does this packing fraction relate to the crossover concentration that separates dilute and semidilute polymer systems?

We thank the referee for his/her insightful comments. The observed crossover occurs at zero polymer concentration, corresponding to a Newtonian fluid. This represents the ideal case in which the rotational diffusion of a swimmer does not depend on its active stress. However, due to statistical fluctuations, the measured values show slight deviations from the ideal values. Following the referee's suggestion, we have replaced the ideal case with the measured results. The updated figure is now presented as Fig. 11. In this regime, hydrodynamic interactions result in symmetric momentum exchange on both sides of the squirmer, leading to a vanishing net torque acting on the swimmer. Consequently, the rotational motion becomes independent of the active stress, yielding an almost unit enhancement. At a packing fraction of $\phi = 0.04a^{-3}$, the scaled monomer concentration is $\phi/\phi_0 = 0.76$ under conditions $k_BT/\varepsilon = 0.0005$ and Re=0.04, indicating that the system lies within the dilute regime. Here, $\phi_0 = N_m/(4/3\pi R_{g0}^3) = 0.052a^{-3}$ represents the polymer overlap concentration. For a higher packing fraction of $\phi = 0.062a^{-3}$, the scaled monomer concentration becomes $\phi/\phi_0 = 1.18$, placing the system in the semidilute regime. However, significant rotational enhancements—ranging from approximately 30 to 100—are observed for pushers/pullers across both concentration regimes. This indicates that the crossover behavior depicted in Figure 11 is not directly attributable to the polymer concentration regime. Rather, it is the system heterogeneity introduced by the presence of polymers that plays a crucial role in the observed rotational enhancement. We have clarified and emphasized this point using red text in the main manuscript (section 3.4).

Reviewer #2:

The paper studies the rotational and translational behavior of a single squirmer in polymer solutions at two specified Reynolds numbers by LB simulations. Eventually, the authors provide remarkable enhancements in the rotation motion and decay in translation motion. On the one hand, the results provide new insight into the influences of Reynolds number and squirmer-polymer boundary conditions on the dynamics, but on the other hand, the reported conclusions are lack of significant and clear findings or deeper physical discussions compared with those of the previous work in the similar topic. Thus I have some points that should be considered before the paper could be considered for publication.

We thank the referee for the time and providing his/her recommendation as these constructive comments have improved the readability and explanations of our manuscript. Below, we address all the comments of the referee in detail.

Comment 1:
My major concerns related to the physical contribution of the manuscript are the new findings compared with those of the authors' previous work (i.e., Ref. [63]). I believe that there are many similarities in these studies. The authors should state the conceptual advance offered in the new manuscript and focus on the differences between the present work and this published one.

We thank the referee for the constructive comments on our work. Both this study and our previous work (Ref. [63]) investigate the behavior of a single swimmer in a flexible polymer solution, with a shared focus on understanding the microscopic mechanisms behind rotational enhancement. However, the two studies explore different aspects of this phenomenon.

Our previous work (Ref. [63]) performed hydrodynamic simulations of a squirmer immersed in a polymer solution using multiparticle collision dynamics (MPC) at Re=1.1. That study focused on revealing the rotational enhancement of a neutral swimmer with surface attraction in a viscoelastic fluid. The amplification is a consequence of two effects, a decrease of the amount of adsorbed polymers by active motion and an asymmetric encounter with polymers on the squirmer surface. The effect of active stress was briefly discussed in the Supplementary Materials, while the transport properties was not addressed.

In contrast, the present work focuses on elucidating the influence of the Reynolds number on both the translational and rotational behavior of squirmers with various active stresses. Here, no surface attraction is employed. Instead, the influence of two boundary conditions, i.e., no-slip and purely repulsive, are examined. The hydrodynamic interactions due to active stress are sufficiently strong to induce profound effective attraction to the polymers. The observed rotational enhancements of pushers and pullers arise from distinct mechanisms driven by active stress. Specifically, polymer wrapping occurs in front of a pusher, while numerous polymers are absorbed in the rear of a puller. Both mechanisms enhance the rotational motion while simultaneously suppressing the swimming speed through forces and torques arising from direct contacts or asymmetric local flows induced by the polymers. One key result is the identification of a critical Reynolds number, Re_c=0.2. Above Re_c, the dominant inertial effects of the fluid efficiently advect polymers into the vicinity of the squirmer, leading to more pronounced rotational enhancement for swimmers under the no-slip boundary condition. However, in the low-Re regime, this effect diminishes because the strong viscous drag of the fluid impedes polymer advection toward the squirmer, resulting in similar rotational enhancements under both boundary conditions.

The comparison between the two studies is discussed in the first paragraph of section '3.4 Effects of Reynolds number' in the main text and is highlighted in red.

Comment 2:
The manuscript discussed the rotational and translational behavior of a single squirmer in polymer solutions at different Reynolds numbers. I think that the rotational behavior has been well reported in Ref. [63] while the translational behavior in the present study has not been discussed in Ref. [63]. The authors considered the mean swimming velocity as the sole translational characteristic, which is relatively weak. As the squirmer experiences 'random' rotation and 'random' walk, the MSD and translational diffusivity of the single squirmer are much more important physical quantities for translational behavior. Could the authors provide new insights?

We thank the referee for the insightful suggestion. In response, we have conducted a detailed investigation of the translational diffusivity of squirmers across a range of Reynolds numbers. Additionally, we examined the effects of temperature, monomer concentration, squirmer–polymer size ratio, and polymer chain length on translational diffusion. Overall, we find that the translational diffusivity generally follows the relation $D_t^a \sim D_t^p + U^2/D_r^a$. Given that the variation in swimming speed U is relatively small, the translational diffusivity is strongly influenced by the rotational motion. Specifically, increased rotational diffusion tends to suppress translational diffusion. For consistency with the rotational diffusion enhancement, the translational diffusion enhancement is characterized by normalizing D_t^a with the diffusion coefficient D_t^p of the passive colloid. Our results indicate that both activity and boundary conditions have clear impacts on the translational enhancements. In particular, pullers with the rear-side polymer suction effect exhibit higher rotational diffusion enhancements and therefore lower translational diffusion enhancements in most cases. Additionally, compared to the repulsive case, the no-slip boundary condition results in stronger rotational enhancement in the finite Reynolds number regime, leading correspondingly to smaller translational diffusion enhancements. The impacts of various factors, including Reynolds number, temperature, monomer concentration, squirmer–polymer size ratio, and polymer chain length are discussed in detail in the revised manuscript (sections 3.1, 3.2, 3.3, and 3.4) and the Supplemental Materials (sections 2.2, 2.5, 2.6, and 2.7), and have been highlighted in red for clarity.

Comment 3:
In my opinion, the effect of flow inertia is one of the key aspects that has advanced beyond Ref. [63]. However, the manuscript presents results for only two Reynolds numbers. The impact of Reynolds number is not discussed in the same section, which makes the text more difficult to follow. Additionally, is there a critical Reynolds number at which a transition occurs in the rotational or translational behavior? What is the Reynolds number range where viscous forces dominate or inertial forces dominate in this work? The authors should conduct a direct comparison of the two Reynolds numbers and explore the Re-dependent rotational and translational behaviors across the range of 0.04–0.8.

We thank the referee for the constructive suggestion. In response, we have conducted a detailed investigation of the transport and rotational properties of squirmers across the Reynolds number range of 0.04–0.8. The main results are summarized below.

The rotational diffusion enhancement generally decreases with increasing Reynolds number, primarily because the rotational diffusion of passive colloids rises as fluid viscosity drops. While passive particles are strongly affected by this change, the rotational diffusion of active swimmers shows smaller variations due to the hydrodynamic and steric interactions with polymers. Pushers and pullers exhibit much stronger enhancements than neutral swimmers, owing to the front-side polymer wrapping and rear-side polymer suction effects, respectively. Interestingly, we identified a critical Reynolds number, Re_c=0.2, above which the difference between the no-slip and repulsive boundary conditions becomes significant but remains minor in the low-Re regime. Notably, pushers are more sensitive to the no-slip boundary condition, whereas pullers maintain stable behavior because the rear-side suction effect is less dependent on viscosity, resulting in distinct trends for different swimmer types.

The swimming velocities of all swimmer types decrease across the entire range of Reynolds numbers. A critical value at Re_c=0.2 marks the threshold above which the no-slip boundary condition largely reduces swimming speeds compared to the purely repulsive case, due to closer polymer–squirmer interactions at finite Reynolds numbers. Below Re_c, the difference between the two boundary conditions is minimal.

The translational diffusion enhancement decreases with increasing Reynolds number, primarily due to the higher diffusivity of passive colloids. Above the critical value Re_c=0.2, the no-slip boundary condition produces stronger differences for pushers and pullers compared to the repulsive case, while these differences remain small at low Re. Neutral swimmers and pushers generally exhibit greater translational diffusion than pullers because pullers experience stronger rotational motion from the rear-side polymer suction effect. However, at finite Re, pushers under the no-slip boundary condition show similar enhancements to pullers due to the strong front-side polymer wrapping, which also reduces their translational diffusivity compared to the repulsive case.

The detailed revisions are included in both the main manuscript (section ‘3.3 Effects of Reynolds number’) and the Supplemental Materials (section 2.5 ‘Additional results on the effects of Reynolds number’). All changes are highlighted in red for clarity.

Comment 4:
I think the conclusion section is too specific and even longer than the introduction section, making it difficult for readers to grasp the key points. I think this is not suitable as a conclusion for Communications Physics. I suggest the authors provide a more concise conclusion and highlight the key contributions in comparison to Ref. [63]. As the author has written 'To thoroughly elucidate the relevance of fluid viscoelasticity, we have investigated the swimming behavior of swimmers at finite and low Reynolds numbers.', my view is that the study highlights the relevance of fluid viscoelasticity but does not thoroughly elucidate the underlying mechanism. For instance, it is essential to provide a simple model or scaling approach to estimate the diffusivity, one that at the least takes the Reynolds number into account.

We thank the referee for pointing out this issue. We have moved the comparison of our work with previous studies to a separate section titled '3.5 Comparisons with Other Works' and have re-summarized the results according to the referee's suggestions in the section titled "4 Summary." In particular, we addressed the role of the Reynolds number in determining the rich dynamic behavior of different types of swimmers. The rotational and transport properties are summarized separately, and the underlying mechanisms leading to the diffusion enhancements are discussed. Finally, concluding remarks and a comparison of our squirmer model with real swimmers are discussed. The revisions can be found in the main manuscript (sections 3.5 and 4). All changes are highlighted in red for clarity.

Regarding the underlying mechanism, our study clearly demonstrates how hydrodynamic and steric interactions between deformed polymers and squirmers influence their rotational motion, which in turn impacts their swimming speed and translational diffusion. Basically, the greater the rotational enhancement, the less the swimmer can maintain persistent translational motion, leading to reductions in swimming speed and translational diffusion. These mechanisms depend on the type of active stress: polymer wrapping occurs in front of a pusher, while numerous polymers are absorbed at the rear of a puller. These interactions generate heterogeneous forces and torques or induce asymmetric local flows that amplify the squirmer's rotational motion. In contrast, the source dipole flow field of a neutral swimmer allows polymers to skim rapidly over its surface, resulting in no substantial enhancement. As shown in Fig. S5 and 6, the diffusion coefficients of swimmers generally show weak dependence on the Reynolds number but are substantially influenced by the active stress and boundary condition. Therefore, it is difficult to use a simple model or scaling approach to describe these systems. On the other hand, both the rotational and translational diffusion enhancements exhibit clear decays as Re increases. These reductions are primarily attributed to the increase in the diffusion coefficient of the passive colloid, which appears in the denominator when calculating the enhancement and thus strongly affects its trend.

Comment 5:
Fig. 10 cannot provide sufficient information. The authors only investigated two values of R/l0, so it is insufficient to represent such a clear linear trend. I suggest the authors add more data points (e.g., R/l0=4 and 5) in the figures.

We thank the referee for the helpful suggestion. To make the results more convincing, we have performed additional simulations for squirmers with radius $R/l_0^n=4.5$. The rotational diffusion enhancements, swimming velocities, and translational diffusion enhancements all exhibit clear linear trends, further supporting the validity of our results. The corresponding Figs. 13 and 15 have been updated accordingly.

Comment 6:
The authors state 'system heterogeneity plays a crucial role in determining the behavior'. It seems that heterogeneity is a key characteristic. However, which parameter can represent the degree of system heterogeneity? What will happen when a squirmer is in rigid polymer solutions? Additionally, is there any relationship between viscoelasticity and heterogeneity? I'm not sure which part is more important.

We thank the referee for raising this question.

Compared to a pure monomer fluid, which exhibits an almost homogeneous monomer distribution throughout the system, the globular structure of polymers—where monomers are bonded via a strong FENE potential—results in a relatively heterogeneous distribution. Consequently, in a relatively dilute polymer solution, when a squirmer approaches the polymers, the flow field generated by the active stress leads to heterogeneous steric and hydrodynamic interactions on the squirmer. Specifically, polymer wrapping occurs at the front of a pusher, while polymers are

absorbed at the rear of a puller. In these cases, the entire polymer chain interacts with the squirmer, and these encounters are typically heterogeneous in a dilute solution. For example, a single polymer may wrap around the front of a pusher at a time, or only one polymer may attach to one side of a puller's rear. These mechanisms create net forces and torques, or induce asymmetric local flows, which enhance the squirmer's rotational motion. In contrast, in a very dense system, since many polymers approach the squirmer simultaneously, their effects can balance out, resulting in weaker enhancement. For example, multiple polymers absorbed symmetrically on both sides of a puller's rear can generate symmetric flows that cancel out net forces or torques acting on the puller. Therefore, the monomer packing fraction ϕ/ϕ_0 can serve as an indicator of system heterogeneity: for a polymer solution, a larger ϕ/ϕ_0 generally indicates a more homogeneous system, while a smaller value implies greater heterogeneity. However, ϕ/ϕ_0 alone does not determine the rotational enhancement of the squirmer. In a very dilute solution, although the system is more heterogeneous, the average time between two squirmer–polymer encounters is relatively long, so the accumulated effect remains weak. As a result, a maximum enhancement can appear at an intermediate monomer packing fraction. In our case, this maximum occurs at $\phi/\phi_0 = 3.58$, beyond which the enhancement for a puller decreases and reaches a plateau for a pusher. Due to computational cost, we cannot simulate systems with $\phi/\phi_0 > 4.7$. If we could, the enhancement for a pusher would likely begin to decrease, and a maximum would be observed for a neutral swimmer. On the other hand, in a pure monomer solution, because monomers are homogeneously distributed, they encounter the squirmer uniformly during swimming, leading to no net force or torque and thus no rotational enhancement. Therefore, the globular structure of the polymer as a whole is essential for this effect. We have added text in the second paragraph of section '3.4 Other effects' of the main manuscript to address this point and highlighted it in red for clarity.

It is a very interesting suggestion to simulate a system with rigid polymers. On one hand, since rigid polymers cannot be deformed, their rod-like structure may lead to different swimmer dynamics compared to flexible chains. However, the system heterogeneity associated with an intermediate monomer packing fraction ϕ/ϕ_0 would still play an important role. Additionally, if semiflexible polymers with strong bending stiffness are considered instead of fully rigid ones, the elasticity of the system may be significantly enhanced, resulting in richer squirmer dynamics. Due to the limited time available for revision, this paper focuses mainly on the effects arising from flexible polymers. Investigating the influence of rigid and semiflexible polymers on the translational and rotational motion of squirmers will be an interesting direction for future work.

In our system, the flexible polymer induces a more pronounced viscous effect rather than an elastic one. Therefore, viscoelasticity is not the primary factor responsible for the rich dynamics observed here. Instead, our work highlights the importance of mechanical and hydrodynamic interactions arising from polymer deformation and system heterogeneity in governing the transport and rotational behavior of the squirmer. As inspired by the referee, the interplay between viscoelasticity and polymer deformation in a system with semiflexible polymers of appropriate bending stiffness could be an interesting topic for future investigation.

Comment 7:
Could the author explain the similarities/differences between the LB simulation used in the present work and the MPCD in Ref. [63]? Do these methods yield the same results in a low-Re case, such as Re = 0.04 or Re = 0?

We thank the referee for raising this important point.

Both methods referenced are classical approaches widely employed in hydrodynamic simulations. In principle, the simulation results should be independent of the specific method used, provided that all relevant physical parameters are properly captured. In the previous work utilizing the multiparticle collision dynamics (MPC) method, the rotational behavior of a neutral swimmer with surface attraction was analyzed in the main text, while the behavior of pushers and pullers without surface attraction was shortly discussed in the supplemental materials. That study was only conducted at a finite Reynolds number of Re = 1.1. In contrast, the present work employs the lattice Boltzmann method to investigate both the rotational and translational dynamics of microswimmers in viscoelastic fluids across a range of Reynolds numbers, specifically Re = 0.8, 0.04. At finite Reynolds numbers (Re∼1), both studies report qualitatively and quantitatively consistent rotational behaviors of microswimmers. In particular, for pushers, polymers tend to accumulate and wrap around the front of the swimmer, whereas for pullers, polymers are predominantly absorbed in the rear. In both cases, these interactions enhance rotational motion and hinder translational movement due to forces and torques resulting from direct polymer-swimmer contacts or asymmetric local flow fields induced by polymer

dynamics. For neutral swimmers, the source dipole flow field rapidly advects polymers toward the rear, thereby minimizing their impact on both rotational and translational dynamics.

It is worth noting that the rotational enhancements for different types of swimmers exhibit overall quantitative agreement between the two methods, though some differences remain. In MPC, pushers show a stronger enhancement of approximately 12, while pullers exhibit a weaker enhancement of about 2. In contrast, both pushers and pullers show an enhancement of around 4 in LBM under similar parameter settings. These discrepancies can be attributed to variations in key simulation parameters, including the Reynolds number (Re = 1.1 in MPC vs. Re = 0.8 in LBM), overlap concentration ($\phi/\phi^* = 2.95$ in MPC and $\phi/\phi^* = 5.64$ in LBM), particle size ($R/l_0^n = 6$ in MPC and $R/l_0^n = 3$ in LBM), and Péclet number ($Pe_t = 679$ in MPC and $Pe_t = 2800$ in LBM). Given these differences, the observed variations in rotational enhancement are within reasonable expectations. The Péclet number $\mathrm{Pe_t}=\mathrm{U_0 l/D}$ and the Reynolds number $\mathrm{Re}=\rho \mathrm{U_0 l}/\eta$. Here, l denotes the size of the swimmer, U_0 is the swimming velocity, ρ is the fluid density, η is the viscosity, and D is the translational diffusion coefficient.

Comment 8:
Does the polymer number affect the results, and is the chosen polymer number sufficient to obtain reliable results?

We thank the referee for pointing out this issue. The rotational enhancements and transport properties indeed show clear dependencies on polymer concentration. These results are already presented in Fig. 11and 12 and discussed in the main text. We summarize the key points here for clarity. System heterogeneity plays a crucial role in determining the rotational behavior of the swimmer. In the dilute polymer regime, only a few polymers interact with the squirmer, resulting in minimal enhancement. As the polymer concentration increases, rotational enhancement becomes more pronounced due to more frequent heterogeneous encounters between the squirmer and surrounding polymers. However, this enhancement eventually reaches a plateau, as the system becomes more homogeneous with higher polymer concentration. Simultaneously, the swimming velocities and translational diffusivities of all swimmer types decrease monotonically with increasing polymer concentration, primarily due to the impediment caused by enhanced rotational motion.

Comment 9:
In Fig. 4, how does the author obtain the contours of number density and velocity for the monomers? Are they time-averaged or instantaneous results?

We thank the referee for raising this question. The contour plots of monomer number density and velocity are generated by time-averaging the local monomer density and velocity fields in the vicinity of the squirmer, in the cylindrical center-of-mass frame of the swimmer, across multiple time frames. We have addressed this point in red in the captions of the corresponding figures (Fig. 4, 7, and 14) in the main text.

Reviewer #3:

The authors model the locomotion of a spherical squirmer in a viscoelastic fluid at finite Reynolds numbers using the Lattice-Boltzmann method (LBM) for simulating the solute and an immersed, inertialess bead-spring model for the polymers. The authors note that while the effects of viscoelasticity and fluid inertia on a squirmer have been studied separately in the past, their combined effects on swimming is not well known. The authors review much of the relevant literature to motivate their study and contribute a useful analysis of an inertial squirmer in a polymeric fluid, especially with regards to the consideration of rotational diffusivity. In particular, the authors demonstrate greatly enhanced rotational diffusion in some situations due to asymmetric squirmer-polymer interactions. However, I am left with several questions and there are a few points where the applicability or physical explanations of the manuscript are not clear. I therefore recommend that the authors address the following points prior to publication, with the "major remarks" having higher priority.

We thank the referee for the time and providing his/her recommendation as these constructive comments have improved the readability and explanations of our manuscript. Below, we address all the comments of the referee in detail.

Comment 1:
I am unsure to what physical systems the authors' results apply. Are there biological or artificial swimmers that fall into the parameter space of active stress and temperature that the authors chose for their study? The choice of temperature / kinetic energy ranges is especially confusing to me. From the labeling of Fig. 2, one can infer the temperatures to be unrealistically hot if the units of k B T are assumed to be Joules (though no units are given). I think the authors should do some work to relate their results to the physical world, as I am not very familiar with LBM, and the manuscript works exclusively in LBM "units".

We thank the referee for raising this important question. In the lattice Boltzmann method, especially with the Ludwig code used in this work, the thermal energy is typically set below $0.0001\mathrm{ma}^2/\tau^2$, where m, a, and τ are the lattice Boltzmann units of mass, length, and time, respectively. If the thermal energy exceeds this limit, the relevant hydrodynamics cannot be properly resolved. Therefore, we chose the thermal energy in the range of $0.000001\mathrm{ma}^2/\tau^2$ and $0.0001\mathrm{ma}^2/\tau^2$. Previously, we used the kinetic energy to normalize the thermal energy, but we now recognize this may not be the most appropriate choice. Instead, we use the polymer bond strength $\varepsilon = 0.002\mathrm{ma}^2/\tau^2$ as a more natural reference, resulting in a dimensionless thermal energy of $k_BT/\varepsilon = 0.0005\sim0.05$. To compare with experimental systems, we defined two widely used dimensionless parameters in active matter simulations: the Péclet number $\mathrm{Pe_t} = \mathrm{U_0 l/D}$ and the Reynolds number $\mathrm{Re}=\rho \mathrm{U_0 l}/\eta$. Here, l denotes the size of the swimmer, U_0 is the swimming velocity, ρ is the fluid density, η is the viscosity, and D is the persistence time. The Péclet number characterizes the competition between translational diffusion and active motion, while the Reynolds number characterizes the competition between inertial and viscous forces. We have conducted a thorough literature review on real swimmers and calculated these two dimensionless numbers for comparison. The results from our simulations and experimental estimates are summarized in tables and provided in the Supplementary Materials. The good agreement between our squirmer model and real swimmers supports the validity of our simulation approach. This comparison has been described in a new subsection (section '2.8 Comparison with real swimmers') added to the Supplementary Materials, and the main manuscript has also been revised accordingly (the last paragraph in section '4 Summary'). All new texts have been highlighted in red for clarity.

Comment 2:
On a similar note, in what sorts of systems would the no-slip or repulsive boundary conditions best apply? What is the reason for considering both cases?

We thank the referee for raising this important question. The no-slip boundary condition allows for momentum exchange between two contacting objects in both the normal and tangential directions. In contrast, a purely repulsive boundary condition permits momentum exchange only in the normal direction. Both types of boundary conditions are commonly used in soft matter simulations. However, when lateral momentum exchange between contacting objects is relevant, the no-slip condition is generally preferred. In our study, our goal was to examine whether the choice of boundary condition influences the hydrodynamic behavior of squirmers in a polymer solution. To this end, we conducted comparative simulations using both boundary conditions across a range of Reynolds numbers. Our results

show that the no-slip boundary condition leads to relatively stronger effect in both transport and rotation properties above a critical Reynolds number Re_c = 0.2, due to additional tangential momentum exchanges between squirmers and monomers. However, this effect becomes negligible in low-Re regime, where viscous drag dominates and suppresses direct contact between squirmers and polymers. For technical convenience, we employed only the repulsive boundary condition when analyzing the effects of polymer concentration, squirmer size, and polymer length on swimmer behavior in the low-Re regime.

Comment3:
There should be a more thorough comparison of the present work with that of Ref. [63], which seems to be a similar study where the flow is presumably at or very near zero Reynolds number. I would expect that the results of these papers should be especially comparable in the Re = 0.04 case, and this appears to be true at least for the neutral squirmer. One key difference seems to be in the use of a Lennard-Jones polymer-boundary interaction in [63] versus the no-slip and repulsive conditions used in the present work. Does this difference in boundary conditions significantly affect the results? Are there any other significant differences? I suggest the authors note such differences in their resubmission to avoid seeming redundant.

We thank the referee for the constructive comments on our work. Both this study and our previous work (Ref. [63]) investigate the behavior of a single swimmer in a flexible polymer solution, with a shared focus on understanding the microscopic mechanisms behind rotational enhancement. However, the two studies explore different aspects of this phenomenon.

Our previous work (Ref. [63]) performed hydrodynamic simulations of a squirmer immersed in a polymer solution using multiparticle collision dynamics (MPC) at Re=1.1. That study focused on revealing the rotational enhancement of a neutral swimmer with surface attraction in a viscoelastic fluid. The amplification is a consequence of two effects, a decrease of the amount of adsorbed polymers by active motion and an asymmetric encounter with polymers on the squirmer surface. The effect of active stress was briefly discussed in the Supplementary Materials, while the transport properties was not addressed.

In contrast, the present work focuses on elucidating the influence of the Reynolds number on both the translational and rotational behavior of squirmers with various active stresses. Here, no surface attraction is employed. Instead, the influence of two boundary conditions, i.e., no-slip and purely repulsive, are examined. The hydrodynamic interactions due to active stress are sufficiently strong to induce profound effective attraction to the polymers. The observed rotational enhancements of pushers and pullers arise from distinct mechanisms driven by active stress. Specifically, polymer wrapping occurs in front of a pusher, while numerous polymers are absorbed in the rear of a puller. Both mechanisms enhance the rotational motion while simultaneously suppressing the swimming speed through forces and torques arising from direct contacts or asymmetric local flows induced by the polymers. One key result is the identification of a critical Reynolds number, Re_c=0.2. Above Re_c, the dominant inertial effects of the fluid efficiently advect polymers into the vicinity of the squirmer, leading to more pronounced rotational enhancement for swimmers under the no-slip boundary condition. However, in the low-Re regime, this effect diminishes because the strong viscous drag of the fluid impedes polymer advection toward the squirmer, resulting in similar rotational enhancements under both boundary conditions.

The comparison between the two studies is discussed in the first paragraph of section ‘3.4 Effects of Reynolds number’ in the main text and is highlighted in red.

Comment4:
I found the explanation of why there is less difference between the no-slip and repulsive boundary conditions at the lower Reynolds number (Re) of 0.04 to be somewhat unsatisfactory. The polymer "motility" is cited, but this is not a great description because the polymers are not self-propelled (motile) in any way. I believe the authors meant to refer to the "mobility" of the polymer, but even this should not be different because the polymers are hydrodynamically inertialess and modeled with Stokeslets independently of Re. I think what is happening has to do with differences in background flow characteristics at one Reynolds number versus the other. In particular, I would expect sharper gradients near the squirmer surface at higher Re, which might lead to increased polymer-boundary interactions. I might be wrong, but I think the authors should reconsider or clarify their explanation of the physical mechanisms behind their results.

We thank the referee for the helpful comment. We have replaced the word “mobility” with “motility” throughout the manuscript. As suggested by the referee, we examined the fluid velocity distribution for squirmers with various active stresses at two Reynolds numbers in a Newtonian fluid. At low Reynolds number, the fluid exhibits lower motility due to higher viscous drag. As a result, polymers advected by the flow are less motile, leading to a reduced number of polymers in the vicinity of the squirmer. Therefore, the influence of the no-slip boundary condition becomes less significant. We have updated the relevant text in the manuscript (page 18) and added a new section (section ‘2.4 Fluid velocity distribution’) in the Supplementary Materials. All new texts have been highlighted in red for clarity.

Comment5:
When investigating the effects of monomer packing fraction, the authors point out that "an increase in polymer concentration enlarges the viscous resistance, which further impairs the transport of squirmers". While I agree that effective viscosity should increase with monomer concentration, this does not necessarily impede the squirmer by itself. In the Newtonian case, squirmer speed only depends on the strength of the first mode of the slip velocity B_1, and it does not depend on viscosity. (The squirmer is mighty and will swim right through honey like it were water, but it will expend much more energy doing so.)

We thank the referee for the insightful comment. We agree that the increased viscous resistance alone does not change the squirmer’s swimming velocity. Therefore, we have deleted that sentence in the main manuscript. However, higher polymer concentrations enhance the squirmer’s rotational motion, which in turn reduces its swimming speed. We have revised the main text accordingly (the first paragraph in page 19), with the changes highlighted in red.

Comment 6:
When investigating the effects of squirmer-polymer aspect ratio in Sec. 2.4, the squirmer radius is varied and the polymer length is held fixed. However, this choice causes the Reynolds number to change, too. Since the objective is to isolate the effect of aspect ratio, it would seem better to instead change the polymer length while keeping squirmer radius fixed (and keeping Reynolds number constant). As a side note, I suggest the authors use a term other than "aspect ratio". In my experience, aspect ratio almost always refers to the ratio of width to height of the same object, not the lengths of two different objects. Something more generic like "size ratio" might fit better.

We thank the referee for raising this point.

To investigate the effect of squirmer size, a smaller value of $B_1=0.003$ was used to prevent excessively strong flows from breaking polymer bonds. This results in a Reynolds number of Re=0.048 for a larger squirmer with $R/l_0^n=6$, and Re=0.024 for a smaller squirmer with $R/l_0^n=3$. These values are all close to Re=0.04, placing them firmly in the low-Reynolds-number regime. As such, the dynamic behaviors of squirmers with different sizes remain comparable and physically relevant. As suggested by the second referee, we have performed additional simulations for squirmers with radius $R/l_0^n = 4.5$. The rotational diffusion enhancements, swimming velocities, and translational diffusion enhancements all exhibit clear linear trends, further supporting the validity of our results. The corresponding Figs. 13 and 15 in the main manuscript have been updated accordingly.

Due to space limitations in the main text, the discussion on the effect of polymer length is provided in the Supplemental Materials. Specifically, for a puller with $R/l_0^n=3$, the rotational enhancement increases by approximately 35% when shorter polymers ($N_m=60$) are used. This moderate increase is primarily attributed to a reduction in the rotational diffusion of the passive colloid. However, this result should not be overemphasized, as the rotational enhancement $\gamma=D_r^a/D_r^p$ is highly sensitive to the values of the rotational diffusion coefficients—small variations in these parameters can lead to noticeable changes in γ. In contrast, the transport properties of squirmers are unaffected by changes in polymer length. A corresponding remark in red referring to the polymer length effect has been added to the page 22 of the main text. The more detailed information can be found in section ‘2.6 Influences of polymer length at Re = 0.04’ in the Supplemental Materials.

We have changed the ‘aspect ratio’ to ‘size ratio’ in both the main text and Supplemental Materials.

Comment 7:
I found the following article [*] (see below), which also considers an inertial squirmer in a Giesekus fluid. The authors should consider addressing this article in their introduction and potentially comparing trends regarding squirmer swimming speed.

[*] Z. Ouyang, Z. Lin, J. Lin, N. Phan-Thien, and J. Zhu, "Swimming of an inertial squirmer and squirmer dumbbell through a viscoelastic fluid," Journal of Fluid Mechanics, vol. 969, p. A34, 2023. doi:10.1017/jfm.2023.593

We thank the referee for pointing out the relevant literature. We have carefully reviewed this work and summarized the key similarities and differences below.

The swimming properties of an inertial spherical squirmer and a squirmer dumbbell moving through an athermal viscoelastic fluid were recently reported by Ouyang, in which the fluid's viscoelasticity was modeled using the Giesekus constitutive equation. The competing effects of fluid inertia and elasticity on swimmer locomotion were thoroughly investigated. At Re=5, all types of swimmers exhibit a monotonic increase in swimming speed with increasing Weissenberg number (Wi, defined in the same way as the Deborah number in our work), reaching plateau values for Wi>8: specifically, U/U_0=1.1 for a neutral swimmer, , U/U_0=2.4 for a pusher with β=-5, and U/U_0=0.97 for a puller with β=5. This behavior may result from the different fluid stresses generated in the extensional flow at the rear of the squirmer. In particular, increasing Wi enhances the convection of vorticity downstream, thereby accelerating both the puller and pusher.

In our case, at Re=0.8, only the neutral swimmer at $k_BT/\varepsilon = 0.0005$ exhibits a slight increase in swimming speed of up to 2%, while the speeds of the pusher and puller both decrease by up to 10%. This discrepancy arises from the rotational enhancement of the swimmers, which impedes their translational motion. In Ouyang's study at Re=0.01, all swimmer types show decreased speeds as Wi increases, with reductions reaching plateau values for Wi>14: approximately 5% for the neutral swimmer and puller, and 12.5% for the pusher. Our system shows similar behavior at Re=0.04 and $k_BT/\varepsilon = 0.0005$, but the reduction is less pronounced: the neutral swimmer and pusher both show decreases of about 2%, and the puller shows a decrease of about 5%. This difference may be attributed to the absence of elastic effects in our flexible polymer model and the additional rotational enhancement caused by the interplay between swimmer active stress and polymer deformation.

The main text has been revised accordingly in the last paragraph of section '3.5 Comparisons with other works' of the main manuscript and highlighted in red for clarity.

Comment 8:
I am curious if there is any handling or polymer tangling in the authors' simulations or if polymers are simply assumed to pass through one another if cross-overs occur. If the latter assumption is used, to the authors have a hypothesis for the effects entanglements might cause? I could imagine polymers getting "tied around" squirmers for relatively long time scales and potentially causing interesting effects.

We thank the referee for this interesting question. In our system, a soft-sphere repulsion is applied between monomers to account for excluded volume effects. Given the short length of polymer bonds, non-connected bonds cannot cross each other. Additionally, the normalized polymer concentrations are ϕ/ϕ^*=4.1 at Re=0.8 and ϕ/ϕ^*=1.2 at Re=0.04, at a low temperature of $K_BT/\varepsilon = 0.0005$. These values indicate that the system lies in the semi-dilute regime, where polymer entanglement is typically absent. Despite the lack of entanglement, we observed a wrapping effect occurring in front of a pusher, which contributes to the enhancement of its rotational motion. The timescale of this phenomenon is approximately 2500τ, where τ is the characteristic time used in our simulations. This timescale is comparable to the time required for the swimmer to travel one body length (about $1800\,\tau$). Therefore, the wrapping effect arises purely from transient encounters between the pusher and polymers during swimming. Polymers do not entangle or remain in contact with the squirmer.

Comment 9:
No error bars are shown on plots that give rotational diffusivity enhancement values, though there are rather significant statistical errors in swimming speed enhancements in many cases. The authors should include these errors or otherwise show that they are not significant.

We thank the referee for this question. The rotational diffusion coefficients are obtained by fitting the rotational correlation function with an exponential decay function. Each correlation function is averaged over 4 to 15 independent simulation runs, and the resulting error bars are shown in Fig. 2. However, since the rotational diffusion coefficient is extracted from a single exponential fit to the averaged correlation function, only one value is obtained. As a result, no error bars are included in the rotational diffusion enhancement graph.

Comment 10:
It seems awkward to have a "Model and simulation methods" subsection under "Results". I would consider moving this subsection into its own standalone section.

We thank the referee for the helpful suggestion. We have set the ‘Model and simulation methods’ as an independent section before the ‘Results’.

Comment 11:
In Sec. 2.1, the authors state that the swimming speed of a squirmer is "... determined by the first surface mode strength B_1", but this is only true either when Re = 0 or β = 0 in a Newtonian fluid. This fact should be stated for the unfamiliar reader.

We thank the referee for the question. The referee is correct. The swimming velocity is derived for a Newtonian fluid under the Re=0 condition and is independent of the active stress parameter β. A slight deviation from the theoretical estimate occurs in the low Reynolds number regime. This issue is now addressed in red in the ‘2 Model and Simulation Methods’ section of the main text.

Comment 12:
While I think it makes sense to keep the details of the Lattice Boltzmann model in the supplemental materials (SM), I suggest including squirmer model description in main text to make the definition of the active stress clear to the reader. The authors use often use the term "no-slip" to refer to a boundary condition at the squirmer surface, but it is really a velocity-fixed slip condition as given by equation (9) in the SM.

We thank the referee for the helpful suggestion. We have added additional descriptions of the squirmer model in the section titled ‘2 Model and Simulation Methods’ of the main text, and the new text has been highlighted in red for clarity. Regarding the boundary condition, we conducted a thorough review of the literature on hydrodynamic simulations of squirmers. To our knowledge, the term “velocity-fixed slip boundary condition” is rarely used. Instead, the term “no-slip boundary condition” is most used, as seen in Phys. Rev. Lett. **132**, 238301 (2024), Phys. Rev. Lett. **112**, 118101 (2014), Soft Matter **16**, 10676 (2020), and J. Fluid Mech. **998**, A2 (2025). Therefore, to maintain consistency with the literature, we have retained the term “no-slip boundary condition” in this work.

Comment 13:
Some notation seems not to be defined in the main text. For example, D_r^{p,ns} and $D_r^{p,ns}$ are used in the main text but the definitions are given in the captions of Figs. 2 and 3, which might not be immediately seen by the reader.

We thank the referee for the helpful suggestion. We have redefined $D_r^{a,ns}$ and $D_r^{p,ns}$ in the main text instead (page 6). Similar issues related to other graphs have also been addressed. All modifications have been highlighted in red for clarity.

Comment 14:
The authors might consider using the equivalent scales on comparable plots for easier visual comparison. For example, the vertical scales on Fig. 3a and 3b are slightly different. Similarly, plots in Figs. 3 and 6 have close enough vertical scales to consider making them the same so trends for the two different Reynolds numbers are most easily compared and contrasted.

We thank the referee for the helpful suggestion. Fig. 3 have been modified to share the same vertical scale. Correspondingly, Fig. 6 has also been updated. To limit the length of the main manuscript, we have moved the discussion of the effect of active stresses on the rotational motion to the Supplemental Materials, where it now appears as an independent section titled ‘2.1 Influence of Active Stress on the Squirmer’s Rotational Motion’. Keeping a balance between visual clarity and aesthetic presentation, we have carefully reviewed all the figures in both the manuscript and the Supplementary Materials and made the necessary adjustments.

Comment 15:

I found a number of English grammatical issues while reading the manuscript (e.g., several incomplete sentences). While they did not severely detract from my understanding of the text, they were prevalent enough to become distracting. The authors should consider editing thoroughly for these issues.

We thank the referee for the helpful suggestion. We have carefully reviewed the manuscript and the Supplementary Materials and improved the language throughout.

Reviewer #1:

I thank the authors for the clarifications regarding the definition of the model and providing a revised SI. My points have been addressed and I can now recommend publication.

We sincerely thank the referee for reviewing the revised version of our manuscript. We are delighted that the clarifications regarding the model definition and the updated Supporting Information have addressed your concerns, and we are grateful for your recommendation for publication.

Reviewer #2:

The authors have addressed most of the concerns I raised, and I have gained new insights from the revised manuscript. I recommend this paper for publication.

We sincerely thank the referee for their positive assessment and for recognizing the improvements made in the revised manuscript. We are pleased that the revisions have provided new insights and that the referee now recommends the paper for publication. We greatly appreciate the constructive feedback, which has helped us enhance the clarity and quality of our work.

Upon further consideration, I would like to offer one final, but I believe important, suggestion. If the authors could provide some fitting expressions or functional forms for the results presented in figures (e.g., Figs. 8, 9, and 10), it would significantly enhance the clarity and utility of their findings.

We thank the referee for this insightful comment. In response, we have fitted the curves presented in Figures 2, 4, 5, 7, 8 and Supplementary Figures 3, 4, 6, 7. These fittings substantially enhance our understanding of the intrinsic physical mechanisms underlying the system. The updated figures and corresponding analyses have been incorporated into both the main manuscript and the Supplementary Materials. All revised texts are highlighted in red for clarity. Please note that the figure numbering has been updated due to format changes requested by the editorial board.

Reviewer #3:

I appreciate the authors' diligence in addressing my many questions and comments. The revised manuscript's significance is now readily apparent and its clarity is significantly improved. I consider it to be a high quality work that I am happy to recommend for publication in the present form.

We sincerely thank the referee for the positive evaluation and thoughtful feedback throughout the review process. We are very pleased that the referee finds the revised manuscript significantly improved in clarity and scientific significance. We greatly appreciate the referee’s recommendation

for publication and their constructive comments, which have helped us strengthen the quality and presentation of our work.